NONCONVEXITY AND THE COST OF HETEROGENEITY IN APPLE CROPS

by

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ABSTRACT

JOHNSON, ELIZABETH B. Nonconvexity and the cost of heterogeneity in apple crops. (Under the direction of H. G. GOLD.)

A return function was developed for the typical North Carolina Red and Golden Delicious apple crop. The data used for determining the grade distributions was collected in Henderson County over a period of four years as a part of the overall Integrated Pest and Orchard Management project. Using the apple grades from the IPOM project and price quotes from the USDA marketing news service a value per bushel of four grades relative to the highest valued grade (Large Tray Pack) was calculated. Size, culling factors and color (for Red Delicious) determine the grade of an apple. The sensitivity of the return function to changes in each of these 3 factors was determined. Results indicated the return function was most sensitive to changes in size.

At present the grower has two choices in marketing his crop. The grower may "pay" to have his crop graded at a packing house or he may have the entire crop processed. The cost of grading is such that a crop composed of approximately 45% fresh quality apples has the same return with either marketing choice. One potential marketing strategy would be for the grower to sort or to differentially pick his crop before marketing. He would then save the sorting

cost on process quality apples. Equations were developed describing the improved value such a practice would reap over the present strategies. Finally, future research on the cost of "presorting" and its associated efficiency was recommended.

BIOGRAPHY

Elizabeth Black Johnson was born in Pinehurst, North Carolina on April 24, 1954, the daughter of Clarence S. and Nita B. Black. She received her elementary and secondary schooling in Candor, N.C. and graduated from East Montgomery High School in June of 1972.

In August, 1974 the author entered Meredith College in Raleigh, North Carolina and in May, 1978 was awarded the Bachelor of Science degree in Biology.

In August of 1979 she entered North Carolina State University to pursue a program of graduate study in Operations Research.

The author is married to Charles S. Johnson.

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A special thanks is extended to the Biomathematics office-mates for their acceptance of and help to an "alien" Operations Research student. Above all the author would like to thank her husband, Chuck, and her parents, Nita and Clarence for their encouragement and support which made this possible.

CONTENTS

Chapter I

INTRODUCTION

A research group within the North Carolina State University Biomathematics program is formulating mathematical models describing the apple orchard as an hierarchical system. This effort is a part of the Integrated Pest and Orchard Management project (IPOM) and of a multi-university consortium for Integrated Pest Management (CIPM) to aid the grower in making management decisions.

1.1 BACKGROUND

A grower generally uses a management plan to maximize his net return. This plan is limited by the grower's knowledge and the nature of farm resources - available capital, labor and land. Its objective, the optimization of net economic return to the grower, can be described by an objective function, which is composed of two components - cost and return. This thesis deals with the return component of such an objective function.

It is often more complicated to describe quantitatively the return from a fruit crop than the return from other crops. While yield and quality determine the return from many crops, the quality of an apple crop is described by a distribution into quality classes or grades rather than by a single quality rating for the entire crop. Five such grades often used are: large tray pack (LTP), small tray pack (STP), bag (BG), slice and sauce (SS) and juice (JU). Apples are sorted into these different grades depending upon 3 factors: size (diameter), color (for Red Delicious) and cullage factors.

When apples are destined for the fresh market, the normal procedure is to deliver the apples to a packing house where they are sorted by grade. The cost of sorting is borne by the grower. The grower is then paid for the apples on the basis of quality distribution. Those apples which are classified as non-fresh or process quality (SS or JU) will be sold to an apple processor but will not bring enough economic return to pay for their sorting cost. That is, process quality apples delivered to a packing house result in a negative economic return. On the other hand the grower has the option of delivering all his apples to the processing plant. There he would be paid as if the entire load of apples were process quality.

The negative return on process quality apples delivered to a packing house creates a breakeven point at about 45% fresh quality apples. For such a mixture the farmer would get aproximately the same return by delivering the crop directly to the processing plant as he would by delivering the crop to a packing house. A graph of the indifference curve is therefore non-convex (Figure 1.1) as opposed to the more common convex indifference curve. This complicates the mathematical characterization of the return function.

A grower/manager makes the decision to market the apple crop as fresh or process after classifying his crop as being above or below the 45% breakeven point. Although at one time growers graded, packaged and marketed their crop, most growers now deliver the ungraded crop either to a packing house or to a processor. (1) It may be possible, however, with certain distributions of apple grades in a crop, that presorting of the crop at harvest time would increase the net return. This initial grading of the crop by the grower would enable the grower to market as fresh only apples graded fresh and to sell process quality apples only to the processing plants. One purpose of this project was to develop a measure of the quality of a crop from a given distribution of the factors which determine apple grades (size, color, and cullage factors) and to determine which, if any, of these distributions would make presorting or differentially picking a profitable management decision.

