* Research supported in part by an NSF Grant No. GP-19568

1 Institut de Statistique, Université de Genève

ANALYSIS OF VARIANCE OF A NON-ORTHOGONAL THREE-FACTOR EXPERIMENT USING A COMPUTER*

P. Vuagnat[1]

Department of Statistics
University of North Carolina at Chapel Hill

Institute of Statistics Mimeo Series No. 803

January, 1972

ANALYSIS OF VARIANCE
OF A NON-ORTHOGONAL THREE-FACTOR EXPERIMENT
USING A COMPUTER*

P. Vuagnat
University of North Carolina at Chapel Hill
and Université de Genève, Suisse

INTRODUCTION

An article of W. L. Stevens [7] originated this work. Stevens has shown by a suitable worked example that the solution of fitting constants to a set of non-orthogonal data can be reduced to an iterative procedure. But this procedure, as he admitted is always lengthy.

We have considered the problem from a much more general point of view; this was possible because we have now at our disposal high-speed and high-capacity computers.

1. Linear models and three-way cross classification with interactions and unequal number of replications.

The three-way lay-out is a particular case of a linear model.

If we use the vectorial notation, the general form of such a model is:

$$\underline{y} = X\underline{\beta} + \underline{e}$$

where $\underline{y}$ is a random observed vector, $\underline{e}$ is a random vector, X is a

* This research was supported in part by a National Science Foundation Grant No. GP-19568.

$(n \times p)$ matrix of known fixed quantities and $\underline{\beta}$ is a $(p \times 1)$ vector of unknown parameters.

In the case of a three-way lay-out with main effects and two-factor interaction, the model is often written in the more explicit form:

$$(2) \quad y_{ijk\ell} = \mu + \alpha_i + \beta_j + \gamma_k + \delta_{ij} + \varepsilon_{ik} + \eta_{jk} + e_{ijk\ell}$$

where

$$\begin{aligned} i &= 1, \ldots, r \\ j &= 1, \ldots, s \\ k &= 1, \ldots, t \\ \ell &= 1, \ldots, n_{ijk}. \end{aligned}$$

n_{ijk} being the number of observations in the ijk^{th} cell.

In this model the number of unknown parameters is equal to:

$$\begin{aligned} (3) \quad p &= 1 + r + s + t + rs + rt + st \\ &= (1+r)(1+s)(1+t) - rst. \end{aligned}$$

But all these parameters are not independent; we have the following restrictions:

$$(4) \quad \begin{aligned} &\sum_{i=1}^{r} \alpha_i = \sum_{j=1}^{s} \beta_j = \sum_{k=1}^{t} \gamma_k = 0 \\ &\sum_{i=1}^{r} \delta_{ij} = \sum_{j=1}^{s} \delta_{ij} = 0 \\ &\sum_{i=1}^{r} \varepsilon_{ik} = \sum_{k=1}^{t} \varepsilon_{ik} = 0 \\ &\sum_{j=1}^{s} \eta_{ij} = \sum_{k=1}^{t} \eta_{jk} = 0. \end{aligned}$$

If we return to the vectorial notation these restrictions can be expressed in the following way:

(5) $L'\underline{\beta} = \underline{0}$

where L' is a $(m \times p)$ matrix.

It can be noticed that the matrices X and L' have a particular pattern: they contain only 0's and 1's as elements.

For calculating the least-squares estimates of $\underline{\beta}$, say $\underline{b}$, we must find the vector $\underline{b}$ that minimizes the quantity:

(6) $S(\underline{b}) = (\underline{y} - X\underline{b})' (\underline{y} - X\underline{b})$

and at the same time satisfies the equations:

(7) $L'\underline{b} = \underline{0}$.

This may be done by using the Lagrange multipliers. We form the function:

(8) $S^*(\underline{b}) = (\underline{y}-X\underline{b})'(\underline{y}-X\underline{b}) + \underline{\lambda}'L'\underline{b}$

where $\underline{\lambda}$ is a $(m \times 1)$ vector of unknown quantities.

The set of equations:

$$\frac{\partial S^*}{\partial \underline{b}} = \underline{0}$$

give us a set of p equations:

(9) $X'X\underline{b} + L\underline{\lambda} = X'\underline{y}$.

With the set of m equations (7), we have a system of $(m + p)$ equations with $(m + p)$ unknowns.

The relations (7) and (9) can be brought together in the following matricial form:

(10) $\begin{pmatrix} X'X & L' \\ L & 0 \end{pmatrix} \begin{pmatrix} \underline{b} \\ \underline{\lambda} \end{pmatrix} = \begin{pmatrix} X'\underline{y} \\ 0 \end{pmatrix}$.

While the matrix X'X is singular, the matrix:

$$\begin{pmatrix} X'X & L' \\ L & 0 \end{pmatrix}$$

is as a rule regular; consequently, the solution of the system (10) can easily be found theoretically; in fact we can write:

$$(11) \quad \begin{pmatrix} \underline{b} \\ \lambda \end{pmatrix} = \begin{pmatrix} X'X & L' \\ L & 0 \end{pmatrix}^{-1} \begin{pmatrix} X'\underline{y} \\ 0 \end{pmatrix}.$$

Practically, the problem is much more difficult because of the dimensions of the matrix:

$$(12) \quad \begin{pmatrix} X'X & L' \\ L & 0 \end{pmatrix} = G.$$

For they become quickly very large when the number of levels of any of the three factor increases. For instance, in the example given by Stevens the numbers of levels are:

$$r = 2, \quad s = 4, \quad t = 4.$$

Then the dimensions of the matrix X'X are given by the relation (3):

$$p = 3 \cdot 5 \cdot 5 - 2 \cdot 4 \cdot 4 = 75 - 32 = 43$$

and the dimension m of the matrix L' can be computed from this formula:

$$m = 2(r+s+t).$$

In our case, we obtain $m = 20$.

Consequently, the dimensions of the matrix G are (63×63).

For this reason, we thought it would be interesting to find a method to compute the inverse matrix G^{-1} from the matrices X'X, L' and L. In the next section we give such a method allowing us to calculate the inverse matrix G^{-1} from the generalized inverses of some singular matrices but with smaller dimensions than G.

2. Generalized inverses and solution of normal equations.

First we will give some definitions because all the authors do not use the same terminology.

Generalized inverse. Let A be a $(m \times n)$ matrix. A matrix A^- is defined to be a generalized inverse of A, if and only if it satisfies:

$$AA^-A = A.$$

Note: Some authors call such a matrix a conditional matrix or c-inverse.

Moore-Penrose inverse. Let A be a $(m \times n)$ matrix. If a matrix A^+ exists that satisfies the four conditions below, we shall call A^+ a Moore-Penrose inverse of A.

1. AA^+ is symmetric
2. A^+A is symmetric
3. $AA^+A = A$
4. $A^+AA^+ = A^+$.

Note: Some authors call such a matrix a generalized inverse or g-inverse.

It is clear that every Moore-Penrose inverse is a generalized inverse; but a generalized inverse may not be a Moore-Penrose inverse.

