ESTIMATION OF AGE AT DEATH DISTRIBUTION FOR A SPECIFIC CAUSE

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SUMMARY

The theory of competing risks is briefly reviewed. *Age at death distribution* is defined for a (hypothetical) population of individuals who are liable to die only from one specific cause (Section 1). A general model of *mixture* of age at death distribution for a joint survival function is introduced (Section 2), and the likelihood function (for estimating parameters) is constructed (Section 3). Of special concern is a model which represents a mixture of *two* survival functions associated with two competing causes: a specific cause (C_1) with liability $\phi\ (0 < \phi \le 1)$ to die from C_1, and 'all other causes' (C_2) with liability 1 (Section 4). Since the joint survival function cannot be uniquely determined from mortality data alone, it is further assumed that the force of mortality of one causes in presence of the other cause is the same as if the other cause were *ignored*. Since C_2 represents a group of many causes, this assumption might be a fair approximation to some situations. It is, in fact, used in actuarial work. Constructing single decrement life table for a specific cause (C_1) from multiple decrement life table, we obtained the marginal distribution, the proportion ϕ, and the age at death distribution for this cause (Section 5). An example for cancer mortality, using the US life tables 1959-61 is presented.

Key words: Competing risks; Mixture of failure distributions; Age at death distribution.

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1. INTRODUCTION

1.1. Suppose that k competing causes of death, $C_1, C_2, \ldots, C_k$, say, are operating in a population. Let $X_1, X_2, \ldots, X_k$ denote (hypothetical) 'times to die' from $C_1, C_2, \ldots, C_k$ respectively.

Let

$$S_{1,\ldots k}(x_1, \ldots, x_k) = \Pr\left\{\bigcap_{\alpha=1}^{k} (X_\alpha > x_\alpha)\right\} \tag{1.1}$$

be the corresponding joint survival function, and

$$S_{\alpha\cdot}(x_\alpha) = S_{1\ldots k}(0, \ldots, x_\alpha, \ldots, 0) \tag{1.2}$$

be the *marginal* survival function. ($F_{\alpha\cdot}(x_\alpha) = 1 - S_{\alpha\cdot}(x_\alpha)$ is the corresponding marginal failure distribution.)

In the theory of competing risks, it is usually assumed that *each death is due to one (so called 'underlying') cause*. Then the time at death, X, say, is

$$X = \min(X_1, X_2, \ldots, X_k) \tag{1.3}$$

and the (observable) overall joint survival function is

$$S_X(x) = S_{1\ldots k}(x, x, \ldots, x) , \tag{1.4}$$

with the force of mortality (hazard rate)

$$a\mu_X(x) = -\frac{dS_X(x)}{dx} \Big/ S_X(x) . \tag{1.5}$$

We can also observe the cause of death.

The (crude) probability of eventually dying from cause C_α *in presence* of all other causes is

$$P_\alpha(x) = \int_x^\infty a\mu_\alpha(t) S_X(t)dt \ , \tag{1.6}$$

where

$$a\mu_\alpha(x) = - \frac{\partial S_{1\ldots k}(x_1,\ldots,x_k)}{\partial x_\alpha} \Bigg|_{\{x_j = x\}} \Bigg/ S_X(x) \ . \tag{1.7}$$

is so called 'crude' hazard rate for cause C_α.

It follows that

$$a\mu_X(x) = a\mu_1(x) + a\mu_2(x) + \ldots + a\mu_k(x) \ , \tag{1.8}$$

Note that

$$\pi_\alpha = P_\alpha(0) = \int_0^\infty a\mu_\alpha(t) S_X(t)dt \tag{1.9}$$

is the proportion of those who died from C_α *in presence* of all other causes, and

$$aS_\alpha(x) = \frac{1}{\pi_\alpha} P_\alpha(x) \tag{1.10}$$

is the (proper) survival function for cause C_α among those who eventually die from C_α in presence of all other causes, that is, the conditional survival function given that death is ultimately due to C_α.

1.2. Because of the condition (1.3), it appears that the joint survival function (1.1) cannot be uniquely identified from mortality data alone: to each model with *dependent* 'times to die' there exists a model with *independent* 'times to die' both yielding the same likelihood (Tsiatis (1975)). In other words, there exists another joint survival function

$$S^*_{1\ldots k}(x_1,x_2,\ldots,x_k) = S^*_{1\cdot}(x_1)S^*_{2\cdot}(x_2)\ldots S^*_{k\cdot}(x_k) \text{ , say,} \qquad (1.11)$$

for which the marginal hazard rates, $\mu^*_{\alpha\cdot}(x)$, satisfy the condition

$$\mu^*_{\alpha\cdot}(x) = a\mu_\alpha(x) \ , \quad \alpha = 1,2,\ldots,k \ , \qquad (1.12)$$

and which, yields the same $S_X(x)$ and $P_\alpha(x)$'s as the joint survival function defined in (1.1). In our further discussion, we assume then that 'times to die' are *independent*.

1.3. Imagine an isolated (hypothetical) population in which C_α is the only cause of death. Let

$$S_\alpha(x) = \Pr\{X_\alpha > x\} \qquad (1.13)$$

be the survival function in this population, and

$$\mu_\alpha(x) = -\frac{dS_\alpha(x)}{dx} \ S(x) \qquad (1.14)$$

be the corresponding force of mortality.