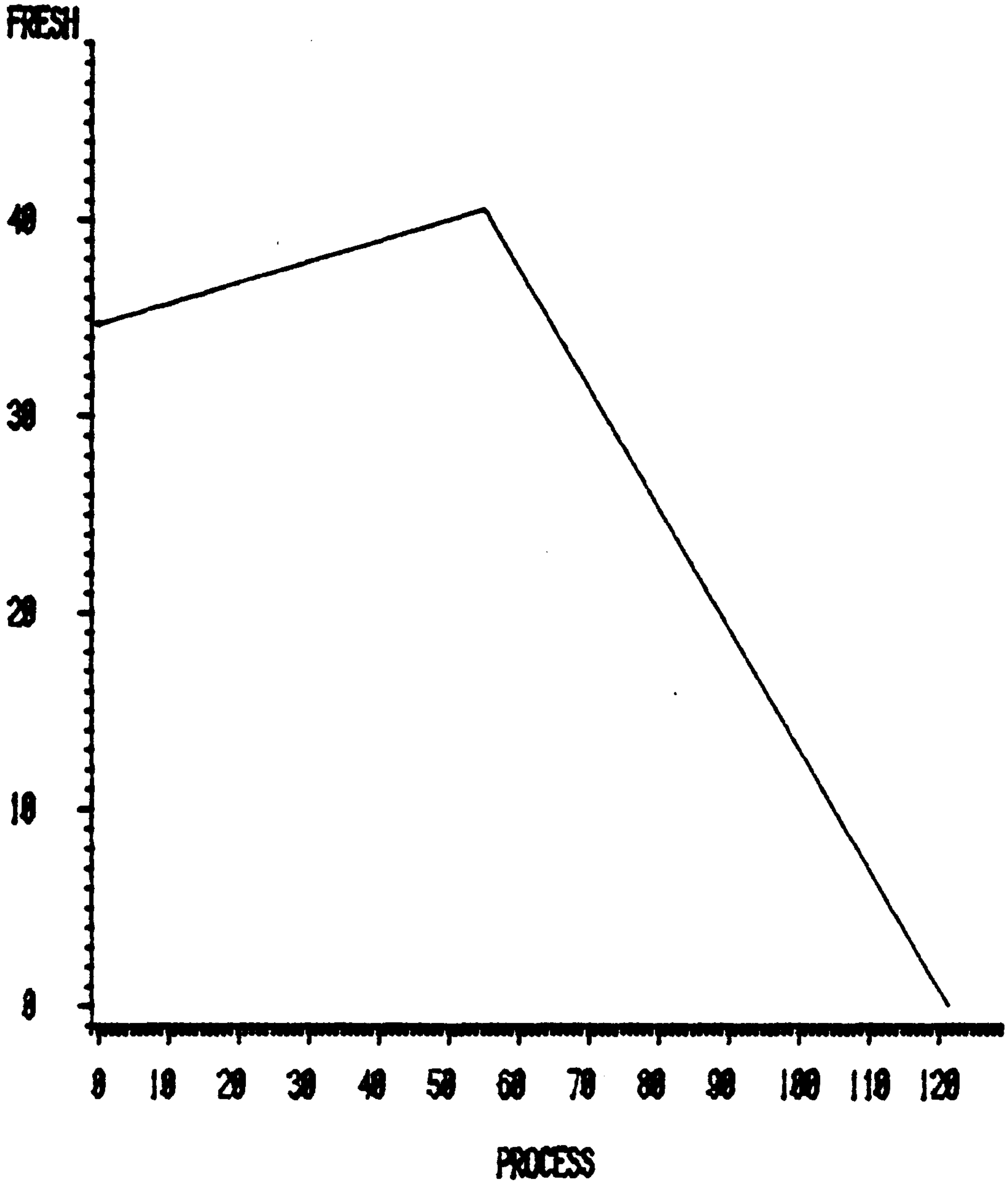

FIGURE 1.1: INDIFFERENCE CURVE IN BUSHELS.

1.2 SOURCE OF DATA

The data used was collected as a part of the overall NCSU IPOM project. (3) For this purpose portions of orchards blocks were chosen at random from an aerial map of Henderson County - North Carolina's major apple producing county. Sample trees within these blocks were randomly chosen. The data pertinent to this study were determined from 25 randomly selected apples collected at harvest from each sample tree by a team of trained technicians. The pertinent data collected on the quality of each sample apple were: diameter, cull (damage from insects, disease, or physical stress-present or not), bitter pit (present or not), cork spot (present or not), red color (percentage of apple surface area with "good" red color), and total color (percentage of apple surface area showing red or pink color). The total yield in bushels was recorded for each sample tree at the time of harvest.

Chapter II

DEVELOPING THE RETURN FUNCTION

For our purposes, an objective function mathematically defines the relationship between a variable (return) which the grower wishes to maximize and certain decision variables which may or may not be directly controllable by the grower. (4) For the model to be valid these decision variables must include those variables which significantly influence economic return. The model should also be flexible as well as accurate in order to use the model for prediction purposes.

The return function is one component in the objective function of a mathematical model. As part of an overall model the return function must meet the requirements for the overall model as well as describe the piece of reality that the return function is modelling. In general, models are vehicles for arriving at some structured view of reality. (5) The return function of an apple crop must represent approximately the actual return on a crop. To achieve this it must embody the characteristics of the real return of a marketed apple crop. This embodiment usually entails simplifications and numerous assumptions. Any simplifications or assumptions are made after evaluating the goals set for developing the model. Because the return function is the mathematical characterization of the economic return from a marketed apple crop it can be used to evaluate different orchard management practices. For the purpose of evaluating the profitability of presorting an apple crop while still in the orchard, a return function must consider the relative value of the apple grades and the relative costs of sorting the crop into the different grades.

2.1 RELATIVE VALUES OF GRADES

The return function of an apple crop provides a measure of crop quality which should be relatively independent of market fluctuations. The value of a given bushel of apples relative to the value of a bushel composed entirely of large tray pack (LTP) apples is less dependent on market fluctuations and therefore better to use for modelling purposes than, for example, the actual price received for a bushel of STP which varies more over the season and is therefore more dependent on market fluctuations. This relative value of a bushel of apples is also approximately related to the actual market value after adjustment using a linear economic index.

Marketing reports obtained from the North Carolina Department of Agriculture and the IPOM's quality and yield data were used to find the relative value of a bushel of each apple grade to that of a bushel of large tray pack (LTP). (2) After assuming the IPOM's harvest dates to be representative of harvest dates in North Carolina, the market prices quoted for the harvest dates were weighted by the yield in the individual grades as determined from the IPOM's yield and quality data.

The marketing reports were not consistent from year to year and certain assumptions were made to compensate. Of the four years 1976-1979 only 1977 and 1979 have complete reports, i.e., in that there are quotes for all five grades. For other years quotes were in price ranges, and the midpoint was used as the price for each harvest date. A general quote for tray pack (TP) was the only price quoted for the 1976 year. Using the midpoint of the TP range to separate the quote into LTP and STP ranges the midpoints for each of these sub-ranges were then used as the price for LTP and STP, respectively, for that date. The price range for 1978 was given for LTP as well as for a general TP. The STP prices for that year were estimated by subtracting the LTP range from the TP range then finding the midpoint of the remaining lower range.

The size classes of LTP were not consistent from year to year; LTP for 1977 Golden Delicious and 1979 Red Delicious were defined as 88-113 apples per bushel, otherwise as 88-100 apples per bushel. Since the market prices quoted were specific for these sizes, these sizes were used to create yield classes.

The IPOM's quality data was used to find the fraction of the yield belonging to each class.

Sample apples were classed into the five grades according to the following criteria:

FRESH (LTP,STP,BG):= 1. Free of culling factors.

2. Total color + Red Color 80.

3. Diameter at least 2.25

LTP:= Diameter = 2.78 (2.734)

STP:= 2.4687 = Diameter 2.78 (2.734)

BG := Diameter 2.4687

PROCESS (SS JU) := Does not meet 1-3 above.

SS:= Diameter = 2.5.

JU:= Diameter 2.5.

