Generating the Matrix t Distribution

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I. Introduction

The matrix t distribution is a generalization of the multivariate Student t distribution, since the marginal distribution of rows or columns are of that form, as are some conditional distributions of columns. Moreover, Zellner (1971) uses the name Generalized Student t distribution. The distribution arises as the marginal posterior distribution of the matrix of regression coefficients in a multivariate normal regression model. Further details can be found in Zellner (1971, Chapter 8 and Appendix B.5), Dickey (1967), Geisser (1965) and Tiao and Zellner (1965).

Since the generation follows a different characterization, the distribution theory is discussed in Section II. The method of computation is given in Section III. The first Appendix contains some details in two lemmas. The code for generating the distribution is given in the second Appendix.

II. Distribution Theory

(A) Generation of L

Let L be a q x q lower triangular matrix whose diagonal elements L_{ii} and lower triangular elements are generated independently as follows ($\alpha \geq q$)

$$L_{ii} \sim X_{\alpha + 1 - i} \qquad i = 1,\ldots,q \qquad (\alpha \geq q)$$

$$L_{ij} \sim N(0,1) \qquad i < j < i < q \,.$$

Above X_ν denotes the chi distribution (square root of chi square) with ν degrees of freedom and N(0,1) denotes the standard normal distribution. The joint density of the element of L is given by

$$f_L(L) = \left[\prod_{i=1}^{q} \frac{2^{-(\alpha+1-i)/2}}{\Gamma[(\alpha+1-i)/2]} L_{ii}^{\alpha-i} e^{-L_{ii}^2/2}\right] \left[(2\pi)^{-q(q-1)/4} \exp\left\{-\sum_{1\le j<i\le q}\sum L_{ij}^2/2\right\}\right]$$

$$= \frac{2^{q-q\alpha/2}\pi^{-q(q-1)/4}}{\prod_{i=1}^{q} \Gamma[(\alpha+1-i)/2]} \prod_{i=1}^{q} L_{ii}^{\alpha-i} \exp\left\{-\frac{1}{2} \mathrm{tr} L^T L\right\} .$$

(B) Multiply C = BL

Let B be a fixed lower triangular and nonsingular q x q matrix. The premultiplication of L by B creates C BL. Rewriting the transformation as $L = B^{-1}C$, the determinant of the Jacobian of $L \to C$ is given by Theoren 2.4.4 of Giri (1977) as $\Pi B|_i$, so the the joint density of the element of C is

$$f_C(C) = \frac{2^{q-q\alpha/2}\pi^{-q(q-1)/4}}{\Pi\Gamma[(\alpha+1-i)/2} \prod_{i=1}^{q} B_{ii}^{i} \prod_{i=1}^{q} (C_{ii}/B_{ii}) \exp\left\{-\frac{1}{2} \mathrm{tr} C^T(BB^T)^{-1}C\right\}$$

$$= \frac{2^{q-q\alpha/2}\pi^{-q(q-1)/4}}{\Pi\Gamma[(\alpha+1-i)/2]} |B|^{-\alpha} \prod_{i=1}^{q} C_{ii}^{\alpha-i} \exp\left\{-\frac{1}{2} \mathrm{tr}\ C^T(BB^T)^{-1}C\right\} .$$

(C) Form $R = CC^T$ with the Wishart distribution.

Notice that C above is also lower triangular. If the product $R = CC^T$ is formed, so C is the Cholesky factor of R, the determinant of the Jacobian of the transformation $C \to R$ is $2^q \Pi C_{ii}{}^{q-i+1}$ (Giri, Theorem 2.4.6). The joint density of the elements of R is then

$$f_r(R) = \frac{2^{-q\alpha/2}\pi^{-q(q-1)/4}}{\Pi\Gamma[(\alpha+1-i)/2]} |BB^T|^{-\alpha/2} |R|^{(\alpha-q-1)/2} \exp\left\{-\frac{1}{2} \mathrm{tr}(BB^T)^{-1}R\right\}$$

which is the density of the Wishart distribution $W_q(BB^T,\alpha)$ with scale matrix BB^T and degrees of freedom parameter α. Driving R from L is $R = CC^T = (BL)(BL)^T = BLL^TB^T$, so LL^T is $W_q(I,\alpha)$ and $BLL^TB^T \sim W_q(BB^T,\alpha)$.

