# PARAMETER ESTIMATION FOR STOCHASTIC DIFFERENTIAL EQUATIONS DRIVEN BY WIENER AND POISSON NOISE

Charles E. Smith and Loren Cobb

Biomathematics Series No. 26

Institute of Statistics Mimeo Series No. 1679
Raleigh, North Carolina

June 1986

NORTH CAROLINA STATE UNIVERSITY
Raleigh, North Carolina

# PARAMETER ESTIMATION FOR STOCHASTIC DIFFERENTIAL EQUATIONS DRIVEN BY WIENER AND POISSON NOISE

Charles E. Smith
Department of Statistics, Biomathematics Program
North Carolina State University
Raleigh, North Carolina 27695-8203, USA

Loren Cobb
Department of Biometry
Medical University of South Carolina
Charleston, South Carolina 29425, USA

**Abstract**

Ensemble and temporal parameter estimators are developed for linear and nonlinear stochastic differential equations driven by both Wiener and Poisson processes. Linear moment recursion relations are obtained for the stationary moments of the process. Consistency and asymptotic normality of the resulting ensemble estimators are demonstrated. The temporal estimators, i.e., using a single temporal record, are shown to coincide with maximum likelihood estimators in the special case of linear systems driven by Wiener noise.

Keywords: stochastic differential equations, parameter estimators, moment recursion relation, nonlinear stochastic systems.

Acknowledgement: Support for this work was furnished by National Science Foundation Grant ISP 80-11451 and Office Naval Research Contract N00014-85-K-0105.

* Author to whom correspondence should be addressed.

## 1. Introduction

This report is concerned with estimating the parameters of a class of linear and nonlinear stochastic differential equations driven by Wiener and Poisson processes. Specifically, we consider two models with a polynomial drift term. The first model (1) has state-dependent noise, while the second (2) has an additive noise term.

The first model is:

$$(1)\qquad dx_t = -\gamma\,(x_t)\,dt + x_t\,dn_t,$$

$$dn_t = \sigma\,dW_t + a\,(dP_t - \lambda\,dt),$$

where $\gamma(x) = \theta_0 + \theta_1 x + \ldots + \theta_d x^d$, $\theta_0 \neq 0$, $\theta_d > 0$, $W_t$ is a standard Wiener process (zero mean and variance t), $P_t$ is a homogeneous Poisson process with intensity $\lambda$ and unit jump size, and $\sigma$, a are constants. We assume the two noise processes, $W_t$ and $P_t$, are independent of each other and of the initial condition $x_0$.

The second model is similar to the first:

$$(2)\qquad dy_t = -\gamma\,(y_t)\,dt + dn_t,$$

$$dn_t = \sigma\,dW_t + a\,(dP_t - \lambda\,dt),$$

with all conditions identical to (1) except that $\theta_0$ is no longer constrained to be nonzero.

Systems of this form, (1) and (2) have been used as models for a number of physical and biological processes (see, for example, Arnold and Lefever, 1981). The polynomial structure of $\gamma$ permits the modelling of systems with multi-model stationary distributions. For example bistable systems can be modelled with $\gamma(x)$ as a cubic polynomial, see Kipnis and Newman (1985), Cobb and Zacks (1985). State-dependent noise, as in (1), has been applied to a variety of nonlinear systems (population growth: Hanson and Tuckwell (1981); optimal harvesting: Ryan and Hanson (1985); economics: Malliaris and

Brock (1982); neural models: Tuckwell (1979), Wilbur and Rinzel (1983), Hanson and Tuckwell (1983), Smith and Smith (1984); overview of Poisson models: Tuckwell (1981)).