It can be stated that the Moore-Penrose inverse of a matrix A is unique while there may exist more than one generalized inverse.

The results we shall use are valid for generalized inverses and, of course, for Moore-Penrose inverses. In our computations, we shall calculate always the Moore-Penrose inverse which is equal to the inverse when the matrix is regular.

Chakravarti [1] gives the following results.

Let A be a matrix having the following particular structure:

$$A = \begin{pmatrix} X'X & L \\ L' & 0 \end{pmatrix}$$

where the column-vectors of L do not belong to the vector space generated by the column-vectors of X'X and A is regular. Then the generalized inverse A^- of A is given by:

$$A^- = \left(\begin{array}{c|c} C^- - C^-LQ^-L'C^- & C^-L - C^-LQ^-Q + C^-LQ \\ \hline Q^-L'C^- & Q^-Q - Q^- \end{array}\right)$$

where:

$$Q = L'C^-L$$

$$C = X'X + LL'.$$

For our problem, we only need the first term of this partitioned matrix, that is to say:

$$A_{11} = C^- - C^-LQ^-L'C^-.$$

Indeed, the values of the vector solution $\underline{b}$ in (11) will be given by:

$$\underline{b} = A_{11}X'y.$$

The calculation of A_{11} only needs matrices with dimensions $(p \times p)$, $(m \times p)$, $(p \times m)$ and $(m \times m)$. To compute the matrices C^- and Q^- we used a method given by Greville [4] permitting to calculate the Moore-Penrose inverse; then we have calculated the matrix:

$$A_{11} = C^+ - C^+LQ^+L'C^+.$$

3. Analysis of variance and tests for interactions and main effects.

The model we chose is given by the expression (2):

$$y_{ijk\ell} = \mu + \alpha_i + \beta_j + \gamma_k + \delta_{ij} + \varepsilon_{ik} + \eta_{jk} + e_{ijk\ell}$$

where the $e_{ijk\ell}$ are independent and normally distributed $\tilde{N}(0,1)$.

First, let us consider the following test:

Test for three-factor interaction

In our model we have not included parameters for the three-fector interaction because it is not necessary to know explicitly the estimations of these parameters to test the hypothesis of no three-factor interaction.

Indeed, if we use the vectorial notation it is known that the sums of squares due to fitting $\underline{\beta}$ is:

$$\underline{b}'X'\underline{y}$$

the total sum of squares is:

$$\underline{y}'\underline{y}$$

and consequently the sum of squares of error is:

$$\underline{y}'\underline{y} - \underline{b}'X'\underline{y}.$$

On the other hand, as we supposed that there is at least one cell with more than one observation, we have for the sum of squares within the cells the expression:

$$\sum_{i,j,k,\ell} (y_{ijk\ell} - \bar{y}_{ijk.})^2 = \sum$$

where:

$$\bar{y}_{ijk.} = \frac{1}{n_{ijk}} \sum_{\ell=1}^{n_{ijk}} y_{ijk\ell} \quad \text{for} \quad n_{ijk} \neq 0.$$

Then we have the following analysis of variance to test the three-factor interaction:

Due to	S.S.	d.f.	M.S.	F
$\underline{b}$	$\underline{b}'X'\underline{y}$	$p - m$		
3-fact. interaction	$\underline{y}'\underline{y} - \underline{b}'X'\underline{y} - \sum$	$n - p + m - u$	$I_{MS}^{(3)}$	$\frac{I_{MS}^{(3)}}{E_{MS}}$
Error	$\sum$	u	E_{MS}	
Total	$y'y$	n		

where:

- n is the total number of observations
- p is the number of parameters in (2)
- m is the number of restrictions
- u is the number of degrees of freedom of the sum of squares for error and is given by the formula:

$$u = \sum_{\substack{i,j,k \\ n_{ijk} \neq 0}} (n_{ijk} - 1).$$

Now let us consider:

Test for two-factor interaction

To test hypothesis of no two-factor interaction we use the method generally employed in the regression analysis to test the nullity of a set of parameters.

Let ω be the hypothesis that any two-factor interaction is null and denote the corresponding model by:

$$\underline{y} = X^*\underline{\beta}^* + \underline{e}^*$$

and by $\underline{b}^*$ the least-squares estimates of the vector $\underline{\beta}^*$; then, we have the following analysis of variance to test the hypothesis ω:

Due to	S.S.	d.f.	M.S.	F
$\underline{b}$	$\underline{b}'X'\underline{y}$	$p - m$		
b*	$\underline{b}^{*\prime}X^{*\prime}\underline{y}$	$p - m' - m$		
2-fact. inter.	$\underline{b}'X'\underline{y} - \underline{b}^{*\prime}X^{*\prime}\underline{y}$	m'	$I^{(2)}_{MS}$	$\frac{I^{(2)}_{MS}}{E_{MS}}$
3-fact. inter.	$\underline{y}'\underline{y} - \underline{b}'X'\underline{y} - \sum$	$n - p - m - u$		
Error	$\sum$	u	E_{MS}	
Total	$y'y$	n		

where m' is the number of degrees of freedom of the considered interaction and can be calculated from this formula:

$$m' = (r'-1)(r''-1)$$

r' and r'' being the numbers of levels of the two considered factors.

At last, let us consider:

Test for main effects

We used exactly the same method for the test for main effect as for the two-factor interaction, but we suppose that the two-factor interactions and the three-factor interaction are null.

Let ω° be the null hypothesis of any of the main effects: if we denote by:

$$\underline{y} = X^\circ\underline{\beta}^\circ + \underline{e}^\circ$$

the model corresponding to the hypothesis ω°, we shall have the following analysis of variance:

10

Due to	S.S.	d.f.	M.S.	F
$\underline{b}$	$\underline{b}'X'\underline{y}$	$p' - m''$		
b°	$\underline{b}^\circ X^{\circ\prime}\underline{y}$	$p' - m'' - m'''$		
Main effect	$\underline{b}'X'\underline{y} - \underline{b}^{\circ\prime}X^{\circ\prime}\underline{y}$	m'''	$(ME)_{MS}$	$\frac{(ME)_{MS}}{E_{MS}}$
Residual	$\underline{y}'\underline{y} - \underline{b}'X'\underline{y}$	$n - p' - m''$	E_{MS}	
Total	$\underline{y}'\underline{y}$	n		

where p' is the number of parameters in the general model with no interaction and is given by the formula:

$$p' = r + s + t.$$

m'' is the number of restrictions in this model and is equal to 3 and m''' is the number of degrees of freedom of the considered factor, that is to say $(r''' - 1)$ if r''' is the number of levels of this factor.

We must also notice that $\underline{b}$ denotes the estimation of $\underline{\beta}$ in the model where all the interactions were supposed null and not the model (2) of the paragraph 1. We have kept the same notation as for the general model (2) in order to simplify

4. Description of the program

First we will give some considerations about the conception of this program.