(D) The Inverted Wishart distribution.

There are two routes available for the inverted Wishart distribution. One is the direct route, for if $R \sim W_q(BB^T,\alpha)$, then $R^{-1} \sim W_q^{-1}(BB^T,\alpha)$. The other is a more circuitous route, but with simpler steps. The first step is to invert the metrix C from in (B) and the second then is to form the inner produce of the inverse.

Let $A = C^{-1}$, where C was formed in part (B), then determinant of the Jacobian of the transformation, as given by Lemma 1 in the first Appendix, is $|A|^{q+1}$, so the joint density of the elements of A is

$$f_S(A) = \frac{2^{q-q\alpha/2}\pi^{-q(q-1)/4}}{\Pi\Gamma[(\alpha+1-i)/2]} |B|^{-\alpha} \prod_{i=1} A_{ii}^{i-\alpha-(q+1)} \exp\left\{-\frac{1}{2}\operatorname{tr}(BB^T)^{-1}(A^TA)^{-1}\right\} .$$

Now forming $G = A^TA = C^{-T}C^{-1} = (CC^T)^{-1} = R^{-1}$ gives the matrix with the Inverted Wishart distribution. Lemma 2 gives the determinant of the Jacobian of the transformation $A \to G$ as $2^q\Pi A_{ii}^{i}$, so that the joint density of the elements of G is

$$f_G(G) = \frac{2^{-q\alpha/2}\pi^{-q(q-1)/4}}{\Pi\Gamma[(\alpha+1-i)/2]} |B|^{-\alpha} \Pi A_{ii}^{-\alpha-(q+1)} \exp\left\{-\frac{1}{2}\operatorname{tr}(BB^T)^{-1}G^{-1}\right\}$$

$$= \frac{2^{-q\alpha/2}\pi^{-q(q-1)/4}}{\Pi\Gamma[(\alpha+1-i)/2]} |Q|^{+\alpha/2} |G|^{-(\alpha+q+1)/2} \exp\left\{-\frac{1}{2}\operatorname{tr}QG^{-1}\right\}$$

where $Q = (BB^T)^{-1}$. The path to G from L gives the algorithmic route for generating from the Inverted Wishart distribution $W_q(Q^{-1},\alpha)$

$$G = C^{-T}C^{-1} = (BL)^{-T}(BL)^{-1} = B^{-T}L^{-T}L^{-1}B^{-1} \; .$$

The best route depends upon whether Q or Q^{-1} is given. If Q is given, then factor $Q = EE^T$ by Cholesky, solve the set of linear equations to get $L^{-1}E$, and take the inner product

$$(L^{-1}E)^T(L^{-1}E) = E^T L^{-T} L^{-1} E = G \ .$$

Alternatively, if Q^{-1} is given, Cholesky factorization of $Q^{-1} = BB^T$, so invert B^T (upper triangular) in place and use it in place of E above (B^{-T} and E are different) when solving and forming the inner product.

Finally, note the following integration result arising from the normalization constant of $f_G(G)$

$$\int \cdots \int_{\{G>0\}} |G|^{-(\alpha+q+1)/2} \exp\left\{-\frac{1}{2} \text{ tr } QG^{-1}\right\} = \frac{\Pi\Gamma[(\alpha+1-i)/2]}{2^{-q\alpha/2}\pi^{-q(q-1)/4}} |Q|^{-\alpha/2} \ .$$

(E) Matrix T distribution

This final piece is also a two step procedure. The first step is the generation of normal deviates, conditional on G. The next step is to permit a general distribution.

First the matrix S (p x q) whose i^{th} row $S_{i.}$ has a multivariate normal distribution with covariance matrix G, that is

$$S_{i.} | G \sim N_q(0,G)$$

independently over i = 1,...,p. Then the joint density of the elements of S, conditional on G, is

$$f_S(S) = (2\pi)^{-pq/2} \exp\left\{-\frac{1}{2} \text{ tr } SG^{-1}S\right\} |G|^{-p/2} \ .$$

Now let T = VS where V is a square fixed matrix. Since $|J_{S\to T}| = |V|^q$ from Giri (1977, Theorem 2.4.2), then the joint conditional density of the elements of T is

$$f_T(T|G) = (2\pi)^{-pq/2} |V|^{-q}|G|^{-p/2}\exp\left\{-\frac{1}{2} \text{ tr } V^{-1}TG^{-1}T^TV^{-T}\right\}$$