The following algorithm was used to find the relative value of STP, BG, SS, and JU to LTP:

1. $t \; (y_{td} \; (w_{td}/w_{tc})) = y_{cd}$.

2. $d \; (y_{td} \; (w_{td}/w_{tc})) = y_{cd}$.

where:

y_{td}:= yield in bushels of tree on a given harvest date.

w_{td}:= weight in grams of apples on tree on a given date.

w_{tc}:= weight on a given tree in a given class.

y_{cd}:= yield in bushels of a class on a given date.

P_{cd}:= price in dollars quoted for a class on a given date.

r_c := relative price forclass to LTP.

Table 2.1 shows the value/bushel relative to LTP for each year and Table 2.2 gives the averaged relative value.

YEAR		STP	BG	SS	JU
1976	Red	.9202	.6467	.2967	.1731
	Gold	.8809	.6786	.2974	.1735
1977	Red	.8935	.7173	.3516	.1820
	Gold	.9137	.6669	.3294	.1715
1978	Red	.9372	.7084	.2065	.1527
	Gold	.9040	.7096	.2082	.1540
1979	Red	.8577	.7559	.2235	.1512
	Gold	.9554	.8788	.3045	.1880

<u>Table 2.1</u> Yearly Relative Values per Bushel

	STP	BG	SS	JU
Red	.9022	.7073	.2696	.1648
Gold	.9135	.7355	.2849	.1715

<u>Table 2.2</u> Averaged Relative Value per Bushel

2.2 RELATIVE SORTING COST

When the crop is composed of 55% process and 45% fresh apples the value of the crop is the same whether sorted or all processed. The 45-55 point is an estimate based on consultation with individuals with extensive experience in apple production and marketing. It is at best an approximation and may vary throughout a given year and from place to place at any given time. We regard the chosen breakeven point as a reasonable estimate. This breakeven distribution was used to determine the sorting cost per bushel of fresh marketed apples relative to LTP using the distribution of classes from the IPOM's quality data and the relative values determined previously. The sorting cost was then determined by taking the difference in the value of a hypothetical crop of 55% process - 45% fresh quality apples and the value of that same distribution considered as entirely process quality.

The results obtained were: RED s=.281, GOLD s=.288. After finding these relative sorting costs and relative values for the 5 grades an average weighted by yield in the grades was found for both fresh and process apples after sorting. A weighted average was also found for process apples only. These values after sorting for Red Delicious variety only were:

FRESH: .576

PROCESS: -.064

Then for nonsorted apples the weighted average was:

PROCESS ONLY: .216

Looking at the negative return for selling the process apples to the packing house it is easy to see that the heterogeneity of an apple crop is costly to the grower. This cost of heterogeneity is defined as the difference in return for a crop that is sent to the packing house already graded and the return from that same crop sent ungraded. Using the IPOM's data this cost of heterogeneity is graphically illustrated in Figure 2.1. The clear area is the averaged relative return for the IPOM blocks for each year following the current marketing practices. The hashed area shows the increment of return that these same blocks could bring above the regular return if they were presorted before marketing.

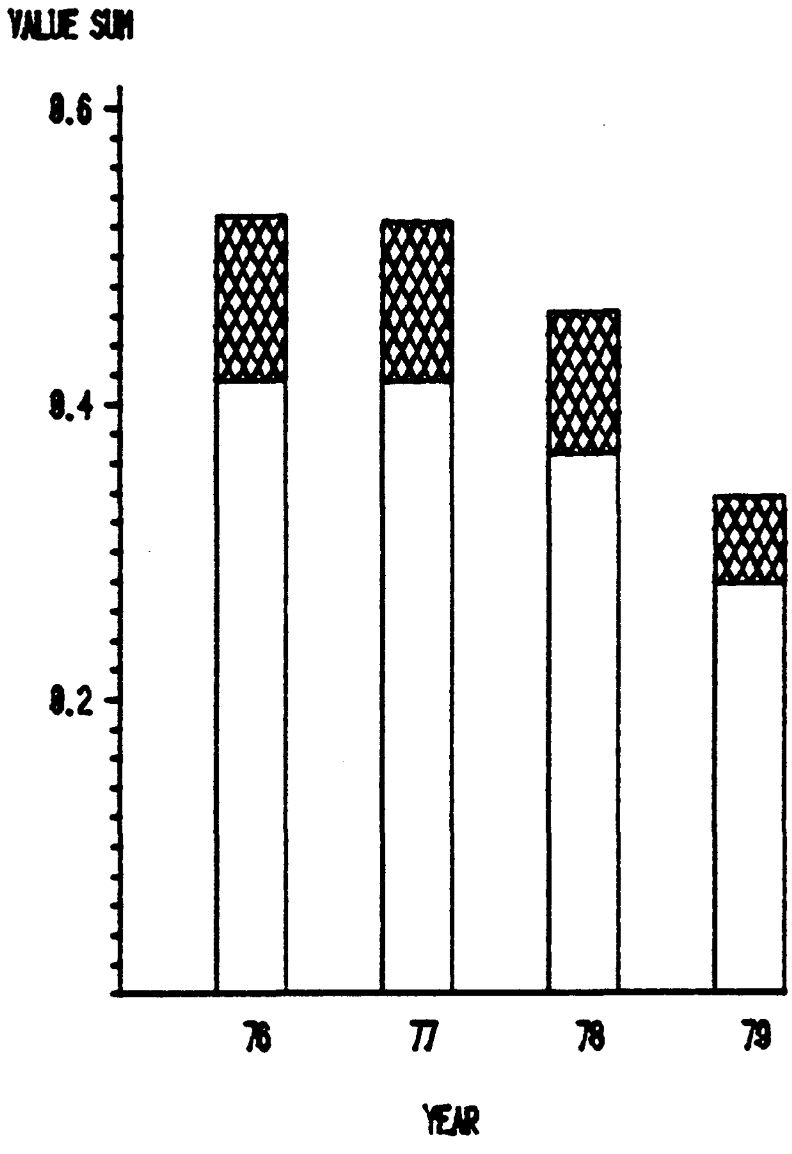

FIGURE 2.1: THE COST OF HETEROGENEITY.

2.3 SENSITIVITY OF THE RETURN FUNCTION

The relative value per bushel for any given distribution of the three quality determining factors may be determined once the value per bushel and the sorting cost for each quality grade relative to LTP is known. This relative value per bushel of different mixtures of the quality grades can be used to test the sensitivity of the return function to variation in fruit diameter, insect cullage and color (for Red Delicious). An analysis of the sensitivity of the return function to variation in the quality determining factors may suggest more efficient and profitable management strategies.