To obtain the various tables of analysis of variance it is necessary to get the various matrices we called X, X^* and X° and also the corresponding restrictions matrices L', $L^{*\prime}$ and $L^{\circ\prime}$.

It can be noted that the matrices X^* and X° can be obtained from the matrix X by suppressing some columns of that matrix.

The matrices $L^{*'}$ and $L^{\circ'}$ can also be obtained from the matrix L', but it is a little more difficult because we must suppress not only some columns but also some rows.

To avoid the long and tedious work of punching the elements of the matrices X, X^*, X°, L', $L^{*'}$ and $L^{\circ'}$ we have written a program permitting

(i) to obtain the matrices X and L' knowing only the numbers r, s and t of levels of the three-factors and the numbers n_{ijk} of observations in each cell.

(ii) to obtain the matrices X^*, X°, $L^{*'}$ and $L^{\circ'}$ from the matrices X and L' respectively for the various tests mentioned above.

As we saw in the paragraph 1, to solve the system of equations given by (10), we must find the inverse of the matrix G given by (12). If the capacity of the computer is sufficient we can use a classical method, but if the dimensions of G are too large we can use the method given in the paragraph 2 employing the generalized inverses.

The main programs corresponding to these two methods are almost the same only the subprograms are different. Although it would have been possible to write a main program valid for the two methods, we have thought that it would be more practical to have two distinct programs.

We tested these two programs with the example given by Stevens. The agreement between the results was very good. The results of the tables of analysis of variance were exactly the same and the values of the components of the vector b disagreed sometimes in the fifth, but mostly in the sixth decimal.

All the quantitites are computed with double precision because the simple precision was not sufficient.

The programs were written in FORTRAN IV G language and the computer used is an IBM System/360 Model 75 computer.

Card preparation

Dimension cards: The dimensions were forseen for the computation of the Steven's example, that is to say for the levels:

$$r = 2 \quad s = 4 \quad t = 4$$

and for a number of observations n equal to 36.

Consequently if the problem has values greater than our problem, the dimension cards must be changed as far as the capacity of the used computer allows it.

Input cards: The *first* card contains in the format (16I5) the following quantities:

IM = r level of the first factor

JM = s level of the second factor

KM = t level of the third factor

N = n total number of observations

ID1M = number greater or equal to the dimensions of G; it must be equal to the dimensions of the matrix A of the program.

The *second* card on to the $\underline{(th-1)}^{th}$ card where h is so defined:

$$h = [rs/16] + 1$$

the symbol [ ] denoting the integer part, contain in the format (16I5) the numbers of observations per cell in the following way:

2^{th} to $(h-1)^{th}$: $n_{111}, n_{211}, \ldots, n_{r11}, n_{121}, \ldots, n_{r21}, \ldots, n_{1s1}, \ldots, n_{rs1}$

h^{th} to $(2h-1)^{th}$: $n_{112}, \ldots, n_{r12}, n_{122}, \ldots, n_{r22}, \ldots, n_{1s2}, \ldots, n_{rs2}$

$\vdots$

$(th-h)^{th}$ to $(th-1)^{th}$: $n_{11t}, \ldots, n_{r1t}, n_{12t}, \ldots, n_{r2t}, n_{1st}, \ldots, n_{rst}$.

The $\underline{th}^{th}$ card and the following cards contain the values of the observations in the format (20F4.0) in the way given below:

$$y_{1111}, \ldots, y_{111n_{111}}, y_{2111}, \ldots, y_{211n_{211}}, \ldots, y_{r111}, \ldots$$
$$\ldots, y_{r11n_{r11}}, y_{1211}, \ldots, y_{121n_{121}}, \ldots, y_{r211}, \ldots.$$
$$\ldots, y_{r21n_{r21}}, \ldots, y_{1s11}, \ldots, y_{1s1n_{1s1}}, \ldots, y_{rs11},$$
$$\ldots, y_{rs1n_{rs1}}, \ldots, y_{rst1}, \ldots, y_{rstn_{rst}}.$$

The number of cards in this last set is equal to:

$$[n/20] + 1 = f.$$

The $(th+f)^{th}$ card and the following cards allow us to obtain the six analyses of variance mentioned in the paragraph 3.

They are punched according to the following pattern in the format (16I5):

$$N_1 \quad N_2 \quad N_3 \quad N_4 \quad N_5 \quad N_6 \quad N_7$$

where the N_i $(i = 1,\ldots,7)$ are so defined:

$N_1 = 1$ always

$N_2 = r$ or 0

$N_3 = s$ or 0

$N_4 = t$ or 0

$N_5 = rs$ or 0

$N_6 = rt$ or 0

$N_7 = st$ or 0

that is to say:

$N_i = 0$ if the parameters in the considered model, resulting from the considered null hypothesis, are 0.

More explicitly, the $(th+f)^{th}$ card corresponding to the complete model given by (2) in the paragraph 1 is:

$$1 \quad r \quad s \quad t \quad rs \quad rt \quad st$$

the three following cards corresponding to the three analyses of variance for the two-factor interactions are:

1	r	s	t	0	rt	st
1	r	s	t	rs	0	st
1	r	s	t	rs	rt	0

the $(th+f+4)^{th}$ card corresponding to the general model with no interactions is:

1 r s t 0 0 0

and the three following cards corresponding to the three analysis of variance for the main factors are:

1	0	s	t	0	0	0
1	r	0	t	0	0	0
1	r	s	0	0	0	0.

Note: By a slight modification or adjunction to the program (because the output print ought to be modified or enlarged), we can easily obtain the values of the parameters for any model supposing the nullity of one or several main effects or interactions.

The last card must always be a blank card.

The output print

The program gives:

i) the values of the parameters for the complete model given by (2).

ii) The three tables of analysis of variance for the test of the three two-factor interactions followed by the values of the parameters in the corresponding null hypothesis.

In each of these tables appears the test of the three-factor interaction.

iii) The three tables of analysis of variance for the test of the three main effects followed by the values of the parameters in the corresponding null hypothesis.

As an example we give the results obtained from the data of the problem given by Stevens in [7].

Time estimates

As we already said, we have used two methods. The method using the generalized inverses is much longer than the method using the classical inversion. Then, if the capacity of the computer allows it, the second method is less expensive. With the Steven's example we got the following times:

First method: 22'30"
Second method: 3'30".

REFERENCES

[1] Chakravarti, I. M. On generalized inverses in a linear associative algebra and their applications in the analysis of a class of designs, 1971, *Institute of Statistics Mimeo Series 766*, University of North Carolina, Chapel Hill.

[2] Graybill, F.A. *An Introduction to Linear Statistical Models*, 1961, McGraw-Hill, New York.

[3] Graybill, F.A. *Introduction to Matrices with Applications in Statistics*, 1969, Wadsworth, Belmont, California.

[4] Greville, T. N. E. Some applications of the pseudoinverse of a matrix, 1960, *SIAM Review*, pp. 15-22.

[5] Kempthorne, O. *The Design and Analysis of Experiments*, 1952, John Wiley, New York.

[6] Scheffé, H. *The analysis of Variance*, 1959, John Wiley, New York.