Next, the joint density of both T and G is

$$f_{T,G}(T,G) = \frac{2^{-\alpha q/2}(2\pi)^{-pq/2}\pi^{-q(q-1)/4}}{\pi\Gamma[(\alpha+1-i)/2]} |Q|^{+\alpha/2}|G|^{-(\alpha+q+p+1)/2}|V|^{-q}$$

$$\times \exp\left\{-\frac{1}{2}\,\mathrm{tr}[Q + T^{T}PT]G^{-1}\right\}$$

where $P = V^{-T}V^{-1} = (VV^{T})^{-1}$. Finally, integrating out G, using the result at the end of (D) gives the marginal density of the elements of T.

$$f_T(T) = \int_{\{G>0\}} f_{T,G}(T,G)\,dG$$

$$= \int_{\{G>0\}} |G|^{-(\alpha+q+p+1)/2} \exp\left\{-\frac{1}{2}\,\mathrm{tr}[Q + T^{T}PT]G^{-1}\right\}dG$$

$$\times \frac{2^{-\alpha q/2}(2\pi)^{-pq/2}\pi^{-q(q-1)/4}}{\pi\Gamma[(\alpha+1-i)/2} |Q|^{\alpha/2}\ |P|^{q/2}$$

$$= \prod_{i=1}^{q} \frac{\Gamma[(\alpha+p+1-i)/2]}{\Gamma[(\alpha+1-i)/2]} (2\pi)^{-pq/2}\ |Q|^{\alpha/2}\ |P|^{q/2}|Q + T^{T}PT|^{-(\alpha+p)/2} .$$

This gives T as having the Matrix T distribution, with degrees of freedom parameter α, and scale matrices P (p x q) and Q (q x q). Notice that S can be obtained from a matrix U (p x q) whose elements are iid N(0,1) random variables, by constructing

$$S = U(L^{-1}B^{-1}) .$$

Finally, note that if $R = T^{T}$, then R (q x p) has the Matrix T distribution with parameter matrices Q^{-1} and P^{-1} (note the reverse order) and degrees of freedom $\alpha + p - q$.

III. Computation

The results of the previous section can be summarized as follows. For L ($q \times q$) generated as in (A) and U ($p \times q$) whose elements are all iid $N(0,1)$, the matrix $T = VUL^{-1}B^{-1}$ has the matrix T distribution with scale matrices $P = (VV^T)^{-1}$ and $Q = (BB^T)^{-1}$.

If the matrices P and Q are given, then the first steps are to compute their Cholesky factors

$$P = DD^T$$

$$Q = EE^T \ .$$

Then $T = D^{-T}UL^{-1}E^T$ is the product required. The order of computation is then

1) generate U
2) solve to get $D^{-T}U$
3) solve to get $(D^{-T}U)L^{-1}$
4) postmultiply $(D^{-T}UL^{-1})E^T$.

If the inverse matrices P^{-1} and Q^{-1} are given instead, then the transpose matrix is computed, following the result at the end of II(D).

Appendix I: Lemmas

Lemma 1. Let C and A be nonsingular lower triangular matrices of order q then the determinant of the Jacobian of the transformation from C to $A = C^{-1}$ is $|L|^{-(q+1)}$.

Proof. Beginning with CA = I, then the fundamental step is

$$\frac{\partial C}{\partial \theta} A + C \frac{\partial A}{\partial \theta} = 0 \tag{A.1.1}$$

so premultiplying by A yields

$$\frac{\partial A}{\partial \theta} = -A \frac{\partial C}{\partial \theta} A \ . \tag{A.1.2}$$

Consider now $\theta = L_{ij}$, then

$$\frac{\partial A}{\partial C_{ij}} = -A(e_i e_j^T)A = -A_{.i}A_{j.} \tag{A.1.3}$$

where $A_{.i}$ denotes the ith column of A and $A_{j.}$ denotes the jth row of A. For an individual element of A, say $A_{\alpha\beta}$, then

$$\frac{\partial A_{\alpha\beta}}{\partial C_{ij}} = \left(\frac{\partial A}{\partial L_{ij}}\right)_{\alpha\beta} = -A_{\alpha i}A_{j\beta} \ .$$

