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MATHEMATICAL PROGRAMS FOR A MARKETING MODEL OF THE FAMILY PLANNING PROCESS

by

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ABSTRACT

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It is widely accepted that the need for effective programs focused on population growth rate problems is a high-priority one in the developing countries. To manage this process, models are needed to allocate the resources provided for family planning services and supplies whether they be for national goals of reduced birth rates or for personal goals of attaining desired family size.

This paper describes some mathematical optimization models based on the view of the family planning process as a marketing/service-providing system. The family planning administrator is faced with a heterogeneous set of decisions dealing with the distribution of funds to population education, various types of allied personnel, facilities and supplies.

A dynamic programming model is given whose structure reflects the allocation of funds to activities for population subgroups at given states in the marketing/ services system. Details on the calculation of flows through the system and corresponding costs are given, and there is a discussion of model requirements, attributes, applications and areas suitable for further work.

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1. THE APPLICATION

Family planning programs can be represented as social processes which move families among parity/decision/service states. This mixture of types of states is characteristic of many health programs and has led investigators to suggest use of Markov models for systems design [1,5,9]. Because massive social and behavioral changes are necessary for the family planning program to succeed, it can be viewed in terms of an innovation and diffusion or "marketing" model [2,3,8]. We believe that the Markov property does not hold for the particular state definition here or in many other health care system models because state changes are dependent on the length of time in a state. Yet the state-to-state flow concept seems logical and potentially powerful and we have set out to find a suitable modeling technique.

This paper presents one way of modeling such programs and the approaches which might be used to seek out the optimal design for family planning programs. Attempts to do so are rare due to the diffuse nature of program experience. Only a few have suggested explicit models, e.g. Reinke [4], and most such efforts have been limited to contraceptive choice. A second obstacle has been the lack of conceptual structures capable of dynamically handling levels and states involving parity, decision-making, and service variables. One objective of our research has been to seek an analytical approach which links a broad range of program decisions

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to the appropriate decision variables in system optimization, e.g., to the number of active contraceptive "acceptors" who continue in a program. This is important because the current norm for many programs is for managers to maximize by period the number of new entrants or acceptors.

As McLaughlin [2] has pointed out, there is a distinct need for appropriate models to train family planning administrators. In particular, they must be persuaded to use a dynamic model as their conceptual basis for designing information and control systems.