[7] Stevens, W. L. Statistical analysis of a non-orthogonal tri-factorial experiment, 1948, *Biometrika, 35*, pp. 346-367.

ACKNOWLEDGMENTS

I am grateful to Professor I. M. Chakravarti for his help and valuable suggestions in the preparation of this work and for a critical reading of this manuscript.

APPENDIX

1. Listing of the program employing generalized inverses and data for the Steven's example.

2. Listing of the program employing the classical inverse and data for the Steven's example.

3. Results obtained with the data of the Steven's example.

DATA SET LISTING

```
C     ANALYSIS OF VARIANCE OF A NON-ORTHOGONAL THREE-FACTOR EXPERIMENT G
      DOUBLE PRECISION F(43),H(43),Y(43),P(43)
      DOUBLE PRECISION A(43,43),B(43,43),BT(43,43),EL(43,43),ELT(43,43)
      DOUBLE PRECISION YT(4,4,4)
      DOUBLE PRECISION CCA,SCD,SCDC,CMDC,CMD,SCF,CMF
      DOUBLE PRECISION SY,SCY,SYT,TCOR,SCC,SCCEL,SCI,CMI,SCA,SC
      DIMENSION NOBS(4,4,4),NX(7),NXX(7)
      READ 1,IM,JM,KM,N,IDIM
    1 FORMAT(16I5)
      DO 21 K=1,KM
   21 READ 1,((NOBS(I,J,K),I=1,IM),J=1,JM)
      READ 55,(Y(I),I=1,N)
   55 FORMAT(20F4.0)
      SY=0.
      SCY=0.
      SCYT=0.
      KOMPT=0
      SCA=0
      IJK=0
      NCEL=0
      DO 70 K=1,KM
      DO 70 J=1,JM
      DO 70,I=1,IM
      IF(NOBS(I,J,K).EQ.0)GO TO 70
      NM=NOBS(I,J,K)
      NCEL=NCEL+1
      YT(I,J,K)=0.
      DO 71 L=1,NM
      IJK=IJK+1
      YT(I,J,K)=YT(I,J,K)+Y(IJK)
      SCY=SCY+Y(IJK)**2
   71 SY=SY+Y(IJK)
      SCYT=SCYT+YT(I,J,K)**2/FLOAT(NM)
   70 CONTINUE
      TCOR=SY**2/FLOAT(IJK)
      SCC=SCY-TCOR
      NDLT=IJK-1
      SCCEL=SCYT-TCOR
      NDLCEL=NCEL-1
      SCI=SCC-SCCEL
      NDLI=NDLT-NDLCEL
      CMI=SCI/FLOAT(NDLI)
   99 READ 1,(NX(I),I=1,7)
      IF(NX(1).EQ.0)GO TO 60
      NC=0
      NCC=0
      MI=1
      KOL=1
      NXT=0
      NXX(1)=0
```

DATA SET LISTING

```
      DO 41 I=2,7
      NXX(I) = 0
      KOL = KOL + NX(I)
      IF(NX(I).EQ.0) GO TO 411
      IF(I.GT.4) GO TO 42
      NCC = NCC+1
      NEP = NCC
      GO TO 41
   42 IF (I-6)43,44,45
   43 NCC = NCC+IM+JM-1
      INT IJ = NCC
      GO TO 41
   44 NCC=NCC+IM+KM-1
      INTIK=NCC
      GO TO 41
   45 NCC=NCC+JM+KM-1
      INTJK=NCC
      GO TO 41
  411 NXX(I)=1
   41 CONTINUE
      IMIN=KOL+1
      IMAX=KOL+NCC
      DO 22 IND=1,7
      NXT=NXT+NXX(IND)
      NC=NC+NX(IND)
      IF(NX(IND).EQ.0) GO TO 22
      IJK=0
      MM=NC
      DO 23 K=1,KM
      DO 23 J=1,JM
      DO 23 I=1,IM
      IF(NOBS(I,J,K).EQ.0) GO TO 23
      NM=NOBS(I,J,K)
      DO 24 L=1,NM
      IJK=IJK+1
      DO 25 M=MI,MM
      MA=M-MI+1
      GO TO (26,27,28,29,30,31,32),IND
   26 BT(IJK,M)=1.
      GO TO 25
   27 IF(MA-I)33,26,33
   33 BT(IJK,M)=0.
      GO TO 25
   28 IF(MA-J)33,26,33
   29 IF(MA-K)33,26,33
   30 IF(MA-((I-1)*JM+J))33,26,33
   31 IF(MA-((I-1)*KM+K))33,26,33
   32 IF(MA-((J-1)*KM+K))33,26,33
   25 CONTINUE
   24 CONTINUE
```

DATA SET LISTING

```
 23   CONTINUE
      DO 46 I=IMIN,IMAX
      K=I-KOL
      IMM=I-IMIN+1
      DO 47 M=MI,MM
      MA=M-MI
      GO TO (34,35,35,35,36,37,38),IND
 34   ELT(K,M)=0.
      GO TO 47
 35   IF(IMM-IND+1+NXT)34,48,34
 48   ELT(K,M)=1.
      GO TO 47
 36   IK=INTIJ-IM-JM+1
      IL=INTIJ
      IS=JM
      IP=INTIJ-JM
      GO TO 39
 37   IK=INTIK-IM-KM+1
      IL=INTIK
      IS=KM
      IP=INTIK-KM
      GO TO 39
 38   IK=INTJK-JM-KM+1
      IL=INTJK
      IS=KM
      IP=INTJK-KM
 39   IN=IMM-IK
      IQ=IMM-IP
      IT=MA/IS+1
      IU=MA+1-IS*(MA/IS)
      IF(IMM.LE.IK.OR.IMM.GT.IL) GO TO 34
      IF(IMM.LE.IP) GO TO 40
      IF(IQ.EQ.IU) GO TO 48
      GO TO 34
 40   IF(IN-IT)34,48,34
 47   CONTINUE
 46   CONTINUE
      MI=MM+1
 22   CONTINUE
      DO 50 I=1,N
      DO 50 J=1,KOL
 50   B(J,I)=BT(I,J)
      CALL MATVMU(B,Y,P,KOL,N,IDIM)
      CALL MATMUL(B,BT,A,KOL,N,KOL,IDIM)
      M=NCC
      DO 7 I=1,M
      DO 7 J=1,KOL
  7   EL(J,I)=ELT(I,J)
      CALL MATMUL(EL,ELT,BT,KOL,M,KOL,IDIM)
      DO 8 I=1,KOL
```