We define the failure distribution

$$F_\alpha(x) = 1 - S_\alpha(x) \qquad (1.15)$$

as the *age at death distribution from cause* C_α *acting alone*. This is, in fact, the distribution of age of 'hypothetical' deaths from C_α.

1.4. In competing risk theory, it is usually assumed that each individual is liable to die from any of the k causes. Therefore, under the additional assumption of independence of 'times to die', we have

$$S_{1\ldots k}(x_1,x_2,\ldots,x_k) = S_1(x_1)S_2(x_2)\ldots S_k(x_k) \ , \qquad (1.16)$$

so that the marginal distributions represent the age at death distributions.

In real populations, however, some individuals are often more prone to die from one (or few) specific cause(s), but have low risk of dying from other causes. The population can be regarded as a *mixture* with respect to risk (liability) to die from different causes.

The topic of the present paper is the estimation of age at death distribution from a specific cause from mortality data of a population which is heterogeneous with respect to liability of dying from different causes.

2. MIXTURE OF SURVIVAL FUNCTIONS

2.1. In an ordinary (human) population, the number of all possible causes is vast. For convenience of the argument, we confine ourselves to three causes: two specific causes C_1 and C_2, and the third C_3, which denotes 'all other causes' except C_1 or C_2. Extension to k causes is straightforward.

Consider a population in which C_1, C_2 and C_3 are operating simultaneously. Since everybody must die sometime, it seems reasonable to assume that everybody is liable to die from 'other causes' (C_3). However, for the specific causes, we assume that only a proportion ϕ_1 is liable to die from C_1 (of course, as well as from C_3); a proportion ϕ_2 from C_2 and C_3; and a proportion ϕ_{12} from C_1, C_2 and C_3, while the remaining proportion $1 - (\phi_1 + \phi_2 + \phi_{12})$ of individuals could die from C_3 only. Further, we assume that (conditional on liability) the 'times to die', X_1, X_2, X_3, are independent. Therefore, the joint survival function for this population is a *mixture*

$$S_{123}(x_1,x_2.x_3) = [\phi_1 S_1(x_1) + \phi_2 S_2(x_2) + \phi_{12} S_1(x_1) S_2(x_2) + (1-\phi_1-\phi_2-\phi_{12})] S_3(x_3) \ . \tag{2.1}$$

We may also write (2.1) in the form

$$S_{123}(x_1,x_2,x_3) = [1 - (\phi_1+\phi_{12})F_1(x_1) - (\phi_2+\phi_{12})F_2(x_2) + \phi_{12}F_1(x_1)F_2(x_2)][1 - F_3(x_3)] \ . \tag{2.1a}$$

The marginal survival distributions are

$$\begin{aligned} S_{1\bullet}(x) &= S_{123}(x,0,0) = 1-(\phi_1+\phi_{12})F_1(x) , \\ S_{2\bullet}(x) &= S_{123}(0,x,0) = 1-(\phi_2+\phi_{12})F_2(x) , \\ S_{3\bullet}(x) &= S_3(x) = 1-F_3(x) \ . \end{aligned} \tag{2.2}$$

Of course, in the general case

$$S_{123}(x_1,x_2,x_3) \neq S_{1\bullet}(x_1)S_{2\bullet}(x_2)S_{3\bullet}(x_3) \ , \tag{2.3}$$

so that *unconditionally*, X_1, X_2, X_3 are *not independent*.

Note that

$$S_{1\bullet}(\infty) = 1-(\phi_1+\phi_{12}) > 0 \quad \text{and} \quad S_{2\bullet}(\infty) = 1-(\phi_2+\phi_{12}) > 0 \ , \tag{2.4}$$

that is, $S_{1\bullet}(x)$ and $S_{2\bullet}(x)$ are *improper* distributions.

2.2. Denoting by $f_\alpha(x) = -\dfrac{dS_\alpha(x)}{dx}$ the probability density function of the random variable X_α, we express (from (1.7)) the 'crude' hazard rate $a\mu_1(x)$ in the form

$$a\mu_1(x) = \frac{f_1(x)[\phi_1+\phi_{12}S_2(x)]}{1-(\phi_1+\phi_{12})F_1(x)-(\phi_2+\phi_{12})F_2(x)+\phi_{12}F_1(x)F_2(x)} . \tag{2.5}$$

A similar expression is obtained for $a\mu_2(x)$ by exchanging the subscripts 1 and 2, while

$$a\mu_3(x) = -\frac{dS_3(x)}{dx} \Big/ S_3(x) . \tag{2.6}$$

2.3. Special cases. Let E_α denote the event that an individual is liable to die from C_α $(\alpha = 1,2,3)$. We have

$$\Pr\{E_1\} = \phi_1 + \phi_{12} \; ; \quad \Pr\{E_2\} = \phi_2 + \phi_{12} \; ; \quad \Pr\{E_3\} = 1 . \tag{2.7}$$

We may identify three special cases:

(i) *The events* E_1 *and* E_2 *are mutually exclusive.*

In this case, $\phi_{12} = 0$, so that (2.1a) takes the form

$$S_{123}(x_1,x_2,x_3) = [1 - \phi_1F_1(x_1) - \phi_2F_2(x_2)][1 - F_3(x_3)] . \tag{2.8}$$