The quality determining factors may be partially controllable by management. A grower might change his insect pest management program to reduce the proportions of apples culled by insect damage in his crop and thereby increase his return. The capital which was expended in this spraying effort, however, might have increased the grower's return even more if it had been used to finance another aspect of crop production, such as more elaborate pruning or thinning of trees to improve the distributions of fruit color or size in the crop.

One method of determining the sensitivity of the return functon to changes in diameter, color and cull is to fix two factors and vary the third by a constant amount. We simulated an increase in each of the three quality determining factors while holding the other two factors constant using the IPOM's sample blocks. A means of increasing these factors without altering any other value determining factors was determined. To increase the diameter without altering size or color, we shifted the size distribution of the entire crop so that 10% of the crop's process quality apples became fresh quality. To decrease the diameter, 10% of the total crop of the fresh quality apples were decreased in diameter to become process quality. The difference in value per bushel of the 2 different distributions, adjusted by the percent changed was used as a measure of sensitivity. The same method was used to determine the sensitivity of the return function to changes in color.

A measure of the sensitivity to culling factors was found by reclassifying 10% of the crop which had been graded cull as noncull or noncull as cull. Instead of shifting the distribution of the entire crop by a factor as before, we randomly selected apples for reclassification to cull or noncull and again finding the difference in the value/bushel we found a measure of sensitivity for cull. The sensitivity results are summarized in Table 3.1.

On a year to year basis the sensitivity of the return function to diameter changes was most significant. Figures 3.4-3.7 illustrate the differences in the sensitivities for the three factors. This sensitivity to size might indicate that a grower could reap a better return from an investment in those management practices which increase the size of apples, for example; thinning.

Factor	Mean	St. Dev.
Color	.3704	.3622
Size	.6203	.2293
Cull	.2443	.1898

Table 3.1

Sensitivity results in relative value/bushel for 10% change, for Red Delicious.

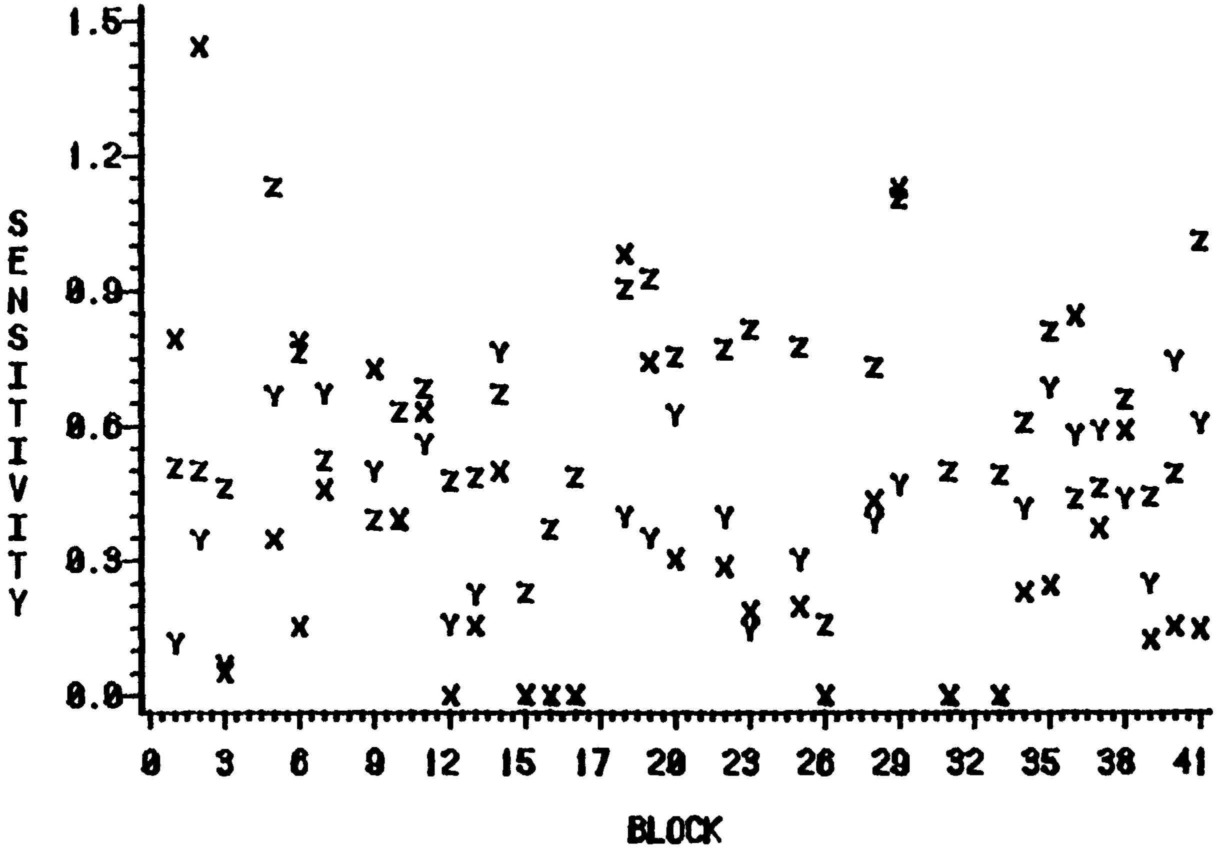

FIGURE 3.4: SENSITIVITY VALUES/BLOCK FOR 1976.
X=COLOR Y=CULL Z=SIZE.