Figure 1. Movement of Families with Respect to Age/Parity Status

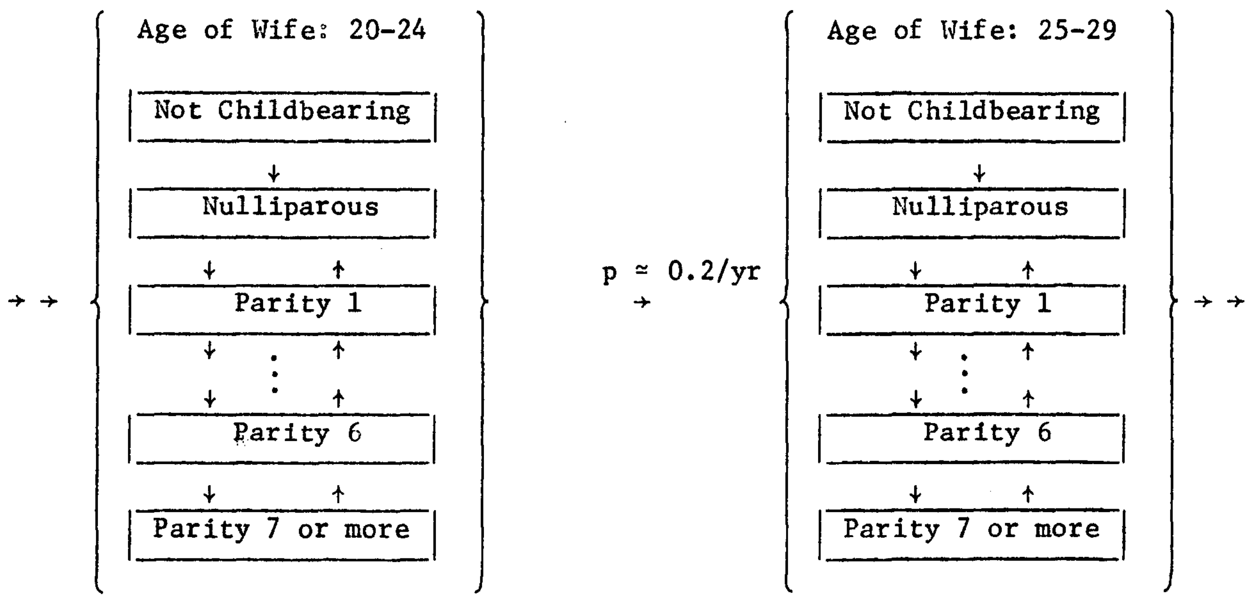
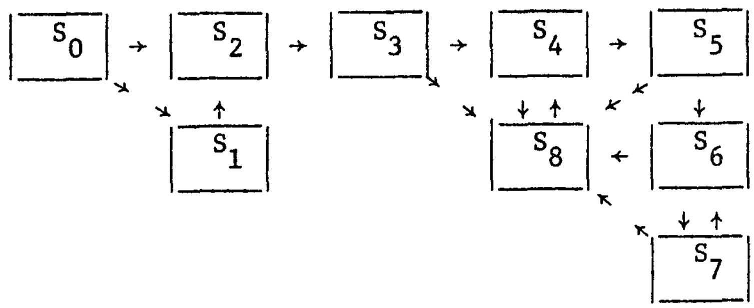


Figure 1, condensed from [2], indicates how parity status can be represented as states and movements between states. Figure 2, also abridged from [2], shows the marketing model with its flows among states of awareness, knowledge-seeking, decisions, service and continuance. Each population of potential acceptor families is usually broken down into age-specific groups by ages of the wife, 15-19, 20-24, 25-29, etc. The birth rates in each group differ markedly and use of average birth rates rather than age-specific birth rates often leads to erroneous conclusions.

Thus the task outlined in terms of Figure 1 is to control movements from one parity state to another within the age-specific group. Figure 2 indicates how individual actions and program actions link into the control of those movements. Later sections of this paper then link alternative program actions and their costs to the desired changes in flows, movements and program outcomes.

Figure 2. Simplified Network Version of the Marketing Model



S_0 = target population of couples under consideration
S_1 = unaware couples
S_2 = aware couples
S_3 = knowledge-seeking couples
S_4 = couples reaching positive decision
S_5 = couples taking initial action
S_6 = couples receiving service
S_7 = couples continuing in program
S_8 = couples reaching negative decision

Much of the underlying basic theory for this paper is developed in [7].

2. A DYNAMIC MODEL

a. Viewpoint and Formulation

We will treat the marketing model as a directed network with a single node representing all successive states (Figure 2). We have deleted the arcs from nodes S_1, S_7 and S_8 to a terminal "Leave the Group" node shown in [2], since those flows typically due to aging and infertility can be considered relatively constant. They can be added later without weakening the arguments herein.

A dynamic programming (DP) model can be constructed for allocating funds to influence the flow rates over a planning horizon in such a way as to optimize the number of couples who continue to use the family planning services, i.e., remain at node S_7. Other criteria are used for some programs and will be mentioned below. The output of any model should be the allocations by node of the budget input to population activities such as education, outreach, paramedical and clinic workers, training, doctors, advertising campaigns, facilities and supplies, etc. The objective of this presentation will be to state these allocations in terms of budget items for the nodes defined in Figure 2. The optimization model presented may be modified for other network interpretations of the system and for other resource definitions.

The planning horizon will consist of a number of time intervals, typically determined by budget periods.