(ii) *The events* E_1 *and* E_2 *are independent.*

Put $\phi_1 + \phi_{12} = \gamma_1$, and $\phi_2 + \phi_{12} = \gamma_2$. In view of independence of events E_1 and E_2 we have $\phi_{12} = \gamma_1\gamma_2$, so that $\phi_1 = \gamma_1(1-\gamma_2)$ and $\phi_2 = \gamma_2(1-\gamma_1)$. The joint survival function (2.1a) takes the form

$$S_{123}(x_1,x_2,x_3) = [1 - \gamma_1F_1(x_1)][1 - \gamma_2F_2(x_2)][1 - F_3(x_3)] . \tag{2.9}$$

This model has been discussed by Hoel (1972). Note that (2.9) does not necessarily coincide with $S^*_{123}(x_1,x_2,x_3)$ defined in (1.11).

(iii) *Each individual is liable to die from any cause.*

Finally, if we assume $\gamma_1 = \gamma_2 = 1$ (or equivalently $\phi_1 = \phi_2 = 0$ and $\phi_{12} = 1$), the joint survival function (2.1a) takes the form

$$S_{123}(x_1,x_2,x_3) = S_1(x_1)S_2(x_2)S_3(x_3) \ . \tag{2.10}$$

3. ESTIMATION OF AGE AT DEATH DISTRIBUTION: PARAMETRIC APPROACH

Suppose that we have a sample of n individuals followed until all had died. Let n_α denote the number of deaths from cause C_α, and $x_{\alpha j}$ be the time at death of the jth individual who died from cause C_α *in presence* of all other causes. The corresponding likelihood is

$$L_\alpha \sim \prod_{j=1}^{n_\alpha} Q'_\alpha(x_{\alpha j}) = \prod_{j=1}^{n_\alpha} a\mu_\alpha(x_{\alpha j})S_X(x_{\alpha j}) \ , \tag{3.1}$$

where

$$Q_\alpha(t) = \pi_\alpha - P_\alpha(x) = \int_0^x a\mu_\alpha(t)S_X(t)dt \ . \tag{3.2}$$

The overall likelihood is

$$L = \prod_{\alpha=1}^{k} L_\alpha \ . \tag{3.3}$$

Likelihood for grouped data can be obtained using the probabilities of dying in specified intervals (multinomial).

Of course, in view of nonidentifiability of the joint survival function, we are not sure whether our estimates of $S_\alpha(x)$'s are correct, even if the model fits the data well, without giving reasons for assuming a special parametric form of joint survival function. (See Section 1.2.)

4. SURVIVAL MODEL WITH TWO CAUSES OF DEATH. NONPARAMETRIC APPROACH

4.1. Nonparametric estimation of *marginal* survival functions can be derived only for models based on independence of (unconditional) 'times to die'. For example, in our case of $k = 3$ causes, with incomplete liability, only model (2.9) can be considered, for the remaining models only bounds can be obtained (Peterson (1975)).

In deriving the nonparametric estimates it is convenient to distinguish only two causes; the specific cause C_1, say, and the 'other causes' (except C_1), C_2, say. Our model (2.1) takes now the form

$$S_{12}(x_1,x_2) = [\phi S_1(x_1) + (1-\phi)]S_2(x_2) \tag{4.1}$$

or

$$S_{12}(x_1,x_2) = [1 - \phi F_1(x_1)][1 - F_2(x_2)] \ , \tag{4.1a}$$

where ϕ $(0 < \phi < 1)$ is the proportion of individuals who are liable to die from C_1 as well as from C_2, while the proportion $(1-\phi)$ is 'susceptible' only to 'other causes', C_2.

It is worthwhile to notice that (4.1) resembles a model discussed by Berkson and Gage (1952). In their problem, the population consists of patients who had an onset of cancer, and $(1-\phi)$ (= c in their notation) corresponds to patients who are 'cured', while ϕ (= $1-c$) are those who died from cancer.

4.2. By life table techniques, we can only obtain the *marginal* distribution, $S_{\alpha\cdot}(x)$ $(\alpha = 1,2)$.

For the specific cause, C_1, we have

$$S_{1\cdot}(x) = 1 - \phi F_1(x) \quad . \tag{4.2}$$

This implies that the proportion of those who are liable to die from C_1 is

$$\phi = 1 - S_{1\cdot}(\infty) \quad . \tag{4.3}$$

Therefore, the *age at death distribution*, $F_1(x)$, can be obtained from the formula

$$F_1(x) = \frac{1-S_{1\cdot}(x)}{1-S_{1\cdot}(\infty)} \ , \tag{4.4}$$

and

$$S_1(x) = \frac{S_{1\cdot}(x)-S_{1\cdot}(\infty)}{1-S_{1\cdot}(\infty)} \ . \tag{4.4a}$$

For 'other causes' C_2, we have, of course,

$$F_2(x) = 1 - S_{2\cdot}(x) = F_{2\cdot}(x) \quad . \tag{4.5}$$

The force of mortality of the *marginal* (improper) survival function $S_{1\cdot}(x)$, is

$$\mu_{1\cdot}(x) = \frac{\phi\, f_1(x)}{1-\phi F_1(x)} = \frac{f_1(x)}{\frac{1-\phi}{\phi} + S_1(x)} \ , \tag{4.6}$$

while the force of mortality of $S_1(x)$ is

$$\mu_1(x) = f_1(x)/S_1(x) \quad . \tag{4.7}$$

Of course,

$$\mu_{1\cdot}(x) \le \mu_1(x) \ , \tag{4.8}$$

4.3. When the data are complete, and the individual times at death, $t_{\alpha j}$ (α = 1,2), are recorded, the *product-limit* estimate (often called the Kaplan-Meier estimate) can be used for estimating the marginal distributions $S_{\alpha\bullet}(x)$. Examples with this kind of data are given by Hoel (1972).