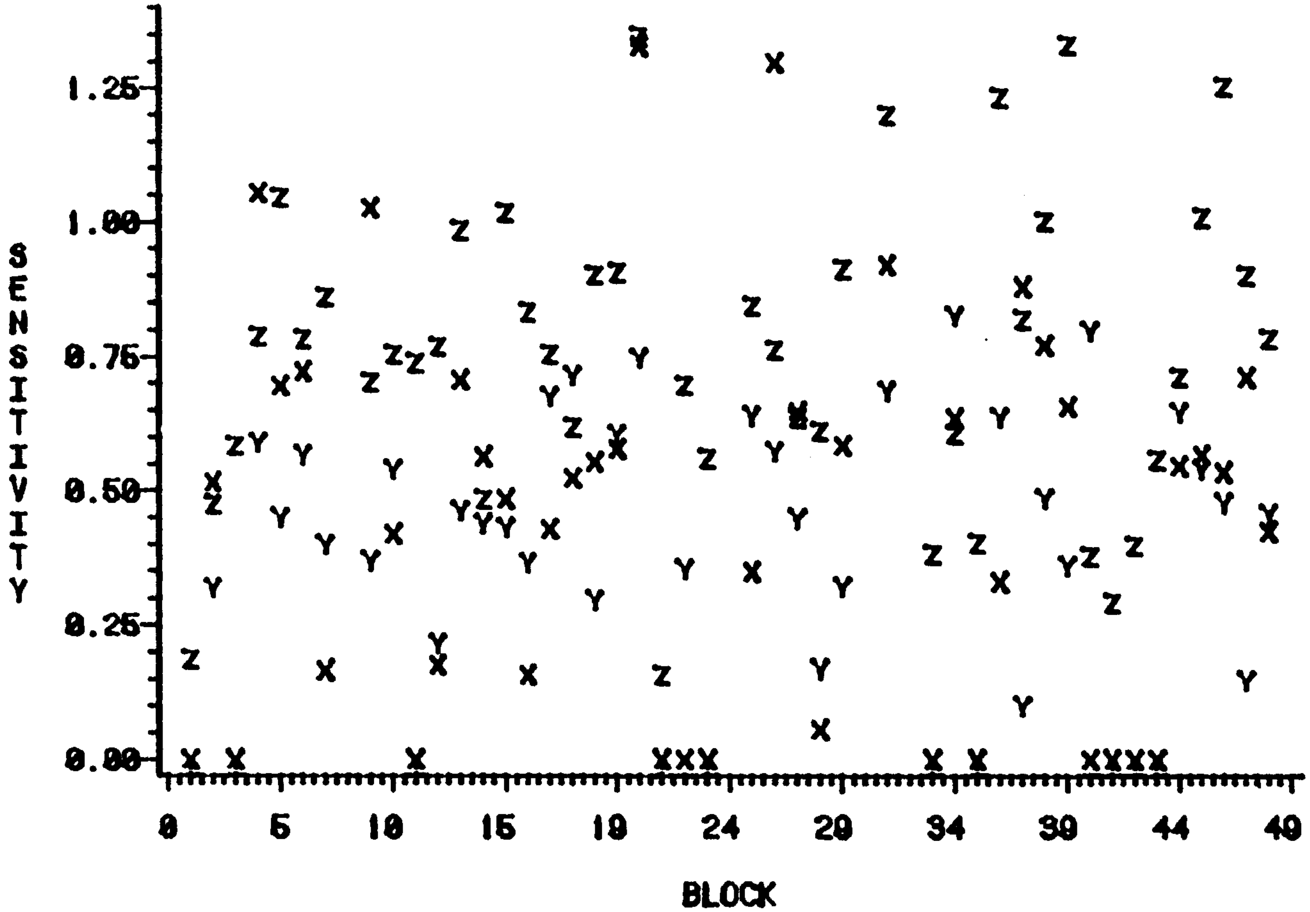

FIGURE 3.5: SENSITIVITY VALUES/BLOCK FOR 1977.
X=COLOR Y=CULL Z=SIZE.

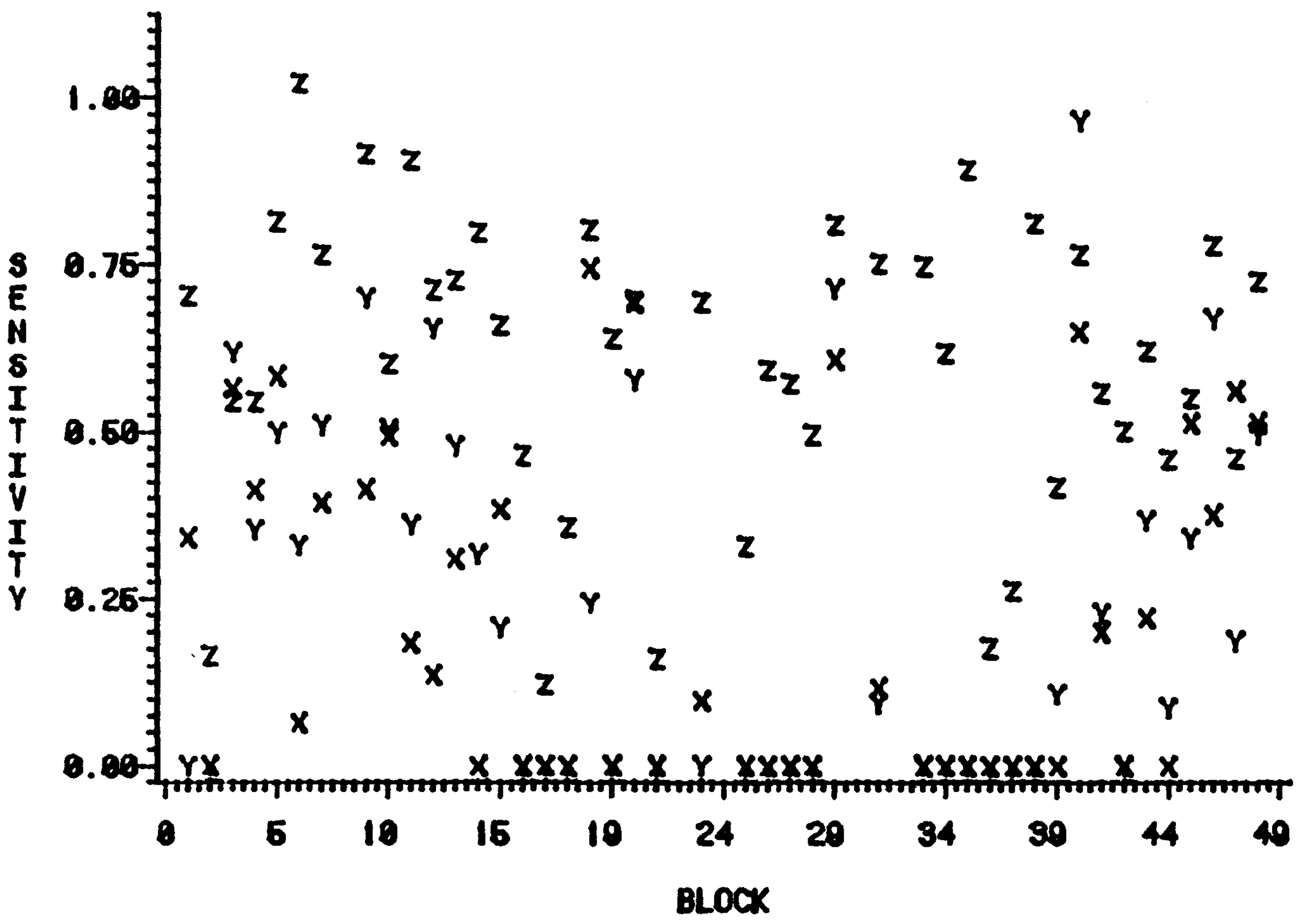

FIGURE 3.6: SENSITIVITY VALUES/BLOCK FOR 1978.
X=COLOR Y=CULL Z=SIZE.