DATA SET LISTING

```
      DO 8 J=1,KOL
  8   A(I,J)=A(I,J)+BT(I,J)
      CALL GINVG(A,B,KOL,KOL,IDIM)
      CALL MATMUL(B,EL,A,KOL,KOL,M,IDIM)
      CALL MATMUL(ELT,A,BT,M,KOL,M,IDIM)
      CALL GINVG(BT,A,M,M,IDIM)
      CALL MATMUL(EL,A,BT,KOL,M,M,IDIM)
      CALL MATMUL(B,BT,A,KOL,KOL,M,IDIM)
      CALL MATMUL(A,ELT,BT,KOL,M,KOL,IDIM)
      CALL MATMUL(BT,B,A,KOL,KOL,KOL,IDIM)
      DO 9 I=1,KOL
      DO 9 J=1,KOL
  9   B(I,J)=B(I,J)-A(I,J)
      CALL MATVMU(B,P,H,KOL,KOL,IDIM)
      SC=0.
      DO 2 I=1,KOL
  2   SC=SC+P(I)*H(I)
      SC=SC-TCOR
      IF(KOMPT.GT.0)GO TO 72
      SCA=SC
      NDLA=KOL-1-NCC
      CCA=SCA/FLOAT(NDLA)
      SCD=SCC-SCA
      NDLD=NDLT-NDLA
      SCDC=SCD-SCI
      NDLDC=NDLD-NDLI
      CMDC=SCDC/FLOAT(NDLDC)
      FI=CMDC/CMI
      CMD=SCD/FLOAT(NDLD)
      PRINT 93
 93   FORMAT(1X,'VALUES OF THE PARAMETERS FOR THE COMPLETE MODEL'//)
      PRINT 6, (H(I),I=1,IMAX)
      PRINT 67
      KOMPT=KOMPT+1
      GO TO 99
 72   SCF=SCA-SC
      GO TO (85,86,87,173,73,74,75),KOMPT
173   SCA=SC
      KOMPT=KOMPT+1
      NDLA=IM+JM+KM-3
      CCA=SCA/FLOAT(NDLA)
      SCD=SCC-SCA
      NDLD=NDLT-NDLA
      CMD=SCD/NDLD
      GO TO 99
 73   NDLF=IM-1
      GO TO 76
 74   NDLF=JM-1
      GO TO 76
 75   NDLF=KM-1
```

DATA SET LISTING

```
76   SCF=SCA-SC
     CMF=SCF/FLOAT(NDLF)
     NDLAF=NDLA-NDLF
     FF=CMF/CMD
     KK=KOMPT-4
     PRINT 77,KK
77   FORMAT(20X,'ANALYSIS OF VARIANCE FOR THE FACTOR',I5/)
     PRINT 90
     PRINT 78,SCF,NDLF,CMF,FF
78   FORMAT(1X,'CONSIDERED FACTOR',8X,F20.6,2X,I4,F20.6,F12.3)
     PRINT 79,SC,NDLAF
     PRINT 80,SCA,NDLA
     GO TO 92
85   NDLINT=(IM-1)*(JM-1)
     INT=12
     GO TO 88
86   NDLINT=(IM-1)*(KM-1)
     INT=13
     GO TO 88
87   NDLINT=(JM-1)*(KM-1)
     INT=23
88   CMF=SCF/FLOAT(NDLINT)
     NDLAF=NDLA-NDLINT
     FF=CMF/CMI
     PRINT 89, INT
89   FORMAT(20X,'ANALYSIS OF VARIANCE FOR INTERACTION', I4/)
     PRINT 90
90   FORMAT(39X,'SS',10X,'DF',10X,'MS',15X,'F'/)
     PRINT 91, INT,SCF,NDLINT,CMF,FF
91   FORMAT(1X,'INTERACTION',I3,11X,F20.6,2X,14,F20.6,F12.3)
     PRINT 79,SC,NDLAF
79   FORMAT(1X,'OTHER FACTORS',12X,F20.6,2X, .4/)
     PRINT 80,SCA,NDLA
80   FORMAT(1X,'PARTIAL TOTAL (A)',8X,F20.6,2X, I4/)
     PRINT 82,SCDC,NDLDC,CMDC,FI
82   FORMAT(1X,'INTERACTION 123 (D)-(C)',2X,F20.6,2X,I4,F20.6,F12.3)
     PRINT 81,SCI,NDLI,CMI
81   FORMAT(1X,'WITHIN (C)',15X,F20.6,2X,I4,F20.6/)
92   PRINT 83,SCD,NDLD,CMD
83   FORMAT(1X,'REST (D)=(B)-(A)',9X,F20.6,2X, I4,F20.6//)
     PRINT 84,SCC,NDLT
84   FORMAT(1X,'TOTAL (B)',16X,F20.6,2X, I4////)
     PRINT 94
94   FORMAT(1X,'VALUES OF THE PARAMETERS"//)
     PRINT 6,(H(I),I=1,IMAX)
 6   FORMAT(1X,8F16.6)
     KOMPT=KOMPT+1
     PRINT 67
67   FORMAT(1H1)
     GO TO 99
```

DATA SET LISTING

```
 60   CALL EXIT
      END

      SUBROUTINE GINVG(A,B,IMAX,JMAX,IDIM)
      DOUBLE PRECISION A(IDIM,IDIM),B(IDIM,IDIM)
      DOUBLE PRECISION BT(43,43),C(43),D(43),G(43),H(43)
      DOUBLE PRECISION E,F,DEN,BB
      E=0.
      DO 4 I=1,IMAX
  4   E=E+A(I,1)**2
      DO 5 J=1,IMAX
  5   B(1,J)=A(J,1)/E
      JC=2
      IL=1
 26   DO 6 I=1,IMAX
  6   H(I)=A(I,JC)
      CALL MATVMU(B,H,D,IL,IMAX,IDIM)
      CALL MATVMU(A,D,C,IMAX,IL,IDIM)
      DO 9 I=1,IMAX
      IF(H(I)-C(I))8,9,8
  9   CONTINUE
  1   FORMAT(1X,10I5)
      DO 10 I=1,IL
      DO 10 J=1,JMAX
 10   BT(J,I)=B(I,J)
      CALL MATVMU(BT,D,C,IMAX,IL,IDIM)
      CALL MATVMU(B,C,G,IL,IMAX,IDIM)
      CALL MATVMU(BT,G,H,IMAX,IL,IDIM)
      F=0.
      DEN=0.
      DO 11 I=1,IMAX
      F=F+A(I,JC)*C(I)
 11   DEN=DEN+A(I,JC)*H(I)
      DO 12 I=1,IMAX
 12   G(I)=(1.+F)*C(I)/DEN
      GO TO 13
  8   CALL MATMUL(A,B,BT,IMAX,IL,IMAX,IDIM)
      DO 14 I=1,IMAX
      DO 14 J=1,IMAX
      IF(I.EQ.J)GO TO 15
      BT(I,J)=-BT(I,J)
      GO TO 14
 15   BT(I,J)=1.-BT(I,J)
 14   CONTINUE
      CALL MATVMU(BT,H,G,IMAX,IMAX,IDIM)
 13   BB=0.
      DO 17 I=1,IMAX
 17   BB=BB+G(I)**2
      DO 18 I=1,IMAX
```