From population data, we construct the multiple decrement life tables. It is customary to construct a single decrement life table associated with a given multiple decrement life table, where a specific cause (C_1) is *eliminated*. This is equivalent to calculating the marginal (proper) distribution $S_{2\bullet}(x)$ of model (4.1). Of course, one may also construct by the same method the marginal distribution $S_{1\bullet}(x) = 1 - \phi F_1(x)$, and then the age at death distribution, $F_1(x)$. This problem is discussed in the next section.

5. APPLICATION TO POPULATION MORTALITY DATA: LIFE TABLES

It is assumed that the reader is familiar with terminology of multiple decrement life tables. Notation is, in fact, not the same in different countries. In the present paper, we try to conform to English notation, but using subscripts rather than superscripts (e.g. $al_{\alpha x}$ instead of $al_x^{(\alpha)}$, etc). We also assume that the survival functions are the 'true' functions, since the life tables are models, not data.

5.1. *Overall survival function,* $S_X(x)$.

The al_x column gives the number of survivors at exact age x out of al_0 newborns. Thus we have

$$S_X(x) = al_x/al_0 \ . \tag{5.1}$$

5.2. *The survival function for* C_1 *in presence of all other causes*, $aS_1(x)$.

Multiple decrement life tables given also columns of $ad_{\alpha x}$ — the number of life table deaths between age x to $x+1$ from cause C_α. Using the $ad_{\alpha x}$ column, we can calculate $al_{\alpha x} = \sum_{y=0}^{x} ad_{\alpha y}$ — the number of individuals present age x who eventually die from C_α. The crude probability, $P_\alpha(x) = al_{\alpha x}/al_0$. Hence $\pi_\alpha = al_{\alpha 0}/al_0$, and the survival function for those who died from C_α *in presence* of all other causes is

$$aS_\alpha(x) = \frac{1}{\pi_\alpha}\frac{al_{\alpha x}}{al_0} = \frac{al_{\alpha x}}{al_{\alpha 0}} . \tag{5.2}$$

5.3. *Age at death distribution*, $F_1(x)$.

Consider two causes, C_1 (specific), and C_2 ('other causes'). Let ℓ_{1x} denote the number of survivors from C_1 in a population in which other causes are *ignored*. Thus $S_{1\cdot}(x) = \ell_{1x}/\ell_{10}$ represents the *marginal* survival function for cause C_1.

Let $ad_x = al_x - al_{x+1}$ denote the number of deaths between age x to $x+1$, and $ad_{2x} = ad_x - ad_{1x}$ — the number of deaths from 'all other' causes (C_2).

The crude conditional probabilities of dying from all causes, from C_1, and from C_2 *in presence* of all causes acting simultaneously, are respectively

$$aq_x = ad_x/al_0 \ ; \quad aq_{\alpha x} = ad_{\alpha x}/al_0 \ , \quad \alpha = 1,2 \ . \tag{5.3}$$

Let q_{1x} denote the conditional probability of dying from C_1 when C_2 is *ignored*.

(i) One method of estimating q_{1x} is to consider, deaths ad_{2x} as if they were *withdrawals*. Thus

$$q_{1x} \doteqdot \frac{ad_{2x}}{al_x - \frac{1}{2}\,ad_{2x}} = \frac{aq_{1x}}{1 - \frac{1}{2}\,aq_{2x}} , \tag{5.4}$$

(e.g. see Jordan (1967), p. 279.)

(ii) There are some other approximations to q_{1x}. Among these the following approximate formula was used in constructing the US Life Tables 1959-61 (1968)

$$q_{1x} \doteqdot aq_{1x}\,\frac{1 - \frac{1}{2}\,aq_{2x}}{1-aq_{2x}} . \tag{5.5}$$

Then ℓ_{1x} is calculated from the formula

$$\ell_{1x} = \ell_{10}(1-q_{10})(1-q_{11})\ldots(1-q_{1,x-1}) , \tag{5.6}$$

where $\ell_{10} = 10^5$ is the commonly chosen radix.

The marginal survival function, $S_{1\bullet}(x)$ is then

$$S_{1\bullet}(x) = 1 - \phi F_1(x) = \ell_{1x}/\ell_{10} . \tag{5.7}$$

Hence (from (4.3))

$$\phi = 1 - \ell_{1\infty}/\ell_{10} = (\ell_{10} - \ell_{1\infty})/\ell_{10} . \tag{5.8}$$

The age at death distribution, $F_1(x)$, is from (4.4)

$$F_1(x) = \frac{1-S_{1\bullet}(x)}{1-S_{1\bullet}(\infty)} = \frac{\ell_{10}-\ell_{1x}}{\ell_{10}-\ell_{1\infty}} , \tag{5.9}$$

and

$$S_1(x) = 1 - F_1(x) = \frac{\ell_{1x}-\ell_{1\infty}}{\ell_{10}-\ell_{1\infty}} . \tag{5.9a}$$

In a similar way we derive $S_{2\bullet}(x)$. In fact, we have $S_2(x) = S_{2\bullet}(x)$.