Let $f(x,t,x_0)$ denote the transition probability density function (pdf) for the Markov process $x_t$, given the initial value $x_0$. With $\theta_d > 0$ the transition pdf approaches a nondegenerate pdf, denoted by $f^*$, as $t \to \infty$. The stationary density $f^*$ is independent of $x_0$. In the next section we show that the moments of $f^*$ satisfy a linear recursion relation which depends only on the parameters $\theta_0, \ldots, \theta_d, \sigma, \lambda, a$. This recursion relation permits the construction of an estimator of the parameters $(\theta_0, \ldots, \theta_d)$ if $\lambda$, a and $\sigma$ are known. This estimator is shown to be consistent and asymptotically normal. Since this estimator is constructed from i.i.d. samples of the ensemble, we refer to it as an "ensemble" estimator. The problem of estimation from a single temporal record is then addressed.

This report extends earlier results on versions of (1) and (2) with Wiener input alone (Cobb et al., 1983; also see Lanska, 1979), and with Poisson input alone (Smith and Cobb, 1982).

## 2. Estimation from observations of an ensemble

In this section, we derive linear moment recursion relations for models (1) and (2) and construct ensemble estimators for $\theta$ based on these relations.

Theorem 1 (Moment Recursion Relations).

Let $m_n$ denote the $n^{th}$ noncentral moment of $f^*$, the stationary pdf. For models (1) and (2) these moments satisfy a recursion relation for $n = 0, 1, 2, \ldots$ that is linear in terms of $m_n$. For model (1), (3a) $$\sum_{i=0}^{d} \theta_i \, m_{i+n} = \psi_{n+1} \, m_{n+1}$$

with

$$\psi_k = \lambda((1 + a)^k - 1)/k + (k-1)\, \sigma^2/2.$$

For model (2),

(3b) $$\sum_{i=0}^{d} \theta_i \, m_{i+n} = n\, \sigma^2 \, m_{n-1}/2 + \sum_{k=0}^{n} \binom{n}{k} \beta_k \, m_{n-k}$$

with

$$\beta_k = \lambda \, a^{k+1}/(k+1).$$

Proof.

Model (1):

Let $\partial_x$ denote the operator for partial differentiation with respect to x. It can be shown (Gihman and Skorohod, 1972, p299) that $f(x,t,x_0)$ satisfies the forward Feller-Kolmogorov equation

(4) $$\partial_t f = \partial_x[\gamma(x)\, f] + \sigma^2/2 \; \partial_x^2[x^2 f]$$

$$-\lambda \, f + \lambda \, f(x/(1+a)) \; / \; |1+a|.$$

The stationary pdf $f^*$ is the solution of $\partial_t f = 0$ in (4). The boundary conditions are $\partial_x^k \, f^*(\pm \infty) = 0$ for k = 0, 1, 2, ... Now let $F_\nu\{f\}$ denote the Fourier transform of a function f, i.e.,

$$F_\nu\{f\} \overset{d}{=} \int_{-\infty}^{\infty} f(x) \exp(-i\nu \, x) \, dx,$$

where $\nu$ is the transform variable, and i is the unit imaginary number.

Taking the Fourier transform of our equation for $f^*$, we obtain

(5) $$F_\nu\{\partial_x[\gamma(x) f^*]\} + \sigma^2/2 \; F_\nu\{\partial_x^2[x^2 f^*]\}$$

$$= \lambda \; (F_\nu\{f^*\} - F_{\nu(1+a)} \{f^*\}),$$

where for the last term we have used the scaling property of the Fourier transform, i.e.,

$$F_\nu\{f(bx)\} = 1/|b| \; F_{\nu/b} \{f(x)\}$$

for some constant b. The LHS is evaluated using

$$F_\nu\{\partial_x f(x)\} = (i\nu) \; F_\nu\{f\}$$

and

$$F_\nu\{x^m f(x)\} = i^m \; \partial_\nu^m \; F_\nu\{f\}$$

to obtain

(6)
$$(i\nu)\; \gamma(i\partial_\nu)\; F_\nu\{f^*\} + \sigma^2/2\; (i\nu)^2\; (i\partial_\nu)^2\; F_\nu\{f^*\}$$
$$= \lambda\; (F_\nu\{f^*\} - F_{\nu(1+a)}\{f^*\}).$$