DATA SET LISTING

```
      G(I)=G(I)/BB
 18   CONTINUE
      DO 19 J=1,IMAX
      DO 20 I=1,IL
 20   B(I,J)=B(I,J)-D(I)*G(J)
 19   B(IL+1,J)=G(J)
      IF(JC.EQ.JMAX)GO TO 21
      JC=JC+1
      IL=IL+1
      GO TO 26
 21   RETURN
      END

      SUBROUTINE MATVMU(A,B,C,IMAX,JMAX,IDIM)
      DOUBLE PRECISION A(IDIM,IDIM),B(IDIM),C(IDIM)
      DO 1 I=1,IMAX
      C(I)=0.
      CO 1 J=1,JMAX
  1   C(I)=C(I)+A(I,J)*B(J)
      RETURN
      END

      SUBROUTINE MATMUL(A,B,C,IM,JM,KM,ID)
      DOUBLE PRECISION A(ID,ID),B(ID,ID),C(ID,ID)
      DO 1 I=1,IM
      DO 1 K=1,KM
      C(I,K)=0.
      DO 1 J=1,JM
  1   C(I,K)=C(I,K)+A(I,J)*B(J,K)
      RETURN
      END

        2   4   4  36  43
        2   1   2   0   1   1   1   1
        2   1   2   1   1   1   1   0
        1   1   1   2   2   0   1   1
        2   0   1   2   2   0   1   1
      43  58  58  73  59  81  62  67  71  93  83  60  75  89  71 101  76 100  91  70
      85  70  58  92  88 106  73  89  89  98  69  72 105 108 109  76
        1   2   4   4   8   8  16
        1   2   4   4   0   8  16
        1   2   4   4   8   0  16
        1   2   4   4   8   8   0
        1   2   4   4   0   0   0
        1   0   4   4   0   0   0

        1   2   0   4   0   0   0
        1   2   4   0   0   0   0
```

DATA SET LISTING

```
C     ANALYSIS OF VARIANCE OF A NON-ORTHOGONAL THREE-FACTOR EXPERIMENT
      DOUBLE PRECISION A(65,65),B(65,65),BT(65,65),P(65),Y(65),X(65)
      DOUBLE PRECISION YT(4,4,4),H(65)
      DOUBLE PRECISION CCA,SCD,SCDC,CMDC,CMD,SCF,CMF
      DOUBLE PRECISION SY,SCY,SYT,TCOR,SCC,SCCEL,SCI,CMI,SCA,SC
      DIMENSION NOBS(4,4,4),NX(7),NXX(7)
      READ 1,IM,JM,KM,N,IDIM
    1 FORMAT(16I5)
      DO 21 K=1,KM
   21 READ 1,((NOBS(I,J,K),I=1,IM),J=1,JM)
      READ 55,(Y(I),I=1,N)
   55 FORMAT(20F4.0)
      SY=0.
      SCY=0.
      SCYT=0.
      KOMPT=0
      SCA=0
      IJK=0
      NCEL=0
      DO 70 K=1,KM
      DO 70 J=1,JM
      DO 70 I=1,IM
      IF(NOBS(I,J,K).EQ.0)GO TO 70
      NM=NOBS(I,J,K)
      NCEL=NCEL+1
      YT(I,J,K)=0.
      DO 71 L=1,NM
      IJK=IJK+1
      YT(I,J,K)=YT(I,J,K)+Y(IJK)
      SCY=SCY+Y(IJK)**2
   71 SY=SY+Y(IJK)
      SCYT=SCYT+YT(I,J,K)**2/FLOAT(NM)
   70 CONTINUE
      TCOR=SY**2/FLOAT(IJK)
      SCC=SCY-TCOR
      NDLT=IJK-1
      SCCEL=SCYT-TCOR
      NDLCEL=NCEL-1
      SCI=SCC-SCCEL
      NDLI=NDLT-NDLCEL
      CMI=SCI/FLOAT(NDLI)
   99 READ 1,(NX(I),I=1,7)
      IF(NX(1).EQ.0)GO TO 60
      NC=0
      NCC=0
      MI=1
      KOL=1
      NXT=0
      NXX(1)=0
      Do 41 I=2,7
```

DATA SET LISTING

```
      NXX(I)=0
      KOL=KOL+NX(I)
      IF(NX(I).EQ.0)GO TO 411
      IF(I.GT.4)GO TO 42
      NCC=NCC+1
      NEP=NCC
      GO TO 41
 42   IF(I-6)43,44,45
 43   NCC=NCC+IM+JM-1
      INTIJ=NCC
      GO TO 41
 44   NCC=NCC+IM+KM-1
      INTIK=NCC
      GO TO 41
 45   NCC=NCC+JM+KM-1
      INTJK=NCC
      GO TO 41
411   NXX(I)=1
 41   CONTINUE
      IMIN=KOL+1
      IMAX=KOL+NCC
      DO 22 IND=1,7
      NXT=NXT+NXX(IND)
      NC=NC+NX(IND)
      IF(NX(IND).EQ.0)GO TO 22
      IJK=0
      MM=NC
      DO 23 K=1,KM
      DO 23 J=1,JM
      DO 23 I=1,IM
      IF(NOBS(I,J,K).EQ.0)GO TO 23
      NM=NOBS(I,J,K)
      DO 24 L=1,NM
      IJK=IJK+1
      DO 25 M=MI,MM
      MA=M-MI+1
      GO TO (26,27,28,29,30,31,32),IND
 26   BT(IJK,M)=1.
      GO TO 25
 27   IF(MA-I)33,26,33
 33   BT(IJK,M)=0.
      GO TO 25
 28   IF(MA-J)33,26,33
 29   IF(MA-K)33,26,33
 30   IF(MA-((I-1)*JM+J))33,26,33
 31   IF(MA-((I-1)*KM+K))33,26,33
 32   IF(MA-((J-1)*KM+K))33,26,33
 25   CONTINUE
 24   CONTINUE
 23   CONTINUE
```

```
      DO 46 I=IMIN,IMAX
      IMM=I-IMIN+1
      DO 47 M=MI,MM
      MA=M-MI
      GO TO (34,35,35,35,36,37,38),IND
 34   A(I,M)=0.
      GO TO 47
 35   IF(IMM-IND+1+NXT)34,48,34
 48   A(I,M)=1.
      GO TO 47
 36   IK=INTIJ-IM-JM+1
      IL=INTIJ
      IS=JM
      IP=INTIJ-JM
      GO TO 39
 37   IK=INTIK-IM-KM+1
      IL=INTIK
      IS=KM
      IP=INTIK-KM
      GO TO 39
 38   IK=INTJK-JM-KM+1
      IL=INTJK
      IS=KM
      IP=INTJK-KM
 39   IN=IMM-IK
      IQ=IMM-IP
      IT=MA/IS+1
      IU=MA+1-IS*(MA/IS)
      IF(IMM.LE.IK.OR.IMM.GT.IL)GO TO 34
      IF(IMM.LE.IP)GO TO 40
      IF(IQ.EQ.IU)GO TO 48
      GO TO 34
 40   IF(IN-IT)34,48,34
 47   CONTINUE
 46   CONTINUE
      MI=MM+1
 22   CONTINUE
      DO 50 I=1,N
      DO 50 J=1,KOL
 50   B(J,I)=BT(I,J)
      CALL MATVMU(B,Y,P,KOL,N,IDIM)
      CALL MATMUL(B,BT,A,KOL,N,KOL,IDIM)
      DO 52 J=IMIN,IMAX
      DO 53 I=1,KOL
 53   A(I,J)=A(J,I)
      DO 54 I=IMIN,IMAX
 54   A(I,J)=0.
      P(J)=0.
 52   CONTINUE
      JMAX=IMAX
```