Notice that we should have $\phi > \pi_1$, because there should be more individuals susceptible to a specific cause than individuals who actually die from this case.

5.4. *Survival function in a population with additional risk.*

In many investigations of survivorship, and especially in clinical trials, it is important to know the mortality pattern (life table) of the population exposed to the *additional risk* of dying from a specific cause such as, for example, cancer, diabetes, tuberculosis, etc.

Follow up studies of such populations are difficult and expensive; usually the population represents a mixture of persons of different ages; patients enter the study at various time points; and it takes a long time (and high cost) to follow each patient until death. It seems that a survival function of the form

$$S_X^{\#}(x) = S_1(x)S_2(x) \tag{5.10}$$

might sometimes be a fair approximation to the survival function for such a population. It can be interpreted as the survival function among individuals who all are exposed to risk of dying from a specific cause (C_1) as well as from other causes (C_2).

Clearly, we will have

$$S_X^{\#}(x) \leq S_X(x) \ , \tag{5.11}$$

where $S_X(x)$ is defined in (5.1).

In life table notation, we have

$$S_X^{\#}(x) = \frac{\ell_{1x}-\ell_{1\infty}}{\ell_{10}-\ell_{1\infty}} \cdot \frac{\ell_{2x}}{\ell_{20}} \ . \tag{5.12}$$

For the method to be efficient, it is rather important that the complete life tables be as complete as possible. Unfortunately, most available multiple decrement life tables are abridged and, what is worse, the last recorded age is very often only 85. The coefficient ϕ is rather sensitive to the greatest age x, for which a value of aq_{1x} is given and clearly age 85 is not sufficiently old age.

However, the US multiple decrement life tables 1959-61 have some columns extending to age 100, still having five year intervals. This should lead to somewhat a better estimate of ϕ though obviously not the best.

EXAMPLE. We apply now the method to *cancer mortality*, using the US multiple decrement life tables 1959-61 (1968) for *White Males*.

Columns 2-4 of our Table 1 are extracted from these tables. Of course, we are now using the notation for the abridged life tables: ${}_{n}ad_{x}$ instead of ad_{x}, etc. The remaining columns (6-12) exhibit various survival functions which have been discussed in this section.

Note that $\pi_1 = P_1(0) = 0.1526$ is the proportion of White Males who ultimately die from cancer in presence of all other causes acting in the population, while $\phi = 1-S_{1\cdot}(\infty) = 1-0\cdot 2358 = 0.7642$ is the proportion of Whilte Males who were under additional risk of dying from cancer.

Figures 1A through 4A represent the following survival functions for White Males: the overall survival functions, $S_X(x)$ (Fig. 1A); the

TABLE 1

Various 'survival' distributions associated with cancer (U.S. Life Tables 1959-61, White Males)

Age group x to x+n	al_x	$_{n}ad_{x}$	$_{n}ad_{1x}$	$_{n}ad_{2x}$	$S_X(x)$	$P_1(x)$	$aS_1(x)$	$S_{1.}(x)$	$S_1(x)$	$S_2(x)$	$S^{\#}_X(x)$
0-1	10,000,000	259,169	723	258,446	1.0000	.1526	1.0000	1.0000	1.0000	1.0000	1.0000
1-5	9,740,831	39,340	4,832	34,508	.9741	.1525	.9995	.9999	.9999	.9742	.9741
5-10	9,701,491	25,684	4,548	21,136	.9701	.1520	.9964	.9994	.9993	.9707	.9700
10-15	9,675,807	25,512	3,522	21,990	.9676	.1515	.9934	.9990	.9986	.9686	.9673
15-20	9,650,295	59,524	4,557	54,967	.9650	.1512	.9911	.9986	.9982	.9664	.9646
20-25	9,590,771	80,195	5,518	74,677	.9591	.1507	.9881	.9981	.9975	.9609	.9585
25-30	9,510,576	70,516	7,333	63,183	.9511	.1502	.9845	.9975	.9968	.9534	.9503
30-35	9,440,060	81,201	9,942	71,259	.9440	.1495	.9797	.9968	.9958	.9471	.9431
35-40	9,358,859	116,121	15,388	100,733	.9359	.1485	.9731	.9957	.9944	.9399	.9346
40-45	9,242,738	189,484	27,634	161,850	.9243	.1469	.9631	.9941	.9922	.9298	.9226
45-50	9,053,254	310,867	51,293	259,574	.9053	.1442	.9449	.9911	.9883	.9135	.9028
50-55	8,742,387	496,077	92,672	403,405	.8742	.1390	.9113	.9854	.9809	.8872	.8702
55-60	8,246,310	697,839	140,748	557,091	.8246	.1298	.8506	.9747	.9669	.8461	.8180
60-65	7,548,471	965,096	197,779	767,317	.7548	.1158	.7583	.9574	.9443	.7884	.7445
65-70	6,583,375	1,200,840	235,224	965,616	.6583	.0959	.6287	.9309	.9096	.7072	.6433
70-75	5,382,535	1,361,796	241,044	1,120,752	.5383	.0724	.4745	.8948	.8624	.6015	.5187
75-80	4,020,739	1,421,412	214,255	1,207,157	.4021	.0483	.3165	.8495	.8030	.4733	.3801
80-85	2,599,327	1,292,853	159,413	1,133,440	.2599	.0269	.1760	.7945	.7311	.3272	.2392
85-90	1,306,474	846,467	80,748	765,720	.1306	.0109	.0715	.7269	.6427	.1799	.1156
90-95	460,007	364,365	24,257	340,108	.0460	.0028	.0186	.6502	.5423	.0710	.0385
95-100	95,642	84,129	3,813	80,316	.0095	.0004	.0027	.5673	.4338	.0170	.0074
100+	11,513	11,513	327	11,186	.0012	.0000	.0002	.4854	.3266	.0024	.0008
∞					.0000	.0000	.0000	.2358	.0000	.0000	.0000