we can now derive the moments $m_n$ by using the well-known fact that

$$m_n = (i\partial_\nu)^n \; F\{f^*\} \text{ when } \nu = 0.$$

Using the $(i\partial_\nu)^n$ operator on both sides of (6), the RHS becomes

$$\lambda\; [(i\partial_\nu)^n \; F_\nu\{f^*\} - (1 + a)^n \; (i\partial_{\nu(1+a)})^n \; F_{\nu(1+a)}\{f^*\}],$$

which simplifies to

$$\lambda[1 - (1 + a)^n]\; m_n \qquad\qquad \text{for } \nu = 0.$$

For the LHS of (6), we use the product rule of differentiation, namely

$$\partial_x^n(u\cdot v) = \sum_{k=0}^{n} \binom{n}{k} \; (\partial_x^{n-k}\, u)\; (\partial_x^k\, v)$$

to obtain $(S_1 + S_2)\; F_\nu\{f^*\}$, where

$$S_1 = (-n)\; (i\partial_\nu)^{n-1}\; \gamma(i\partial_\nu) + 0(\nu),$$

and

$$S_2 = \sigma^2/2\; n(n-1)\; (i\partial_\nu)^n + 0(\nu).$$

When $\nu = 0$ the LHS becomes

$$-n \sum_{k=0}^{d} \theta_k m_{k+n-1} + \sigma^2/2 \, n(n-1) \, m_n$$

so, combining these results and shifting the index by 1, we obtain

$$\sum_{k=0}^{d} \theta_k m_{k+n} = \lambda /(n+1) \, [(1+a)^{n+1} - 1] \, m_{n+1} + n \, \sigma^2/2 \, m_{n+1}$$

$$= \psi_{n+1} m_{n+1}, \quad \text{for } n = 0, 1, 2, \ldots$$

the desired result (3a).

For model (2), the corresponding Feller-Kolmogorov equation is

$$\partial_t f = \partial_x[\gamma(x)f] + \sigma^2/2 \, \partial_x^2 f + \lambda \, (f(x - a) - f).$$

Except for the term $\sigma^2/2 \, \partial_x^2 f$, this equation is identical to the one in Smith and Cobb (1982, p703). The result follows from a minor adjustment of their development.

Remark. These results carry over easily to finite sums of independent Wiener and Poisson noise.

A concrete example gives some feeling for the mechanics of the use of the relationship. Let $\gamma(x) = \theta_0 + \theta_1 x + \theta_2 x^2 + \theta_3 x^3$, a cubic polynomial. We now obtain four equations for the four unknowns $\theta_0$, $\theta_1$, $\theta_2$ and $\theta_3$ in terms of the parameters of the noise and the zeroth through sixth moments of $f^*$.

Ensemble estimators of $\theta$ can be derived from the moment recursion relations as follows. Suppose that $\{X_k\}$, $k = 1, \ell$ are independent random variables, each with density $f^*$. The moment recursion relations for both (1) and (2) reduce to

$$\hat{M}\,\hat{\theta} = \hat{W}\,B,$$

where

$$\theta = [\theta_0, \theta_1, \ldots, \theta_d]',$$

with (') indicating transpose;

$$\hat{M}_{ij} = \hat{m}_{i+j-2} = 1/\ell \sum_{k=1}^{\ell} X_k^{i+j-2},$$

$$\hat{W}_{ij} = m_i\ \delta_{ij}, \qquad \text{(Model 1)}$$

with $\delta_{ij}$ being the Kronecker delta,

$$= \begin{cases} 0 & \text{for } j > i \\ \binom{i-1}{j-1}\ m_{i-j} & \text{for } j \leq i. \end{cases} \qquad \text{(Model 2)}$$

and

$$B_j = \psi_j \qquad \text{(Model 1)}$$

$$= \begin{cases} \beta_{j-1} & \text{for } j \neq 2, \\ \beta_1 + \sigma^2/2 & \text{for } j = 2. \end{cases} \qquad \text{(Model 2)}$$

Theorem 2.