To represent the flow of the members of the target population through the system, we first assume that the members of a particular age/parity-specific group all start at node S_0. There also may be a buildup of members of that group from previous periods at nodes S_1 through S_8. During the period, assumed to be of sufficient length, they all flow through the network in response to the activities of the program by making transitions from node to node. We will employ the following notation:

$C_n \equiv$ size of the target population of couples at period n of an N-period planning horizon, i.e., the initial flow to the source node S_0 at the n-th period.

For example, the new age/parity group of married couples in the age category 30-34 where the wife has two living children could constitute C_n.

Members of that population in the system during the preceding period and left at the nodes are not included in C_n. The latter are called residues and will be defined subsequently. The periods are numbered in reverse order with the initial period designated as N and the final period as 1.

$p_{ij}(n) \equiv$ the proportion of the population at node S_i which flows (makes a transition) to node S_j during period n.

(2.1) $p_{ij}(n) \in [0,1]$ and $\sum_k p_{ik}(n) = 1$; $i,j = 0,1,\ldots,8$; $n = N, N-1, \ldots, 1$.

In the remainder of this paper, we will assume that $p_{84}(n) = p_{76}(n) = 0$ for all n, so that the network will be acyclic. Hence the corresponding transition proportion incidence matrix, representing feasible transitions, has all subdiagonal elements equal to zero. One application of this acyclical case is to the set of decisions on acceptance of vasectomies. Appropriate modifications, however, can be made to deal with the cyclic case; see [7].

Representative service inputs for the various transition proportions might be:

$p_{12}(n)$ - population education, mass media, leaflets
$p_{23}(n)$ - posters, outreach workers, mass media
$p_{34}(n)$ - posters, outreach workers, village meetings
$p_{45}(n)$ - mobile units, outreach workers, training local leaders
$p_{56}(n)$ - paramedical interviewers, physicians, contraceptive clinics, mobile units
$p_{67}(n)$ - incentives, outreach workers, physician interviewers
$p_{38}(n)$ - outreach workers, films, posters
$p_{48}(n)$, $p_{58}(n)$ - outreach workers, physician interviewers, mobile units
$p_{68}(n)$, $p_{78}(n)$ - outreach workers, films, dispensaries
$p_{01}(n)$, $p_{02}(n)$ - are fixed and might be estimated by sampling.

We assume that all members of the target population are aware or unaware so that $p_{01}(n) + p_{02}(n) = 1$.

One objective of such a model is to select the transition proportions in each period so as to maximize the cumulative flow to node S_7 over all N periods while simultaneously minimizing the cumulative flow to node S_8 under conditions of constrained resources. To make this latter objective precise we choose the technique of specifying a trade-off parameter similar to that used in capital budgeting problems. The model below relates the transition proportions to costs in such a way that the optimal allocation of funds producing the maximal flow is determined.

There is a question as to whether to maximize the flow for each period separately or over the total N-period horizon. At the start of a family planning program, the number of acceptors is small and grows during subsequent periods. The administrator would, therefore, like to know whether or not to allocate more funds to population education and recruitment of new acceptors (corresponding to the early nodes) during the initial periods and when to change that strategy to support heavy potential flows through initial action, service and continuance nodes.

Single period maximization would not take these temporal attributes of strategies into account. Thus we suggest a model which encompasses the period-to-period differences by the use of a DP formulation. To do this we need notation to represent the population groups left at the nodes from period-to-period.

$y_i(n) \equiv$ residue from all previous populations under consideration who have flowed to, and remained at, node S_i at the end of period n+1; $n = N-1,\ldots,1$. $i = 1,\ldots,7$ for the acyclic case.

$y_0(n) \equiv C_n$, $n = N,\ldots,1$.

$\overline{y}(n) \equiv (y_0(n), y_1(n),\ldots, y_7(n))$ is the vector of residues.