DATA SET LISTING

```
      DO 4 I=1,KOL
    4 A(I,IMAX+1)=-P(I)
      DO 5 I=IMIN,IMAX
    5 A(I,IMAX+1)=0.
      L=IMAX-1
      CALL SOLSL(A,X,L,IDIM)
      SC=0.
      DO 7 I=1,KOL
      H(I)=P(I)*X(I)
      SC=SC+H(I)
    7 CONTINUE
      SC=SC-TCOR
      IF(KOMPT.GT.0)GO TO 72
      SCA=SC
      NDLA=KOL-1-NCC
      CCA=SCA/FLOAT(NDLA)
      SCD=SCC-SCA
      NDLD=NDLT-NDLA
      SCDC=SCD-SCI
      NDLDC=NDLD-NDLI
      CMDC=SCDC/FLOAT(NDLDC)
      FI=CMDC/CMI
      CMD=SCD/FLOAT(NDLD)
      PRINT 93
   93 FORMAT(1X,'VALUES OF THE PARAMETERS FOR THE COMPLETE MODEL'//)
      PRINT 6,(X(I),I=1,IMAX)
      PRINT 67
      KOMPT=KOMPT+1
      GO TO 99
   72 SCF=SCA-SC
      GO TO (85,86,87,173,73,74,75),KOMPT
  173 SCA=SC
      KOMPT=KOMPT+1
      NDLA=IM+JM+KM-3
      CCA=SCA/FLOAT(NDLA)
      SCD=SCC-SCA
      NDLD=NDLT-NDLA
      CMD=SCD/NDLD
      GO TO 99
   73 NDLF=IM-1
      GO TO 76
   74 NDLF=JM-1
      GO TO 76
   75 NDLF=KM-1
   76 SCF=SCA-SC
      CMF=SCF/FLOAT(NDLF)
      NDLAF=NDLA-NDLF
      FF=CMF/CMD
      KK=KOMPT-4
      PRINT 77,KK
```

DATA SET LISTING

```
 77  FORMAT(20X,'ANALYSIS OF VARIANCE FOR THE FACTOR',I5/)
     PRINT 90
     PRINT 78,SCF,NDLF,CMF,FF
 78  FORMAT(1X,'CONSIDERED FACTOR',8X,F20.6,2X,I4,F20.6,F12.3)
     PRINT 79,SC,NDLAF
     PRINT 80,SCA,NDLA
     GO TO 92
 85  NDLINT=(IM-1)*(JM-1)
     INT=12
     GO TO 88
 86  NDLINT=(IM-1)*(KM-1)
     INT=13
     GO TO 88
 87  NDLINT=(JM-1)*(KM-1)
     INT=23
 88  CMF=SCF/FLOAT(NDLINT)
     NDLAF=NDLA-NDLINT
     FF=CMF/CMI
     PRINT 89,INT
 89  FORMAT(20X,'ANALYSIS OF VARIANCE FOR INTERACTION',I4/)
     PRINT 90
 90  FORMAT(39X,'SS',10X,'DF',10X,'MS',15X,'F'/)
     PRINT 91,INT,SCF,NDLINT,CMF,FF
 91  FORMAT(1X,'INTERACTION',I3,11X,F20.6,2X,I4,F20.6,F12.3)
     PRINT 79,SC,NDLAF
 79  FORMAT(1X,'OTHER FACTORS',12X,F20.6,2X,I4/)
     PRINT 80,SCA,NDLA
 80  FORMAT(1X,'PARTIAL TOTAL (A)',8X,F20.6,2X,I4/)
     PRINT 82,SCDC,NDLDC,CMDC,FI
 82  FORMAT(1X,'INTERACTION 123 (D)-(C)',2X,F20.6,2X,I4,F20.6,F12.3)
     PRINT 81,SCI,NDLI,CMI
 81  FORMAT(1X,'WITHIN (C)',15X,F20.6,2X,I4,F20.6/)
 92  PRINT 83,SCD,NDLD,CMD
 83  FORMAT(1X,'REST (D)=(B)-(A)',9X,F20.6,2X,I4,F20.6//)
     PRINT 84,SCC,NDLT
 84  FORMAT(1X,'TOTAL (B)',16X,F20.6,2X,I4////)
     PRINT 94
 94  FORMAT(1X,'VALUES OF THE PARAMETERS'//)
     PRINT 6,(X(I),I=1,IMAX)
  6  FORMAT(1X,8F16.6)
     KOMPT=KOMPT+1
     PRINT 67
 67  FORMAT(1H1)
     GO TO 99
 60  CALL EXIT
     END

     SUBROUTINE SOLSL(A,X,L,ID)
     DOUBLE PRECISION A(ID,ID),X(ID)
```

DATA SET LISTING

```
      M=L+1
      J=M+1
      DO 1 K=1,L
      KP=K+1
      DO 4 I=KP,M
      P=A(K,K)
      C=A(I,K)
      IF(C)3,4,5
    5 IF(C+P)4,4,6
    3 IF (C+P)7,4,4
    6 IF(C-P)4,4,8
    7 IF(C-P)8,4,4
    8 DO 9 JJ=K,J
      E=A(I,JJ)
      A(I,JJ)=A(K,JJ)
    9 A(K,JJ)=E
    4 CONTINUE
      DO 1 I=KP,M
      DO 1 JJ=KP,J
    1 A(I,JJ)=A(I,JJ)-A(I,K)*A(K,JJ)/A(K,K)
      X(J)=1.
      DO 10 I=1,M
      NI=J-I
      X(NI)=0.
      DO 11 JJ=1,I
      NJ=J+1-JJ
   11 X(NI)=X(NI)-A(NI,NJ)*X(NJ)
   10 X(NI)=X(NI)/A(NI,NI)
      RETURN
      END

      SUBROUTINE MATVMU(A,B,C,IMAX,JMAX,IDIM)
      DOUBLE PRECISION A(IDIM,IDIM),B(IDIM),C(IDIM)
      DO 1 I=1,IMAX
      C(I)=0.
      DO 1 J=1,JMAX
    1 C(I)=C(I)+A(I,J)*B(J)
      RETURN
      END

      SUBROUTINE MATMUL(A,B,C,IM,JM,KM,ID)
      DOUBLE PRECISION A(ID,ID),B(ID,ID),C(ID,ID)
      DO 1 I=1,IM
      DO 1 K=1,KM
      C(I,K)=0.
      DO 1 J=1,JM
    1 C(I,K)=C(I,K)+A(I,J)*B(J,K)
      RETURN
      END
```