Under models (1) and (2) the ensemble estimator $\hat{\theta} = \hat{M}^{-1}\hat{W}B$ is consistent and $\sqrt{\ell}\ (\hat{\theta} - \theta)$ is asymptotically normal $N(0,V)$, where V is a dxd matrix that satisfies

$$[MVM]_{ij} = E\{([\hat{W}B]_i - [\hat{M}\theta]_i)\ ([\hat{W}B]_j - [\hat{M}\theta]_j)\}$$

Proof: (following Cobb et al., 1983, Theorem 2)

Consistency: Let $p_a(x) = a_0 + a_1x + \ldots + a_dx^d$. Because x is a random variable with a continuous nondegenerate density, we have $E[p_a^2(x)] = a'M\,a > 0$ for any vector $a \neq 0$. From this it follows that M is positive definite and invertible. Furthermore, $\hat{M}$ is invertible w.p. 1 by a similar argument, as long as $n > d$. Since $\hat{M} \xrightarrow{p} M$ and $\hat{W} \xrightarrow{p} W$, we also have $(\hat{M})^{-1} \xrightarrow{p} M^{-1}$ and $(\hat{M})^{-1} \hat{W} \xrightarrow{p} M^{-1} W$. Consistency $(\hat{\theta} \xrightarrow{p} \theta)$ follows immediately because B is nonstochastic.

Normality: We have $\sqrt{\ell}\,(\hat{M} - M) = O_p(1)$ and $\hat{\theta} - \theta = o_p(1)$. Rewrite $\sqrt{\ell}\, M\,(\hat{\theta} - \theta)$ as follows, collecting terms of similar order in probability:

$$\sqrt{\ell}\; M\,(\hat{\theta} - \theta) = \sqrt{\ell}\,(\hat{W}B - \hat{M}\theta) - \sqrt{\ell}\,(\hat{M} - M)\,(\hat{\theta} - \theta)$$

Each entry of the second term on the right-hand side is $O_p{}'(1)\; o_p(1) = o_p(1)$, where ('), indicates transpose. Thus

$$\sqrt{\ell}\, M\,(\hat{\theta} - \theta) - \sqrt{\ell}\,(\hat{W}B - \hat{M}\theta) \xrightarrow{p} 0,$$

and

$$\sqrt{\ell}\,(\hat{\theta} - \theta) - \sqrt{\ell}\, M^{-1}\,(\hat{W}B - \hat{M}\theta) \xrightarrow{p} 0,$$

since $M^{-1}$ is invertible. The vector $\sqrt{\ell}\, M^{-1}\,(\hat{W}B - \hat{M}\theta)$ can be written as

$$\sum_{k=1}^{\ell} h(X_k)/\sqrt{\ell}\ ,$$ where $h(x)$ is a vector of polynomials in x. Note

that $E[h(X)] = 0$, due to the linear moment recursion relations. Let $V_{ij} = E[h_i(X)\, h_j(X)]$. Then $\sqrt{\ell}\,(\hat{\theta} - \theta)$ is asymptotically $N(0,V)$ by the multivariate Central Limit Theorem.

### 3. Estimation from continuous observations

In this section we present a method for estimating the coefficients of (1) and (2) given continuous observations of the stochastic process $x_t$ on [0, T]. Explicit maximum likelihood estimation is not possible because the noise process is a mixture of Wiener and Poisson noise. We shall instead construct an estimator which reduces to the MLE when there is no Poisson noise, and which always satisfies a minimum mean squared error criterion. This estimator is best developed in a slightly more general context. Let $\theta = [\theta_0, \theta_1, \ldots, \theta_d]'$ and $g(x) = [1, x, \ldots, x^d]'$ with $\theta_0 \neq 0$, $\theta_d < 0$, and d odd. Suppose that $x_t$ is a stochastic process which satisfies the stochastic differential equation