Now consider $\partial A_{\alpha\beta}/\partial C_{ij}$ for block (α,i), whose partial derivatives involve $A_{\alpha i}$ and a subset of A^T. Since A is lower triangular, $A_{\alpha i} = 0$ for $i > \alpha$, then only diagonal blocks $\alpha = i$ need be considered. The diagonal blocks are of the form

$$\frac{\partial A_{\alpha .}}{\partial C^T_{\alpha .}} = -A_{\alpha\alpha}\left(A^{(\alpha)}\right)^T$$

where $A^{(\alpha)}$ is the $\alpha \times \alpha$ submatrix formed by the first α rows and columns of A. Since the determinant of the Jacobian matrix is the product of the determinants of the diagonal blocks, each one of the form

$$A^{\alpha}_{\alpha\alpha} |A^{(\alpha)}| = A^{\alpha}_{\alpha\alpha} \prod_{\beta=1}^{\alpha} A_{\beta\beta}$$

the final result is

$$|J_{C\to A}| = \prod_{\alpha-1}^{q} A^{\alpha}_{\alpha\alpha} \prod_{\beta=1}^{\alpha} A_{\beta\beta} = \prod_{\alpha=1}^{q} A^{q+1}_{\alpha\alpha} = |A|^{q+1} = |C|^{-(q+1)} .$$

Lemma 2. Let A be a square and lower triangular q x q matrix. The determinant of the Jacobian of the transformation from A to $G = A^{T}A$ is

$$2^{q} \prod_{i=1}^{q} A^{i}_{ii} .$$

Proof. Begin by writing

$$G_{\alpha\beta} = \sum_{\kappa=\alpha}^{q} A_{\kappa\alpha} A_{\kappa\beta}$$

for $\beta \leq \alpha$, then

$$\frac{\partial G_{\alpha\beta}}{\partial A_{ij}} = \begin{cases} 0 & j \neq \alpha \text{ and } j \neq \beta \\ 2A_{i\alpha} & j = \alpha = \beta \\ A_{i\beta} & j = \alpha \neq \beta \end{cases} .$$

Consider block (β,j) with rows $\beta\beta$ to $q\beta$ and columns jj to qj. If $\beta > j$, then this block is all zero. Since then the Jacobian matrix is block triangular, consider only diagonal blocks $\beta = j$, which are of the form

$$\begin{matrix} 2A_{jj} & A_{j+1,j} & \cdots & A_{qj} \\ 0 & A_{j+1,j+1} & \cdots & A_{q,j+1} \\ 0 & & \ddots & \\ 0 & 0 & & A_{qq} \end{matrix}$$

Since the determinant of the Jacobian is just the product of the determinants of the diagonal blocks, each of which are