The conditions (2.1) imply that the transition proportion matrix is stochastic; hence, we may omit redundant proportions and write a vector of all $p_{ij}(n)$ representing feasible transitions in the system in period n.

$$\overline{p}(n) \equiv (p_{02}(n), p_{12}(n), p_{23}(n), p_{34}(n), p_{38}(n), p_{45}(n), p_{48}(n), p_{56}(n), p_{58}(n), p_{67}(n), p_{68}(n), p_{78}(n)).$$

We will show below how to compute the cumulative flows to nodes S_7 and S_8. These will depend on $\overline{y}(n)$ and $\overline{p}(n)$ and, temporarily, are denoted by $G_7(\overline{y}(n), \overline{p}(n))$ and $G_8(\overline{y}(n), \overline{p}(n))$ respectively.

To well-define the objective function, we specify the trade-off between the two objectives of maximizing flow to S_7 while minimizing flow to S_8 as λ_n and write $g_n(\overline{y}(n), \overline{p}(n)) \equiv G_7(\overline{y}(n), \overline{p}(n)) - \lambda_n G_8(\overline{y}(n), \overline{p}(n))$. g_n is an adjusted flow and λ_n is an arbitrary but fixed non-negative constant defined by the decision-maker for each period n. Fur further details on the use of λ_n and an alternate approach, see [7].

The above variables may now be labelled by standard DP nomenclature:

Stages:	time periods of the planning horizon, $n = N,\dots,1$.
States:	the residue vector, $\overline{y}(n)$.
Decision Variables:	the transition proportion vector, $\overline{p}(n)$.
Stage Returns:	the adjusted flow parameterized by λ_n, $g_n(\overline{y}(n), \overline{p}(n))$.
Stage Transformation:	$\overline{y}(n) = t_{n+1}(\overline{y}(n+1), \overline{p}(n+1))$; $n = N-1,\dots,1$.

The last transformation is defined by the calculation of the present period residues, $\overline{y}(n)$, from those in the previous period, $\overline{y}(n+1)$, (known) and the corresponding $\overline{p}(n+1)$.

Let $\mathcal{Q}_n[\overline{y}(n)]$ be the maximizing n-stage adjusted flow; that is, $\mathcal{Q}_n[\overline{y}(n)]$ is the total adjusted flow if an optimal policy is followed with n periods remaining beginning with the current residues $\overline{y}(n)$. The adjusted flow is decomposable, hence:

$$(2.2)\quad \mathcal{Q}_n[\overline{y}(n)] = \max_{\overline{p}(n)} \{G_7(\overline{y}(n), \overline{p}(n)) - \lambda_n G_8(\overline{y}(n), \overline{p}(n)) + \mathcal{Q}_{n-1}[t_n(\overline{y}(n), \overline{p}(n))]\}$$

where $n = N,\dots,1$ and $\mathcal{Q}_0[t_1(\overline{y}(1), \overline{p}(1))] \equiv 0$.

To complete the formulation of the DP model, we must state how the cumulative flows and the costs of affecting flows relate to the residues and the transition proportions and specify the form of the budget constraints.

b. Construction of Flow and Cost Functions

The cumulative flow is determined by the transition proportions, the residues left at nodes from previous periods and the initial population at node S_0 at the beginning of each period. Given a target population C_n at node S_0, we can compute the flow to node S_7 without residues and excluding those who go on to S_8 as follows:

$$(2.3)\quad C_n[p_{01}(n)\, p_{12}(n) + p_{02}(n)]\, p_{23}(n)\, p_{34}(n)\, p_{45}(n)\, p_{56}(n)\, p_{67}(n)[1-p_{78}(n)]$$
$$= C_n I_2(n)[1-p_{78}(n)] \prod_{j=3}^{7} p_{j-1,j}(n) ,$$

where
$$I_2(n) = p_{01}(n)\, p_{12}(n) + p_{02}(n) .$$

The expression (2.3) is appropriate only when the stochastic node constraints are explicit: $\sum_j p_{ij}(n) = 1$, for all S_i. The transition proportions $p_{ij}(n)$ are exactly the node-to-node flow percentages of interest to the family planning administrator. However, the additional constraints add computational complexity to the DP formulation. Fortunately, there is a transformation of the original set of transition proportions to a set of modified proportions which need no further constraints for mathematical programming models.