$$dx_t = \theta' g(x_t)\, dt + \sigma(x)\, dn_t,$$

where $\sigma(x)$ is a smooth non-anticipatory function, $\sigma(x) \neq 0$ if $x \neq 0$, and $n_t$ is, as before, a mixture of an arbitrary number of independent Wiener and compensated Poisson processes, such that the process $x_t$ is ergodic. We shall make use of a weighting function introduced by Lipster and Shiryayev (1977, p. 274):

$$v(x) = \begin{cases} 0 & \text{if } \sigma(x) = 0 \\ 1/\sigma^2(x) & \text{otherwise.} \end{cases}$$

Consider the function $\Delta Q_t(x,c)$ defined by

$$\Delta Q_t(x,c) = [(\Delta x_t - c'g(x_t)\, \Delta t)/\sigma(x_t)]^2 - [\Delta x_t/\sigma(x_t)]^2,$$

where the right hand side is evaluated at $x_t = x$, and where $\Delta x_t \stackrel{d}{=} x_{t+\Delta t} - x_t$. The function $\Delta Q_t(x,c)$ has a graph that for $\omega \in \Omega'$ and $t \in [0, T-\Delta t]$, is a paraboloid with a single minimum. The ratio $\Delta Q_t(x,c)/\Delta t$ converges in probability as $\Delta t \to 0$ to the stochastic differential

$$dQ_t(x,c) = -2\, c'\, g(x_t)\, v(x_t)\, dx_t + c' [g(x_t) g(x_t)'] c\, v(x_t)\, dt$$

with the RHS evaluated at $x_t = x$.

The mean squared error of c for the stochastic process $x_t$ on [0,t] is given by

$$\begin{aligned} MSE(c,T) &= \int_0^T dQ_t(x_t,c) \\ &= -2c' \int_0^T g(x_t)\, v(x_t)\, dx_t + c'[\int_0^T g(x_t)g(x_t)'\, v(x_t)\, dt]\, c. \end{aligned}$$

Let $\hat{\theta}_T(\omega)$ be the solution of $\nabla_c$ MSE(c,T) = 0 for fixed $\omega \in \Omega$.

Thus

$$\hat{\theta}_T(\omega) = [\int_0^T g(x_t)g(x_t)'\, v(x_t)\, dt]^{-1}\, [\int_0^T g(x_t)\, v(x_t)\, dx_t].$$

In the special case in which $\sigma(x) = x$, $d = 1$, and $\lambda = 0$, we have an estimator $\theta_T$ which coincides with the maximum likelihood estimator (see Basawa and Rao (1980, Theorem 5.1) for example). In the general case the consistency of the estimator depends crucially on the ergodicity of the process.

**4. Discussion**

In the preceding sections we have considered both ensemble and temporal estimation. There are interesting connections between the two methods. If the systems described by (1) and (2) are ergodic and satisfy the appropriate mixing conditions, then the moment recursion relations of Section 2 can be used with temporal moments (instead of ensemble moments) to generate estimates of the parameters. It is interesting to observe that, in the case of Wiener noise only, this method coincides with the "minimum contrast" procedure of Lanska (1979). Lanska showed that the minimum contrast estimates are strongly consistent and asymptotically normal.

With respect to the question of existence and uniqueness of solutions for (1) and (2), note that the usual Lipschitz condition is not met by the function $\gamma(x)$, except when d = 1. However if we modify $\gamma$ so that it reads

$$\gamma(x) = \begin{cases} \gamma(A) + \gamma_x(A)\ (x - A) & \text{for } x > A, \\ \gamma(x) & \text{for } B \leq x \leq A, \\ \gamma(B) + \gamma_x(B)\ (x - B) & \text{for } x < B, \end{cases}$$

where $\gamma_x$ denotes differentiation with respect to x. The solutions of the modified process coincide exactly with the solutions of (1) and (2) up until a random hitting time $\tau$, when $x_t$ first reaches A or B. Using this construction and a theorem of Kallianpur and Wolpert (1984), a heuristic argument for the existence and uniqueness of solutions for (1) and (2) in the presence of both Wiener and Poisson noise is obtained.