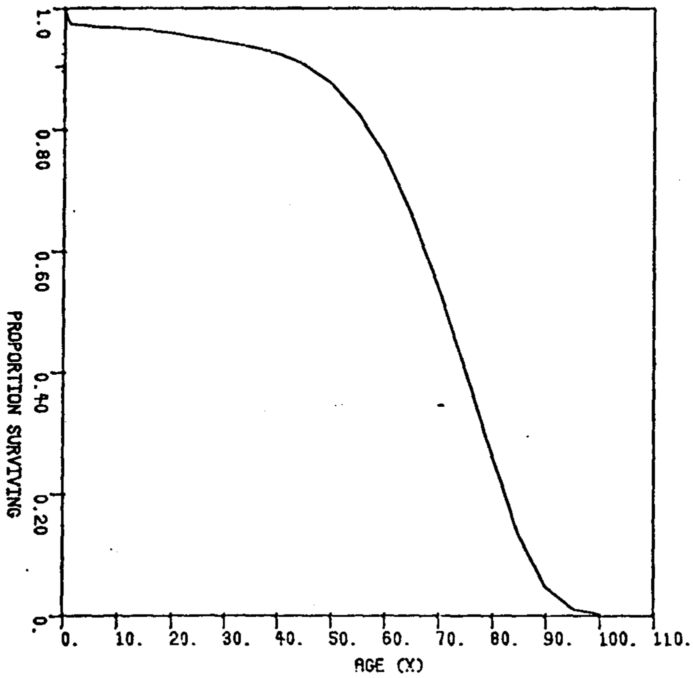

Fig. 1A. Survival function $S_X(x)$

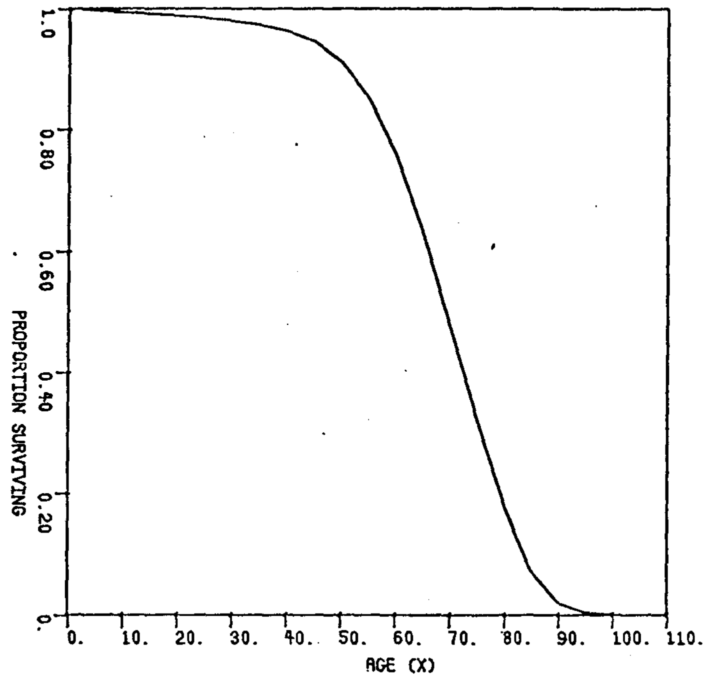

Fig. 2A. Survival function $aS_1(x)$

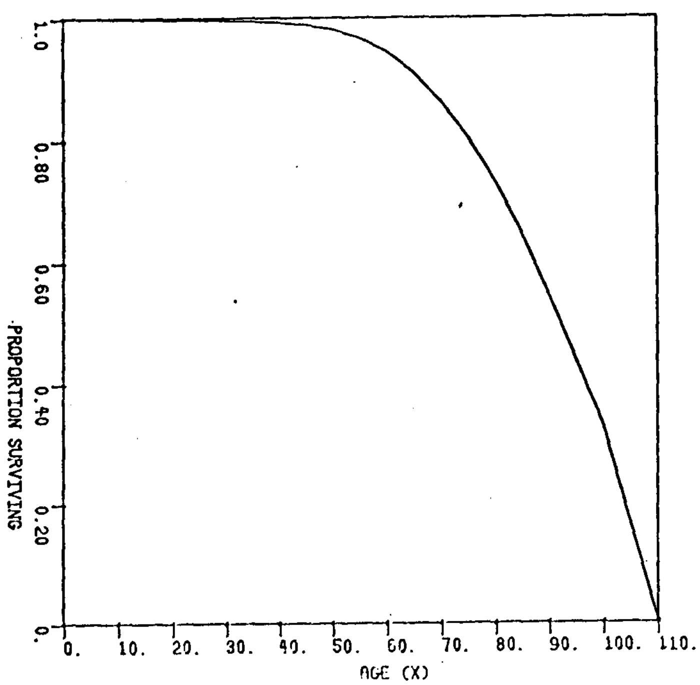

Fig. 3A. Survival function $S_1(x)$

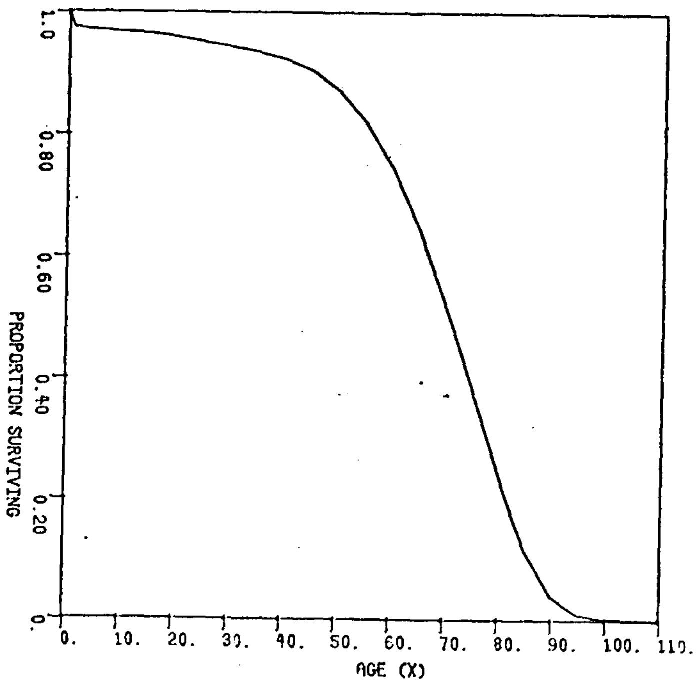

Fig. 4A. Survival function $S_X^{\#}(x)$

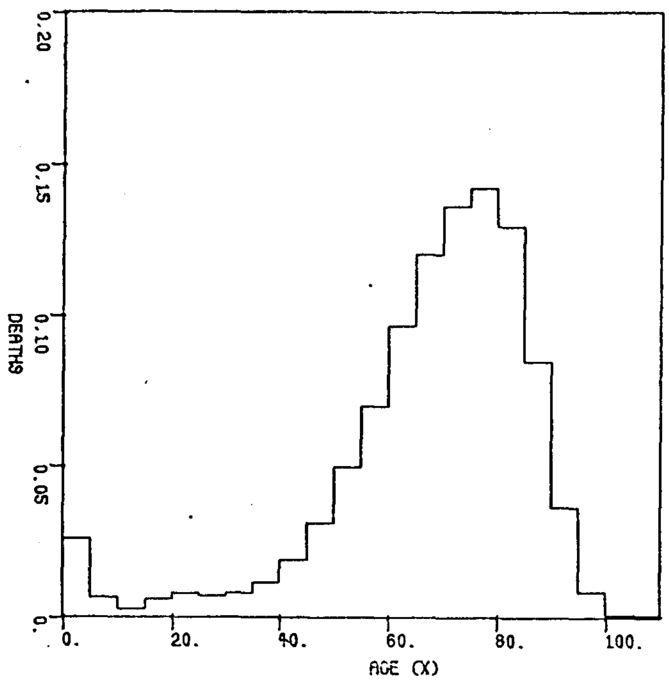

Fig. 1B. 'Curve of deaths' $H_X(x)$

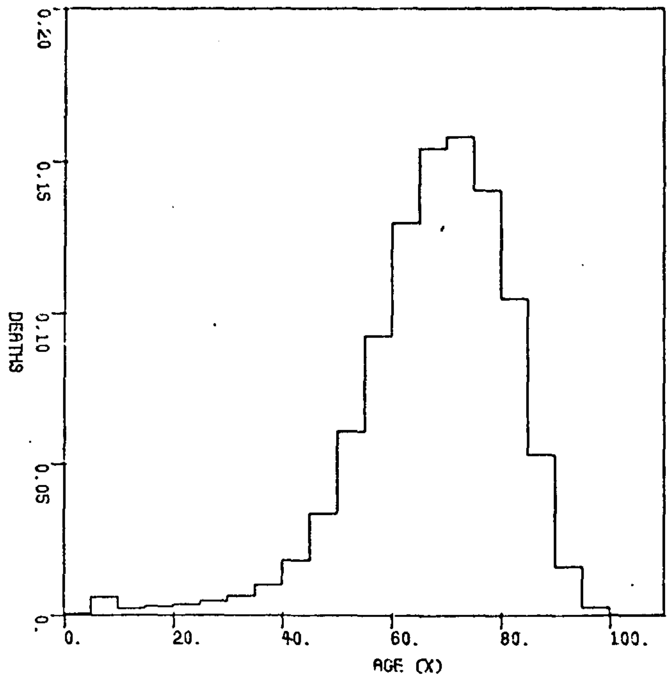

Fig. 2B. 'Curve of deaths' $aH_1(x)$

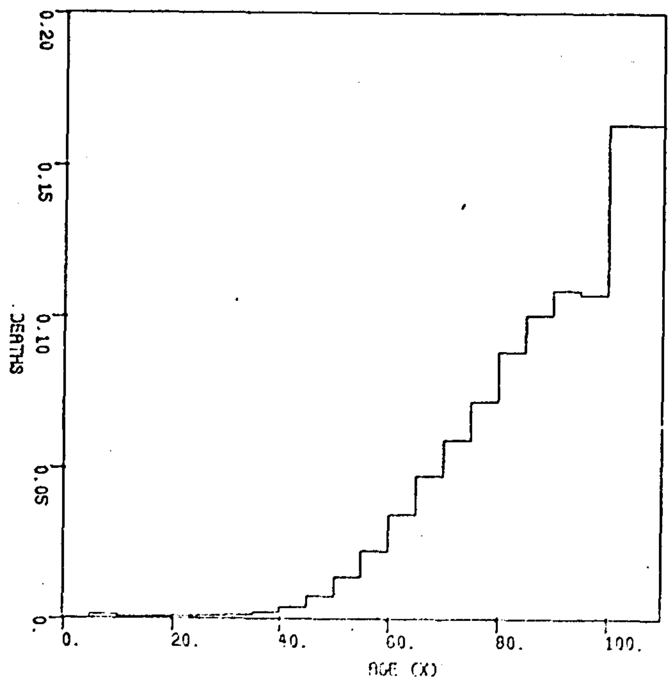

Fig. 3B. Age at death distribution $H_1(x)$

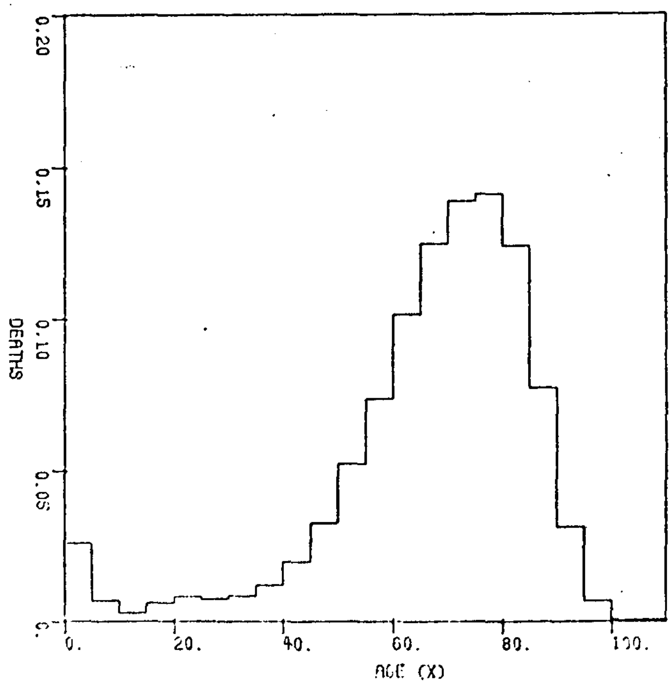

Fig. 4B. 'Curve of deaths' $H^{\#}_X(x)$

survival function of those who eventually die from cancer in presence of all other causes, $aS_1(x)$ (Fig. 2A); the survival function from cancer acting alone, $S_1(x)$ (Fig. 3A); — this is the complement to the age at death distribution from