Since $p_{01}(n) + p_{02}(n) = 1$, the nodes with multiple successors requiring constraints are nodes $S_i, i = 3,\ldots,6$, which have successors S_{i+1} and S_8. For each of these write:

$$p_{i,i+1}(n) = [1-q_{i8}(n)]q_{i,i+1}(n)$$
$$p_{i8}(n) = q_{i8}(n) \quad i = 3,\ldots,6;\ q_{ij}(n) \in [0,1].$$

The residue proportion will be:

$$p_{ii}(n) = [1-q_{i8}(n)][1-q_{i,i+1}(n)]$$

and the three terms already sum to one.

For consistency of notation, let all other $p_{ij}(n) = q_{ij}(n)$. Then (2.3) becomes:

$$C_n I_2(n) \prod_{j=3}^{7} q_{j-1,j}(n)[1-q_{j8}(n)] \ .$$

A similar argument leads to the flow to node S_8:

$$\begin{aligned}(2.4)\quad & C_n I_2(n) q_{23}(n)\{q_{38}(n)+[1-q_{38}(n)]q_{34}(n)q_{48}(n)\\ &+[1-q_{38}(n)]q_{34}(n)[1-q_{48}(n)]q_{45}(n)q_{58}(n)\\ &+[1-q_{38}(n)]q_{34}(n)[1-q_{48}(n)]q_{45}(n)[1-q_{58}(n)]q_{56}(n)q_{68}(n)\\ &+[1-q_{38}(n)]q_{34}(n)[1-q_{48}(n)]q_{45}(n)[1-q_{58}(n)]q_{56}(n)[1-q_{68}(n)]q_{67}(n)q_{78}(n)\}\\ =\ & C_n I_2(n) q_{23}(n) \sum_{k=3}^{7} q_{k8}(n) \prod_{j=3}^{k-1} q_{j,j+1}(n)[1-q_{j8}(n)] \ ,\end{aligned}$$

where we use the convention $\prod_{j=3}^{2}(\ldots) \equiv 1$. The total residue for this period (that portion of $\sum_{j=1}^{6} y_j(n-1)$ resulting from period n) is:

$$\begin{aligned}(2.5)\quad & C_n\Big[[1-q_{02}(n)][1-q_{12}(n)] + I_2(n)[1-q_{23}(n)]\\ &+ I_2(n)q_{23}(n)\{[1-q_{38}(n)][1-q_{34}(n)]+[1-q_{38}(n)]q_{34}(n)[1-q_{48}(n)][1-q_{45}(n)]\\ &+ [1-q_{38}(n)]q_{34}(n)[1-q_{48}(n)]q_{45}(n)[1-q_{58}(n)][1-q_{56}(n)]\end{aligned}$$

$$+ [1-q_{38}(n)]q_{34}(n)[1-q_{48}(n)]q_{45}(n)[1-q_{58}(n)]q_{56}(n)[1-q_{68}(n)][1-q_{67}(n)]\}\Big]$$

$$= C_n\Big[q_{01}(n)q_{11}(n)+I_2(n)\{q_{22}(n)+q_{23}(n)\sum_{k=3}^{6}[1-q_{k,k+1}(n)][1-q_{k8}(n)]\prod_{j=3}^{k-1}q_{j,j+1}(n)[1-q_{j8}(n)]\}\Big]$$

with the same convention $\prod_{j=3}^{2}(\ldots) \equiv 1$.

The reader may check the above derivation by summing the first expressions in (2.3), (2.4) and (2.5) to yield C_n.

As indicated, the transformation from $p_{ij}(n)$ to $q_{ij}(n)$ was to improve the mathematical serviceability of the model. For purposes of exposition we will now revert to the $p_{ij}(n)$ notation.