In the foregoing we have assumed that the noise process has already been characterized, and that we need only to estimate the parameters of $\gamma(x)$. Jointly estimating all parameters is a problem that has been addressed in some special cases by Lansky (1983) and Habib (1985).

**ACKNOWLEDGEMENTS**

We would like to thank several of our colleagues, particularly Shelly Zacks, Ian McKeague and David Dickey, for their comments on a draft of this manuscript.

**REFERENCES**

ARNOLD, L. AND LEFEVER, R., Eds. (1981) Stochastic Nonlinear Systems in Physics, Chemistry, and Biology. Proceedings of the Workshop Bielefeld,Fed. Rep. of Germany, Oct. 5-11, 1980. Springer-Verlag, Berlin.

BASAWA, I. V. AND PRAKASA RAO, B.L.S. (1980) Statistical Inference for Stochastic Processes. Academic Press, New York.

COBB, L., KOPPSTEIN, P., CHEN, N. H. (1983) Estimation and moment recursion relations for multimodal distributions of the exponential family. J. Amer. Stat. Assoc. **78**, 124-130.

COBB, L. AND ZACKS, S. (1985) Applications of catastrophe theory for statistical modeling in the biosciences. J. Amer. Stat. Assoc. **80**, 793-802.

GIHMAN, I. I. AND SKOROHOD, A. V. (1972) Stochastic Differential Equations. Springer-Verlag, New York.

HABIB, M. K. (1985) Parameter estimation for randomly stopped diffusion processes and neuronal modeling. Institute of Statistics Mimeograph Series #1492, Department of Biostatistics, University of North Carolina at Chapel Hill.

HANSON, F. B. AND TUCKWELL, H. C. (1981) Logistic growth with random density independent disasters. Theoret. Popn. Biol. **19**, 1-18.

HANSON, F. B. AND TUCKWELL, H. C. (1983) Diffusion approximations for neuronal activity including synaptic reversal potentials. J. Theoret. Neurobiol. **2**, 127-153.

KALLIANPUR, G. AND WOLPERT, R. (1984) Weak convergence of solutions of stochastic differential equations with applications to nonlinear neuronal models. Center for Stochastic Processes Technical Report 60, Department of Statistics, University of North Carolina at Chapel Hill.

KIPNIS, C. AND NEWMAN, C. M. (1985) The metastable behaviour of infrequently observed, weakly random, one-dimensional diffusion processes. SIAM J. Appl. Math. **45**, 972-982.

LANSKA, V. (1979) Minimum contrast estimation in diffusion processes. J. Appl. Prob. **16**, 65-75.

LANSKY, P. (1983) Inference for the diffusion models of neuronal activity. Math. Biosci. **67**, 247-260.

LIPSTER, R. S. AND SHIRYAYEV, A. N (1977) Statistics of Random Processes I: General Theory. Springer-Verlag, New York.

MALLIARIS, A, G. AND BROCK, W. A. (1982) Stochastic Methods in Economics and Finance. North-Holland, Amsterdam.

RYAN, D. AND HANSON, F. B. (1985) Optimal harvesting with exponential growth in an environment with random disasters and bonanzas. Math. Biosci. **74**, 37-57.

SMITH, C. E. AND COBB. L. (1982) The stationary moments of Poisson-driven nonlinear dynamical systems. J. Appl. Prob. **19**, 702-706.

SMITH, C. E. AND SMITH, M. V. (1984) Moments of voltage trajectories for Stein's model with synaptic reversal potentials. J. Theoret. Neurobiol. **3**, 67-77.