cancer, $F_1(x)$; finally, the survival function among all those who are subject to the additional risk of dying from cancer, $S_X^{\#}(x)$ (Fig. 4A).

Figures 1B through 4B are the corresponding proportionate distributions of deaths in the form of histograms. We call them 'curves of deaths' and, for simplicity, they are denoted as: $H_X(x)$, $aH_1(x)$, $H_1(x)$ and $H_X^{\#}(x)$, respectively.

We notice that the overall survival function $S_X(x)$ (Fig. 1A) and the survival function with additional risk of cancer $S_X^{\#}(x)$ (Fig. 4A) are similar. (Only a slight difference is noticeable for ages above 60.)

The corresponding 'curves of deaths' are also similar (Figures 1B and 4B).

Figures 2A and 3A (and also figures 2B and 3B) describe the mortality from cancer in *presence* and in *absence* of other causes, respectively. There are remarkable differences between $aS_1(x)$ (Fig. 2A) and $S_1(x)$ (Fig. 3A), which become more apparent in the corresponding graphs of $aH_1(x)$ (Fig. 2B) and $H_1(x)$ (Fig. 3B). The peak of deaths from cancer *in presence* of other causes is between ages 65 to 75 (Fig. 2B), while in their *absence* is beyond age 100 (Fig. 3B). This indicates that a fairly high proportion of White Males who would have died from cancer at older ages is not, in fact, observed, because they died from other causes before reaching extreme old age.

Of course, we should always keep in mind the assumptions under which multiple decrement life tables are constructed. Especially, we should be aware of the assumption $\mu_{\alpha\cdot}(x) = a\mu_{\alpha}(x)$ $(\alpha = 1,2)$, which cannot be tested (or proved) and on which the construction of single decrement tables and the derivation of age at death distributions from specific causes is based.

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