$$2 \prod_{\alpha=j}^{q} A_{\alpha\alpha}$$

then the result is

$$|J_{A\to G}| = \prod_{j=1}^{q} \left(2 \prod_{\alpha=j}^{q} A_{\alpha\alpha}\right) = 2^q \prod_{i=1}^{q} A_{ii}^{i} \ .$$

```
//GMXTRV JOB NCS.ES.B4912,SOFT,CLASS=S
C      TEST    PROGRAM    FOR    GMXTRV        --        ALGO//GMXTRV    JOB
NCS.ES.B4912,SOFT,CLASS=S
C  TEST PROGRAM FOR GMXTRV  --  ALGORITHM FOR GENERATING RANDOM
C  MATRICES FROM THE MATRIX T DISTRIBUTION
C  SEE "GENERATING FORM THE MATRIX T DISTRIBUTION" MIMEO SERIES £1692
C
      REAL P(55),Q(55),T(10,10),STORE(55),TVCOV(100,100),TV(100)
C
      INTEGER IP,IQ,NREP,I,J,IPQ,L
C
 20   FORMAT(I4)
 21   FORMAT(1X,7F10.6)
 22   FORMAT(I12,F12.6)
 23   FORMAT(1X,10E12.4)
C
C                   READ NREP  AND DEGREES OF FREEDOM PARAMATER
      READ(1,22) NREP,ALPHA
      WRITE(3,22) NREP,ALPHA
C                   INITIALIZE RANDOM NUMBER GENERATOR
      P(1) = RAN(7372541)
C
      READ(1,20) IP
      DO 1 I = 1,IP
      L = (I*(I-1))/2
      READ(1,21) (P( L+J ),J=1,I)
      WRITE(3,21) (P( L+J ), J=1,I)
  1   CONTINUE
C
      READ(1,20) IQ
      DO 2 I = 1,IQ
      L = (I*(I-1))/2
      READ(1,21) (Q( L+J ),J=1,I)
      WRITE(3,21) (Q( L+J ), J=1,I)
  2   CONTINUE
C
C                   COMPUTE CHOLESKY FACTORS FOR THE MATRICES P AND
Q
C
      CALL CHLSKZ(P,IP,DET,IDET)
      WRITE(3,22) IDET,DET
C
      CALL CHLSKZ(Q,IQ,DET,IDET)
      WRITE(3,22) IDET,DET
C
C                   PRINT OUT FACTORS TO MAKE SURE
      DO 3 I = 1,IP
      L = (I*(I-1))/2
      WRITE(3,23) (P(L+J),J=1,I)
  3   CONTINUE
C
      DO 4 I = 1,IQ
      L = (I*(I-1))/2
      WRITE(3,23) (Q(L+J),J=1,I)
```

```
  4   CONTINUE
C                       INITIALIZE COVARIANCE MATRIX
      IPQ = IP*IQ
      DO 6 I = 1,IPQ
      DO 5 J = 1,I
      TVCOV(I,J) = 0.
  5   CONTINUE
  6   CONTINUE
C
C             GENERATE NREP MATRICES AND COPUTE THE SAMPLE COVARIANCE
MATR
C            TO CHECK THE ALGORITHM
C
      DO 9 KREP = 1,NREP
C
      CALL GMXTRV(T,IP,IQ,ALPHA,P,Q,STORE)
C
      L = 0
      DO 8 I = 1,IP
      DO 7 J = 1,IQ
      L = L + 1
      TV(L) = T(I,J)
  7   CONTINUE
  8   CONTINUE
C
      DO 12 I = 1,IPQ
      DO 11 J= 1,I
      TVCOV(I,J) = TVCOV(I,J) + TV(I)*TV(J)
  11  CONTINUE
  12  CONTINUE
C
  9   CONTINUE
C                       DIVIDE BY NREP AND PRINT IT OUT
      DO 18 I = 1,IPQ
      DO 17 J = 1,I
  17  TVCOV(I,J) = TVCOV(I,J) / NREP
      WRITE(3,23) (TVCOV(I,J),J=1,I)
  18  CONTINUE
      STOP
      END
      SUBROUTINE GMXTRV(T,IP,IQ,ALPHA,PCHFAC,QCHFAC,STORE)
C    GENERATES  RANDOM  MATRICES  OF  ORDER  IP  BY  IQ  HAVING  THE
GENERALIZED
C  STUDENT'S T DISTRIBUTION OR MATRIX T DISTRIBUTION
C  WITH PARAMETER MATRICES P AND Q AND DEGREES OF FREEDOM ALPHA
C
C    T        REAL MATRIX  HAVING THE DESIRED MATRIX T DISTRIBUTION
C             (OUTPUT)  DIMENSIONED BELOW (10,10) ONLY FOR CONVENIENCE
C
C    IP       NUMBER OF ROWS IN THE T MATRIX, ORDER OF MATRIX P
C
C    IQ       NUMBER OF COLUMNS IN THE T MATRIX, ORDER OF MATRIX Q
C
C    ALPHA    DEGREES OF FREEDOM PARAMETER
```

```
C
C      PCHFAC   CHOLESKY  FACTOR  OF  PARAMETER  MATRIX   P  =  PCHFAC  *