The above functions are polynomials in the variables $p_{02}(n)$, $p_{12}(n), \ldots, p_{78}(n)$ of $\overline{p}(n)$. To represent the flow to an arbitrary node along a feasible path, or paths, we use the following notation. If there is a single immediate predecessor S_i to node S_j, let $\overline{p}_{ij}(n)$ be the vector of necessary transition proportions for flow up to node S_j through S_i. For example, $\overline{p}_{34}(n) = (p_{02}(n), p_{12}(n), p_{23}(n), p_{34}(n))$. Then letting $f_{n67}(\overline{p}_{67}(n))$ be the polynomial by which C_n is multiplied in (2.3), the flow to node S_7 is:

$$C_n f_{n67}(\overline{p}_{67}(n)) \ .$$

Similarly, the flows to each of the other nodes, still not considering residues, are of the form $C_n f_{nij}(\overline{p}_{ij}(n))$ for nodes S_j with unique predecessors S_i and $C_n f_{n\cdot j}(\overline{p}_{\cdot j}(n)) \equiv \sum_{i:i\to j} f_{nij}(\overline{p}_{ij}(n))$ for nodes with more than one predecessor, where $\sum_{i:i\to j}$ means the sum is taken over all arcs (S_i, S_j) such that node S_i is a predecessor of node S_j. Hence the expression in (2.4) is $C_n f_{n\cdot 8}(\overline{p}_{\cdot 8}(n))$.

In each period, the flow which reaches node S_7 contains a portion of C_n as well as portions of the residues at preceding nodes. Hence, these last must be considered in the construction of the cumulative flow. To do this we make a homogeneity assumption that all couples at a node are treated alike with respect to proportion transiting. More detail on this assumption and suggestions for removing it as a restriction are given in [7].

The following example illustrates how to find the cumulative flow in period n at nodes with unique predecessors. We refer to nodes S_0, S_1, S_2 and S_3 of Figure 2. The cumulative flow to node S_3 is the sum of the flows to S_3 from S_0, the flow to S_3 from residues at S_1 and S_2, and the residue at S_3. Using notation similar to the previous DP formulation:

$$F_{n23}(\overline{y}(n), \overline{p}_{23}(n)) = C_n f_{n23}(\overline{p}_{23}(n)) + [y_1(n)p_{12}(n) + y_2(n)]p_{23}(n) + y_3(n) .$$

In general, we write $F_{nij}(\overline{y}(n), \overline{p}_{ij}(n))$ for the cumulative flow to node S_j through S_i.

If there is not a unique predecessor, the cumulative flow to node S_j is:

$$F_{n\cdot j}(\overline{y}(n), \overline{p}_{\cdot j}(n)) \equiv \sum_{i:i\to j} F_{nij}(\overline{y}(n), \overline{p}_{ij}(n)) - M_j y_j(n)$$

where $M_j \equiv$ (number of predecessors of j) - 1.

The population left at node S_7, $y_7(n)$, includes continuing couples from all previous periods. For calculation of the flow to node S_8, we may want to use direct proportions or, if we are setting targets for a non-serviced subpopulation, we may omit $y_7(n)$ or include only a fixed fraction $y_7^*(n) = \tau_n y_7(n)$ where $\tau_n \in (0,1)$.

The flow to node S_7 with residues and excluding those who go on to S_8 is:

$$(2.6)\quad F_{n67}(\overline{y}(n), \overline{p}_{67}(n)) = C_n f_{n67}(\overline{p}_{67}(n)) + [1-p_{78}(n)][\sum_{i=1}^{6} y_i(n) \prod_{j=i+1}^{7} p_{j-1,j}(n) + y_7(n)].$$

The flow to node S_8 with residues is:

$$(2.7)\quad F_{n\cdot 8}(\overline{y}(n),\overline{p}_{\cdot 8}(n)) = C_n f_{n\cdot 8}(\overline{p}_{\cdot 8}(n)) + \sum_{i=1}^{7} y_i(n) \sum_{j=\max(3,i)}^{7} p_{j8}(n) \prod_{k=i}^{j-1} p_{k,k+1}(n) ,$$

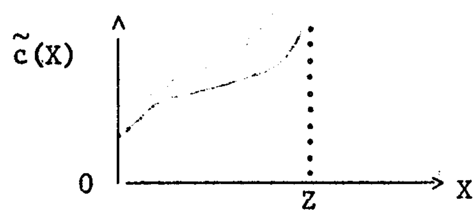
with the convention $\prod_{k=h}^{h-1} (\ldots) \equiv 1, \quad h = 3,\ldots,7.$

Recall that for the above expressions we must either use the transformation from $p_{ij}(n)$ to $q_{ij}(n)$ or stochastically constrain the $p_{ij}(n)$.