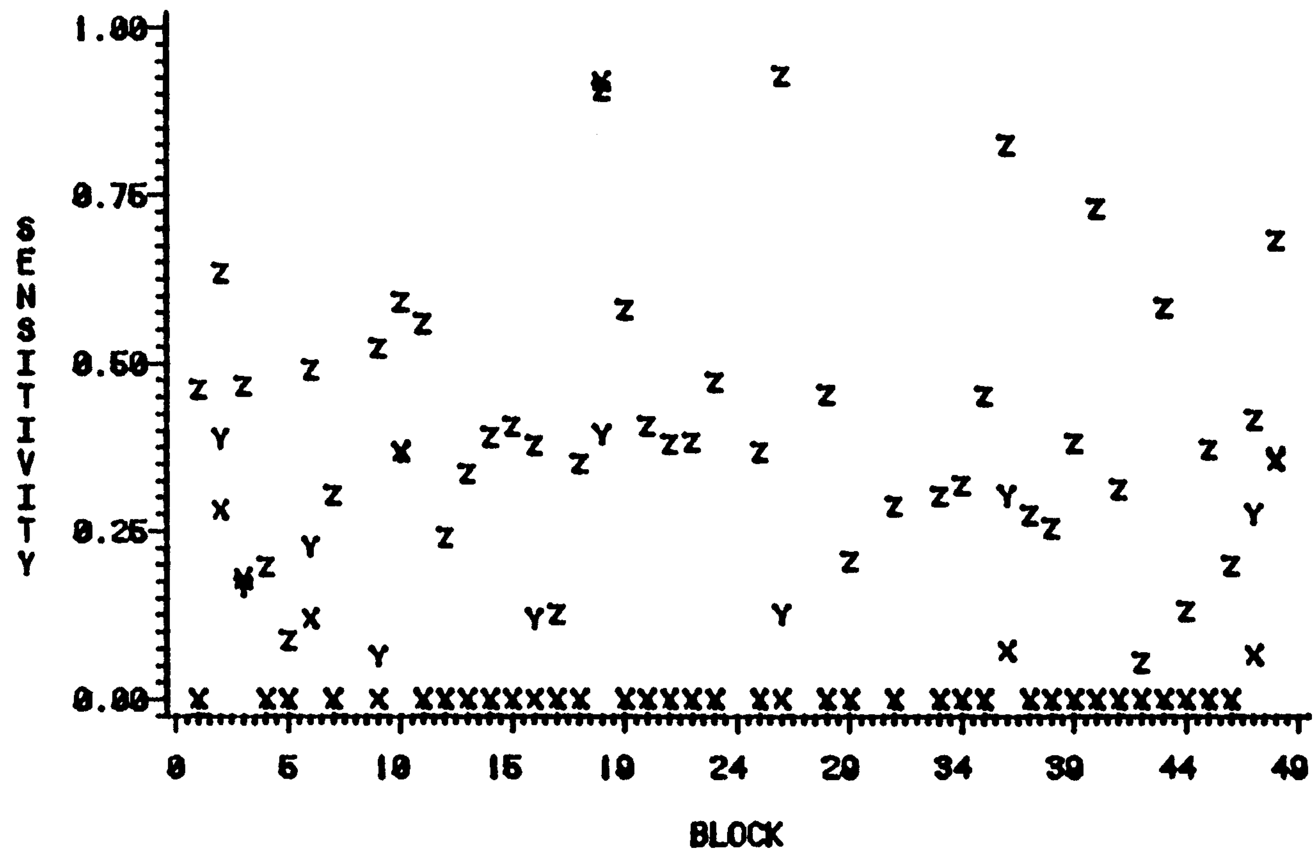

FIGURE 3.7: SENSITIVITY VALUES/BLOCK FOR 1979.
X=COLOR Y=CULL Z=SIZE.

Chapter III

MANAGEMENT STRATEGY OF PRESORTING

There are several ways in which the cost of crop heterogeneity can be reduced. The apples can be picked differentially into the 2 classes, fresh and process, or the segregation of fresh from process can take place after harvesting by sorting techniques.

If the decision is to differentially pick, then a further decision must be made on the level of differential picking. For example, does the farmer select certain trees to be harvested as fresh and others as process, or does he select a particular block of trees for fresh market and another group for processing only. Clearly the costs of differential picking as well as the accuracy of the "presort" will vary depending upon the level chosen for the differential picking.

Presorting techniques will most likely involve some initial investment in machinery to grade the crop as well as additional labor costs to run the machinery. Choosing to presort will require the grower to decide which of the quality determining factors to use in sorting his crop. The crop can be mechanically sized with little additional labor or sized and graded for cullage and color. The crop can be graded, therefore, using any combination of the three quality determining factors.

3.1 DEVELOPING THE PRESORT MODEL

The decision to differentially pick (presort) an apple orchard or not must be made on the basis of:

1. The cost of differential picking (presorting).
2. The accuracy of differential picking (presorting).
3. The actual class distribution within the orchard.

Two general cases may be identified using the 45-55 breakeven level. In the first case more than 45% of the crop would probably be graded as fresh apples. Therefore the entire crop would be sent to the packing house to be sorted and marketed according to the grades. In the second case fresh grade apples comprise 45% or less of the crop, and so the crop is then entirely sent to processing.

This "percent fresh" criterion can be estimated within a certain error factor through sampling techniques. For our purposes we have the IPOM's data samples.

The accuracy or efficiency of differentially picking an apple orchard is related to degree of effort, and therefore of cost. Increasing the accuracy may require pickers to take longer to harvest a crop in order to sort fresh apples from process apples. Alternatively, increased accuracy may be purchased by grower investment in more accurate sorting machinery, thus increasing the cost of presorting the crop. Increased accuracy is also associated with increased gross return because the packing house sorting cost for process apples is reduced. A graph of sorting efficiency as a function of sorting cost indicates that, up to a point, by adding more capital for presorting an increase in the efficiency of the sort may be realized. (Figure 3.1) The sorting efficiency associated with a specific sorting expenditure is needed to find the optimal cost of presorting the crop.