DATA SET LISTING

```
     2   4   4  36  65
     2   1   2   0   1   1   1   1
     2   1   2   1   1   1   1   0
     1   1   1   2   2   0   1   1
     2   0   1   2   2   0   1   1
  43  58  58  73  59  81  62  67  71  93  83  60  75  89  71 101  76 100  91  70
  85  70  58  92  88 106  73  89  89  98  69  72 105 108 109  76
     1   2   4   4   8   8  16
     1   2   4   4   0   8  16
     1   2   4   4   8   0  16
     1   2   4   4   8   8   0
     1   2   4   4   0   0   0
     1   0   4   4   0   0   0
     1   2   0   4   0   0   0
     1   2   4   0   0   0   0
```

VALUES OF THE PARAMETERS FOR THE COMPLETE MODEL

77.546249	10.621645	-10.621653	-7.019826	-1.198977	0.710620	7.508178	-11.323932
4.770985	0.990214	5.562733	-1.174429	-4.395984	5.746488	-0.176073	1.174430
4.395982	-5.746489	0.176072	-11.072997	0.336730	3.841893	6.894376	11.073005
-0.336731	-3.841894	-6.894377	-5.660567	0.107945	8.983373	-3.430749	5.823996
-4.972384	-2.981629	2.130016	4.567067	5.472151	-9.457112	-0.582111	-4.730496
-0.607714	3.455364	1.882845	0.000000	0.000000	-0.000000	0.000000	-0.000000
-0.000000	-0.000000	-0.000000	0.000015	-0.000018	-0.000007	-0.000007	0.000001
0.000004	-0.000001	0.000002	-0.000003	0.000000	0.000002	-0.000001	

ANALYSIS OF VARIANCE FOR INTERACTION 12

	SS	DF	MS	F
INTERACTION 12	223.375240	3	74.458413	1.497
OTHER FACTORS	8883.873818	19		
PARTIAL TOTAL (A)	9107.249057	22		
INTERACTION 123 (D)-(C)	165.250943	4	41.312736	0.831
WITHIN (C)	447.500000	9	49.722222	
REST (D) = (B)-(A)	612.750943	13	47.134688	
TOTAL (B)	9720.000000	35		

VALUES OF THE PARAMETERS

78.255362	9.619641	-9.619649	-7.633335	-3.298811	4.062500	6.869639	-13.630360
3.744644	2.444644	7.441073	-8.869641	0.880355	2.680355	5.308925	8.869649
-0.880355	-2.680355	-5.308928	-4.241667	0.800000	7.433333	-3.991666	3.923809
-3.867858	-2.301191	2.245238	2.812499	2.437498	-7.062502	1.812499	-2.494641
0.630359	1.930359	-0.066069	0.000000	0.000000	-0.000000	0.000000	-0.000000
-0.000000	-0.000000	-0.000000	0.000000	-0.000000	-0.000000	0.000000	0.000000
-0.000000	-0.000000						

ANALYSIS OF VARIANCE FOR INTERACTION 13

	SS	DF	MS	F
INTERACTION 13	854.299408	3	284.766469	5.727
OTHER FACTORS	8252.949649	19		
PARTIAL TOTAL (A)	9107.249057	22		
INTERACTION 123 (D)-(C)	165.250943	4	41.312736	0.831
WITHIN (C)	447.500000	9	49.722222	
REST (D) = (B)-(A)	612.750943	13	47.134688	
TOTAL (B)	9720.000000	35		

VALUES OF THE PARAMETERS

78.358590	9.460224	-9.460232	-5.813134	-6.261365	5.266412	6.808079	-16.511994
4.057450	2.467803	9.986742	-2.869317	0.456439	1.539774	0.873105	2.869320
-0.456438	-1.539774	-0.873106	-5.230432	-0.133208	5.486746	-0.123105	0.498106
-1.126895	-0.259470	0.888258	4.386998	0.817552	-7.092808	1.888256	0.345329
0.442550	1.865532	-2.653408	0.000000	0.000000	-0.000000	-0.000000	0.000000
0.000001	0.000000	0.000000	-0.000002	-0.000001	-0.000001	-0.000000	0.000000
0.000001	0.000002						

ANALYSIS OF VARIANCE FOR INTERACTION 23

	SS	DF	MS	F
INTERACTION 23	573.553269	9	63.728141	1.282
OTHER FACTORS	8533.695789	13		
PARTIAL TOTAL (A)	9107.249057	22		
INTERACTION 123 (D)-(C)	165.250943	4	41.312736	0.831
WITHIN (C)	447.500000	9	49.722222	
REST (D) = (B)-(A)	612.750943	13	47.134688	
TOTAL (B)	9720.000000	35		

VALUES OF THE PARAMETERS

78.074119	9.354844	-9.354851	-6.859219	-3.757429	3.933074	6.683567	-13.796994
3.533980	2.052747	8.210268	-2.020564	-1.067710	1.700805	1.387469	2.020565
1.067710	-1.700805	-1.387470	-7.847813	1.487878	1.791514	4.568422	7.847820
-1.487879	-1.791515	-4.568423	0.000000	0.000000	-0.000000	-0.000000	0.000000
0.000001	0.000000	0.000000	0.000015	-0.000015	-0.000006	-0.000006	-0.000001

ANALYSIS OF VARIANCE FOR THE FACTOR 1

	SS	DF	MS	F
CONSIDERED FACTOR	2537.633921	1	2537.633921	38.437
OTHER FACTORS	5333.795246	6		
PARTIAL TOTAL (A)	7871.429166	7		
REST (D) = (B)-(A)	1848.570834	28	66.020387	
TOTAL (B)	9720.000000	35		

VALUES OF THE PARAMETERS

80.830720 -6.221669 -7.883800 8.294279 5.811187 -16.583868 4.493353 1.489702
10.600814 0.000000 0.000000

ANALYSIS OF VARIANCE FOR THE FACTOR 2

	SS	DF	MS	F
CONSIDERED FACTOR	1543.541779	3	514.513926	7.793
OTHER FACTORS	6327.887387	4		
PARTIAL TOTAL (A)	7871.429166	7		
REST (D) = (B)-(A)	1848.570834	28	66.020387	
TOTAL (B)	9720.00000	35		

VALUES OF THE PARAMETERS

77.108486 9.209456 -9.209463 -16.622747 2.932809 3.312689 10.377253 0.000000
-0.000000

ANALYSIS OF VARIANCE FOR THE FACTOR 3

	SS	DF	MS	F
CONSIDERED FACTOR	3650.368100	3	1216.789367	18.430
OTHER FACTORS	4221.061067	4		
PARTIAL TOTAL (A)	7871.429166	7		
REST (D) = (B)-(A)	1848.570834	28	66.020387	
TOTAL (B)	9720.000000	35		

VALUES OF THE PARAMETERS

78.229611 8.874068 -8.874075 -8.379236 -4.581796 6.458357 6.502668 0.000000
0.000000