TUCKWELL, H. C. (1979) Synaptic transmission in a model for stochastic neural activity. J. Theor. Biol. **77**, 65-81.

TUCKWELL, H. C. (1981) Poisson processes in Biology. In Stochastic Nonlinear Systems, eds. L. Arnold and R. Lefever. Distributed by Springer-Verlag, Berlin, 162-173.

WILBUR, J. A. AND RINZEL, J. A. (1983) A theoretical basis for large coefficient of variation and bimodality in neuronal interspike interval distributions. J. Theor. Biol. **105**, 345-368.

# REPORT DOCUMENTATION PAGE

| | |
|---|---|
| 1a. REPORT SECURITY CLASSIFICATION: UNCLASSIFIED | 1b. RESTRICTIVE MARKINGS |
| 2a. SECURITY CLASSIFICATION AUTHORITY | 3. DISTRIBUTION/AVAILABILITY OF REPORT: Approved for Public Release: Distribution Unlimited |
| 2b. DECLASSIFICATION / DOWNGRADING SCHEDULE | |
| 4. PERFORMING ORGANIZATION REPORT NUMBER(S): Mimeo Series #1679 | 5. MONITORING ORGANIZATION REPORT NUMBER(S) |
| 6a. NAME OF PERFORMING ORGANIZATION: N. C. State University | 6b. OFFICE SYMBOL (If applicable): 4B855 |
| 7a. NAME OF MONITORING ORGANIZATION: Office of Naval Research, Department of the Navy | |
| 6c. ADDRESS (City, State, and ZIP Code): Dept. of Statistics, Box 8203, Raleigh, N. C. 27695-8203 | 7b. ADDRESS (City, State, and ZIP Code): 800 North Quincy Street, Arlington, Virginia 22217-5000 |
| 8a. NAME OF FUNDING/SPONSORING ORGANIZATION: Office of Naval Research | 8b. OFFICE SYMBOL (If applicable): ONR |
| 9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER: N00014-85-K-0105 | |
| 8c. ADDRESS (City, State, and ZIP Code): Dept. of the Navy, 800 North Quincy Street, Arlington, Virginia 22217-5000 | 10. SOURCE OF FUNDING NUMBERS: PROGRAM ELEMENT NO. / PROJECT NO. / TASK NO. / WORK UNIT ACCESSION NO. |

11. TITLE (Include Security Classification)
Parameter Estimation for Stochastic Differential Equations Driven by Wiener and Poisson Noise

12. PERSONAL AUTHOR(S)
Charles E. Smith and Loren Cobb

| 13a. TYPE OF REPORT | 13b. TIME COVERED | 14. DATE OF REPORT (Year, Month, Day) | 15. PAGE COUNT |
|---|---|---|---|
| Technical | FROM ____ TO ____ | June, 1986 | 14 |

16. SUPPLEMENTARY NOTATION

| 17. COSATI CODES | | | 18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number) |
|---|---|---|---|
| FIELD | GROUP | SUB-GROUP | |
| | | | |
| | | | |

19. ABSTRACT (Continue on reverse if necessary and identify by block number)

Ensemble and temporal parameter estimators are developed for linear and nonlinear stochastic differential equations driven by both Wiener and Poisson processes. Linear moment recursion relations are obtained for the stationary moments of the process. Consistency and asymptotic normality of the resulting ensemble estimators are demonstrated. The temporal estimators, i.e., using a single temporal record, are shown to coincide with maximum likelihood estimators in the special case of linear systems driven by Wiener noise.

| | |
|---|---|
| 20. DISTRIBUTION/AVAILABILITY OF ABSTRACT: ☒ UNCLASSIFIED/UNLIMITED ☐ SAME AS RPT. ☐ DTIC USERS | 21. ABSTRACT SECURITY CLASSIFICATION: UNCLASSIFIED |
| 22a. NAME OF RESPONSIBLE INDIVIDUAL | 22b. TELEPHONE (Include Area Code) / 22c. OFFICE SYMBOL |