To further examine the efficiency we model the sorting process in terms of two idealized cases. In case 1, examination of a given apple produces enough information to correctly classify the apple with probability of one. In case 2, the examination produces no useful information, so that classification of the apple is random, with probability determined by the proportion of fresh and process in the crop. Within the context of this model, sorting efficiency is indexed by p, the probability of case 1, i. e., the probability of perfect information. The probability of case 2; that is, of no useful information is (1-p). Then letting P(Event A Event B) be the probability of Event A given Event B, then the probabilities may be assigned as follows:

P(Fresh Sure) = # Fresh/ # Total = F/T

P(Process Sure) = # Process/ # Total = P/T

P(Fresh Unsure) = .5

$$P(\text{Process}|\text{Unsure}) = .5$$

and then the probability of classifying an apple as fresh is:

$$P(Fr) = p*P(\text{Fresh}|\text{Sure}) + (1-p)*P(\text{Fresh}|\text{Unsure})$$
$$= p*(F/T) + (1-p)*.5$$

Using these probabilities the expected numbers of fresh, classified correctly and incorrectly are:

$$E(\text{Fr correct}) = .5*F*(p+1)$$

$$E(\text{Fr incorrect}) = T*(1-p)*.5*(1-F/T)$$

From the expected number of fresh apples which are classified either correctly or incorrectly, and the weighted average relative value of fresh apples, after sorting, an equation for the gross return of the presorted crop was developed:

$$GR = .579*.5*F*(p+1) - .064*(T*((1-p)*.5*(1-F))$$
$$+ .21*(T - (.5*(p+1) + T*((1-p)*.5*(1-F/T))).$$

Without presorting, and depending on the percent fresh, the corrected value/bushel is the maximum of the following:

$$GR = F*.576 - (T-F)*.064$$

$$GR = .216*T.$$

The economic benefit from presorting or differentially picking the crop is the increase in value per bushel gained by presorting the crop. This increase in value is the difference between the gross return for the presorted crop and the corrected value per bushel for the non-sorted crop. Two functions describe this economic benefit corresponding to the two cases where the percent of fresh apples in the crop is either above or below the 45% breakeven point. This improved value may be negative and is a function of p and the distribution of fresh apples:

Gross Improved Value(ImpVal) = f(p, F).

There is a cost associated with improving the value of the crop by differentially picking or presorting the crop. This cost affects the return from the crop. The return thus becomes, after accounting for these costs:

F>=45: ImpVal = .042*F*p - .321*F + .204*T + .140*T*p.

F <45:ImpVal = .042*F*p + .322*F + .140*T + .140*T*p .
- .216*T.

Thus far p has been described as the probability of "being sure" when classifying an apple as either fresh or process. Examination of p as a function of expenditure indicates that the relationship between the probability of an accurate classification and the sorting expenditure is the same as the efficiency-capital relationship. In other words, the more capital invested in the sort the greater the probability of certainty - up to a point. An approximation of this relationship is:

p = (c-a)/(A+c-a) where c<a and 0<a<1.
c= cost of presorting
a, A= parameters of the function.

The improved value due to presorting the crop then becomes:

$$\text{ImpVal} = f((c-a)/(A+c-a),\ F) - c.$$

and for the 2 cases:

$$F>=45:\ -c^2 + ca - CA - .279Fc + .344c + .279Fa - .321FA + .204A - .344a.$$

$$F<45:\ -c^2 + ca - cA + .364Fc + .064c - .364Fa + .322FA - .064a - .076A + .140.$$

Depending upon the case - either more than 45% fresh or less than 45% fresh - and the potential capital 'c', these equations may have a value (ImpVal) larger than zero in which case presorting the apple crop should be profitable. Using these equations as a decision making tool a manager could compare different sorting techniques to determine the more profitable investment. To provide further information on the costs and its relationship to p, a and A we have developed graphs of the feasibility boundaries of a and A at various values of the fraction of fresh quality apples (figures 3.2 and 3.3). For example, in figure 3.2 for 45% fresh quality the solid line outlines a feasibility region whereby any point under the curve or on the curve represents values of a and A at which some positive improved value could be realized with presorting. From these figures we can see that as the efficiency decreases (A increases) the feasibility of 'a' becomes more sensitive to a correct estimate on the fraction of fresh quality apples in the crop. In making a decision around these points a grower would need more accurate determinations of

F and so might increase the sampling that goes into deciding the F value for his crop.