The goal of this construction is to relate the cost of flowing couples through a node to the transition proportions. Let $Z_i(n)$ be the size of the population at node S_i at some period n. Then $Z_i(n)$ is the cumulative flow at node S_i and we want to determine the cost of flowing $X_{ij}(n) = Z_i(n)\ p_{ij}(n)$ to node S_j, where S_j is a successor of S_i. For a small flow there may be a positive set-up cost of influencing the start of the flow; then there is an increase in cost as the flow grows larger and, as $p_{ij}(n)$ approaches 1, the cost to flow $X_{ij}(n)$ may increase sharply.

It can be shown that functions satisfying the above criteria may be approximated by strictly increasing polynomials which may then be used in the DP formulation. Figure 3 is an example of such a function; details on the approximation and a numerical example are given in [7].

Figure 3. A Cost Function for the Node-to-Node Flow



The above argument is for flows to desirable nodes ($i \neq 8$). For S_8, the same cost function types may be used with $X_{i8}(n) = Z_i(n)\zeta_{i8}(n)$, where $\zeta_{i8}(n) = 1-p_{i8}(n)$, $i = 3,\ldots,7$. Here, higher allocations decrease the flow to S_8.

The costs are defined, in general, for each period and each transition as a function of the cumulative flows. If a node S_j has a unique predecessor S_i, the cost will be a function of $p_{ij}(n)$ and $F_{nij}(\bar{y}(n),\ \bar{p}_{ij}(n))$. Costs are associated only with flows *into* node S_j; hence $y_j(n)$ is omitted from cost calculation. Thus costs are actually a function of

$$F_{nij}(\bar{y}(n),\ \bar{p}_{ij}(n)) - y_j(n) = p_{ij}(n)F_{n\cdot i}(\bar{y}(n),\ \bar{p}_{\cdot i}(n)).$$

If an approximating cubic polynomial is $c_{nij}[x] = \sum_{k=0}^{3} a_{nijk}\ x^k$, then the cost is:

$$\sum_{k=0}^{3} a_{nijk}[p_{ij}(n)]^k\ F^k_{n\cdot i}(\bar{y}(n),\ \bar{p}_{\cdot i}(n))\ .$$

The cost of flow to a node without a unique predecessor is:

$$\sum_{i:i\to j} c_{nij}[p_{ij}(n)\ F_{n\cdot i}(\overline{y}(n),\ \overline{p}_{\cdot i}(n))]$$

and the cost of all flow in the network is the sum of the flow costs between all nodes.

There may, of course, be further constraints on flow and/or costs at individual nodes or for individual arcs, such as bounds derived from limitations on interviews, acceptor renewals and returns per time period, personnel and facilities.

Suppose there is a budget of B_n for period n and the costs are cubic polynomials. The form of the DP, using (2.2), (2.6) and (2.7) is:

$$Q_n[\overline{y}(n)] = \max_{\overline{p}(n)} \{F_{n67}(\overline{y}(n),\ \overline{p}_{67}(n)) - \lambda_n F_{n\cdot 8}(\overline{y}(n),\ \overline{p}_{\cdot 8}(n)) + Q_{n-1}[t_n(\overline{y}(n),\overline{p}(n))]\}$$

subject to

(2.8) $$\sum_{j=2}^{8} \sum_{i:i\to j} \sum_{k=0}^{3} a_{nijk}[p_{ij}(n)]^k\ F^k_{n\cdot i}(\overline{y}(n),\ \overline{p}_{\cdot i}(n)) \le B_n$$

$n = 1,\ldots,N$ and $Q_0[t_1(\overline{y}(1),\ \overline{p}(1))] = 0.$

There is a dual formulation which minimizes cost while guranteeing target levels of continuance for the program; see [7].