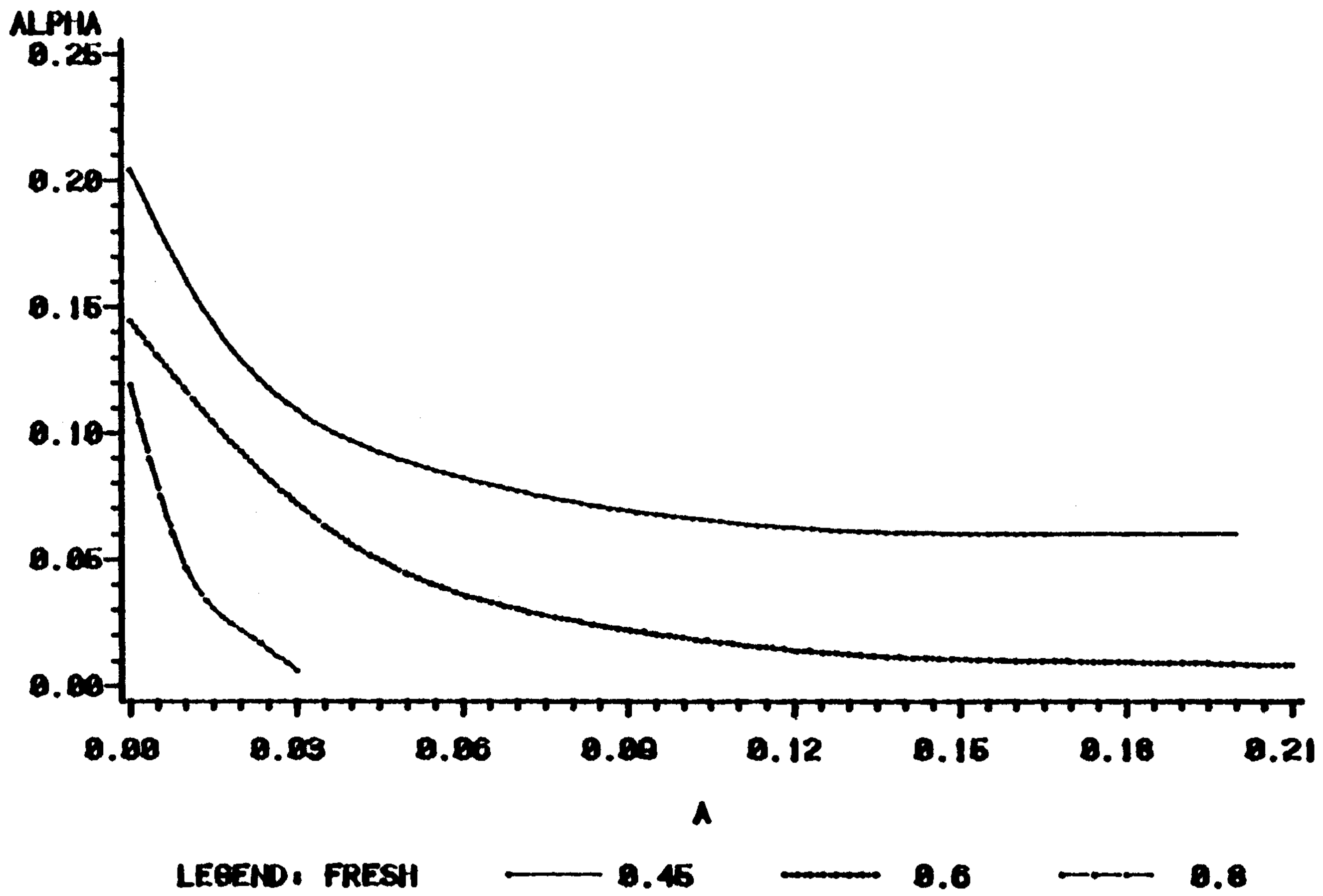

FIGURE 3.2: BOUNDARY CURVES OF ALPHA-A FEASIBILITY FOR FRESH FRACTION .45 OR GREATER.

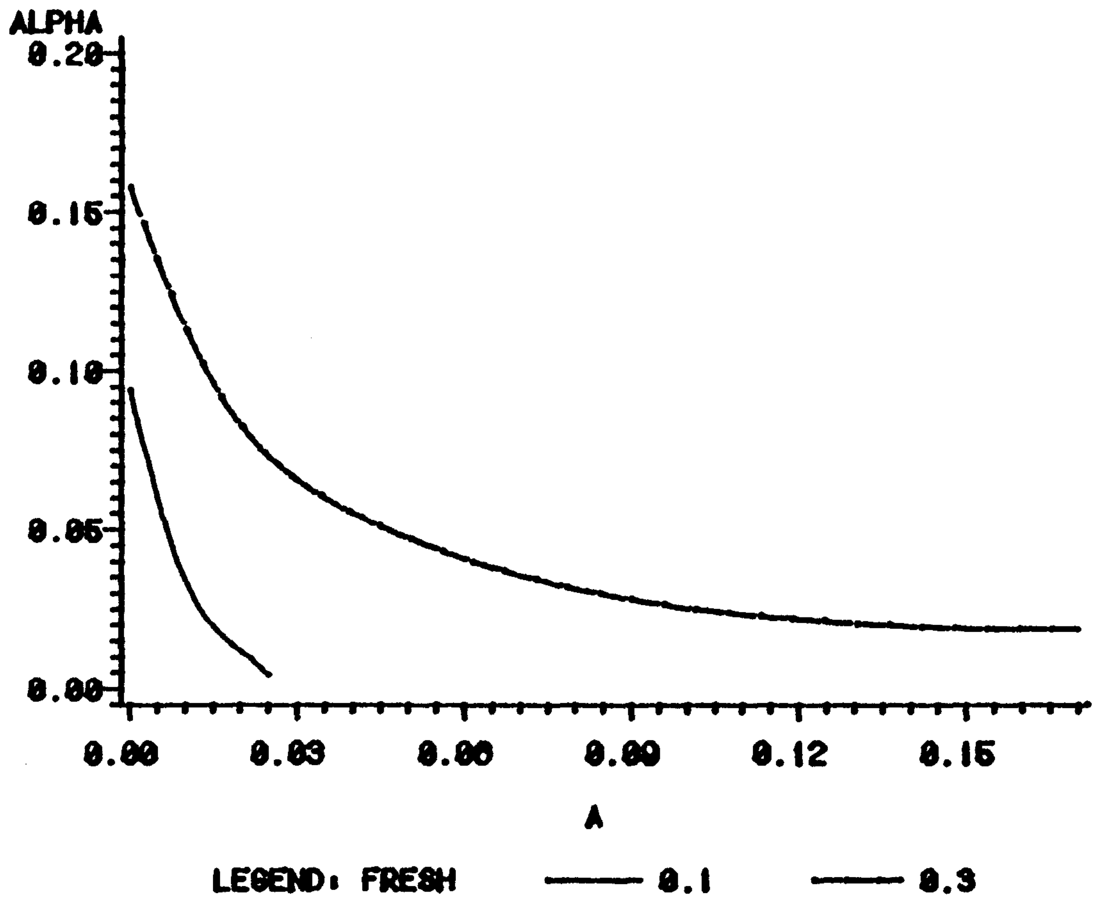

FIGURE 3.3: BOUNDARY CURVES OF ALPHA-A FEASIBILITY FOR FRESH FRACTION LESS THAN .45.

3.2 DISCUSSION

There are several levels of differential picking suggested as well as several potential ways of grading the harvested crop before marketing. Deciding on a method or level will require grower-management time and thus will add to the cost of whichever method is chosen. Just how a decision should be made and the costs of both the decision and the accuracy of the sort gained from the decision need further study.

In the equations presented, 'c', the cost of specific levels of efficient apple classification, was not estimated. Further study of presorting as an apple marketing practice should consider specific methods of differentially picking or presorting the crop as well as the mechanization costs involved, then 'c' can be estimated.

There is not only a cost in assigning the crop to a case (more than 45% fresh or less) but also a cost of incorrectly assigning the crop to a case. We have not included the cost of assigning a crop to a case. These costs need to be estimated.

Once the costs with corresponding levels of efficiency or accuracy are estimated, a method of finding the optimal 'c' needs to be developed for the two cases.

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