To use the optimization model, the administrator and analyst must supply estimates for the cost functions, use the DP algorithm to produce a set of transition proportions $p_{ij}(n)$ (actually $q_{ij}(n)$) then construct the cumulative flows from the formulae above. These may then be substituted into the cost functions to determine nodal expenditures and sensitivity analysis may be performed.

In the construction of the cost functions, submodels may be specified to determine: (i) the effectiveness of distribution at each node, (ii) the interaction among nodes and (iii) an appropriate parameterization to reflect the effects of possible changes during the planning period. Examples might be: (i) suballocations of various personnel and contraceptive supplies, (ii) adjustment for the effect of a single budgeted item, like a population education program, on more than one flow and (iii) the perceived consequences of new or improved birth control devices such as annual shots, monthly pills or IUD's with increased efficacy and the corresponding group receptiveness.

3. COMMENTS AND CONCLUSION

a. Strengths and Weaknesses of the Model

This model is a complex one, especially as extended in [7] to include the necessary cyclic case. But it has a number of strengths, namely:

1. The dynamic formulation reflects the changes in the life cycle of the program and, in particular, the different preferences of the potential acceptors. Such a model should spur program managements to stay current with changes in the population's attitudes and behavior.

2. The variables and functions: C_n, $\overline{p}(n)$, $c_{nij}(x)$, etc. are ones which the behavior-oriented decision-maker should be dealing with anyway.

3. While complex, the analytic formulation of much of the model allows for relatively efficient calculation of many values; in particular, sensitivity analysis may be used to look at tradeoffs and constraint effects more efficiently than corresponding all-simulation approaches. The dual formulation can be used to determine percentages of the target population to be served.

4. Disaggregation of the population can be handled by the same approach used for parity and age-specific groupings. This could be based on health status, economic status, religion, costs, etc.

5. This model focuses attention on a wide range of program choices instead of emphasizing just the traditional ones of contraceptive choice and of crude birth reduction targets, areas which are well researched.

While complexity limits the planning horizon length, this is a realistic feature of any model for program planning. Policy changes will change parameters quite rapidly. Cost data estimates will become much more heterogeneous as the number of periods is extended and social and behavioral changes are quite likely to negate long-term estimates.

Certainly, the reader can raise questions, which can be called either weaknesses or areas that need further work. They are:

1. Adequate development of the cyclic case - some work on this is presented in [7].

2. Linkage to classical program output measures - the program does not optimize directly on the stated objectives of most programs, such as crude birth rates, couple-years of protection, age-specific fertility rates or births averted. But standard formulae are available to transform acceptors and continuers into these values, and they can be reflected by appropriate modifications of the objective function. Schieber [6] provides a useful summary and bibliography in this area.

3. The selection of the best contraceptive is omitted - this has been discussed in [2] and [8]. The basic assumption is that program resource consumption is more a function of the task of convincing couples to practice contraception than for providing services and that the technique adopted is an adopter decision; that the program inputs to that choice are primarily medical, not administrative; and that the choices are heavily cultural and custom dependent. Also, contraceptive choice outputs from other models, [4, 10], may be used as input submodels for our nodes S_6 and S_7.

4. What about interacting inputs? - obviously, an advertising campaign might affect several flows at once, so that there can be interactions. The seriousness of this criticism will have to await some experience in working with real data and experiments with adjusted coefficients.

5. The approach may be infeasible in terms of computational difficulty or data availability. Within the limits of available resources, we plan to continue work on those questions. Computational tests with sample networks are under way. As the title of [7] suggests, however, we do not see the application as being limited to this specific network or this specific problem area.

b. Conclusion

We see the family planning program usefully modeled in a DP formulation as more appropriate than the alternative models and that the problem as formulated can be worked through. Because of the nature of the variables being dealt with, there seems to be considerable potential here for the development of a suitable tool for program planning, program evaluation and administration training, either in situ or in a gaming situation.

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