PCHFAC' =
C             STORED IN SYMMETRIC FORMAT
C      QCHFAC   CHOLESKY  FACTOR  OF  PARAMETER  MATRIX   Q  =  QCHFAC  *
QCHFAC' =
C             STORED IN SYMMETRIC FORMAT
C
C    STORE   REAL VECTOR NEEDED FOR TEMPORARY STORAGE
C
C
C   J  F  MONAHAN  (AUGUST  1986)  DEPT  OF  STATISTICS,  NCSU,  RALEIGH,  NC
USA
C
C  REFERENCE
C           "GENERATING FROM THE MATRIX T DISTRIBUTION" INSTITUTE OF
C           STATISTICS MIMEO SERIES NUMBER 1692
C
      REAL T(10,10),ALPHA,PCHFAC(1),QCHFAC(1),AL,STORE(1),S
      INTEGER IP,IQ,I,J,K,IM1,IPM1,II,JJ,JP1
C
C                     BEGIN BY FILLING UP U (STOED IN T) WITH NORMALS
C
      DO 2 I = 1,IP
      DO 1 J = 1,IQ
      T(I,J) = UNSK(I+J)
  1   CONTINUE
  2   CONTINUE
C                     GENERATE FACTOR OF WISHART MATRIX L IN STORE
      DO 4 I = 1,IQ
      L = ( I*(I-1) )/2
      AL = ALPHA + 1.0 - FLOAT(I)
      STORE(L+I) = GCHIRV(AL)
      IF( I .EQ. 1 ) GO TO 4
      IM1 = I - 1
      DO 3 J = 1,IM1
      STORE(L+J) = UNSK(J)
  3   CONTINUE
  4   CONTINUE
C
C                     PREMULTIPLY BY CHOLESKY FACTOR D INVERSE TRANSP
      DO 7 J = 1,IQ
      T(IP,J) = T(IP,J) / PCHFAC( (IP*(IP+1))/2 )
      IF( IP .EQ. 1) GO TO 7
      IPM1 = IP - 1
      DO 6 II = 1,IPM1
      I = IP - II
      IP1 = I+1
      S = 0.
      DO 5 K = IP1,IP
      S = S + PCHFAC( (K*(K-1))/2 + I )*T(K,J)
  5   CONTINUE
      T(I,J) = ( T(I,J) - S ) / PCHFAC( (I*(I+1))/2 )
  6   CONTINUE
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```
  7   CONTINUE
C                              NOW POSTMULTIPLY BY L INVERSE BY SOLVING
BACKWARDS
      DO 10 I = 1,IP
      T(I,IQ) = T(I,IQ) / STORE( (IQ*(IQ+1))/2 )
      IF( IQ .EQ. 1 ) GO TO 10
      IQM1 = IQ - 1
      DO 9 JJ = 1,IQM1
      J = IQ - JJ
      JP1 = J + 1
      S = 0.
      DO 8 K = JP1,IQ
  8   S = S + T(I,K)*STORE( (K*(K-1))/2 + J )
      T(I,J) = ( T(I,J) - S ) / STORE( (J*(J+1))/2 )
  9   CONTINUE
  10  CONTINUE
C                      FINALLY POSTMULTIPLY BY CHOLESKY FACTOR E
      DO 13 I = 1,IP
      DO 12 J = 1,IQ
      L = (J*(J-1))/2
      S = 0.
      DO 11 K = 1,J
      S = S + T(I,K)*QCHFAC( L + K )
  11  CONTINUE
      T(I,J) = S
  12  CONTINUE
  13  CONTINUE
      RETURN
      END
      REAL FUNCTION GCHIRV(A)
C
C  ALGORITHM TO GENERATE RANDOM VARIABLES WITH THE CHI DISTRIBUTION
C  WITH A DEGREES OF FREEDOM,  FOR A GREATER THAN OR EQUAL TO ONE
C
C  J F MONAHAN (MAY, 1986) DEPT OF STATISTICS, NCSU, RALEIGH, NC USA
C
      REAL A
      REAL ALPHA,ALPHM1,BETA,U,V,VMIN,VDIF,Z,ZZ,RNUM,W,S,EMHLF,VMAXU,
     *     RSQRT2,EMHLF4,EQTRT2,C
C
      DATA EMHLF/ .6065307 /, VMAXU/ .8577639 /, RSQRT2/ .7071068 /
      DATA EMHLF4/ .1516327 /, EQTRT2/ 2.568051 /, C/ 1.036961 /
C
      DATA ALPHA/ 0. /
C                     IS THIS ALPHA THE SAME AS THE LAST ONE?
      IF( A .EQ. ALPHA ) GO TO 1
C                     DO A LITTLE SETUP
      ALPHA = A
      ALPHM1 = ALPHA - 1.
      BETA = SQRT( ALPHM1 )
C                     GET DIMENSIONS OF BOX
      VMAXU = EMHLF * ( RSQRT2 + BETA )/( .5 + BETA )
      VMIN =  -BETA
      IF( BETA .GT. 0.483643 ) VMIN = EMHLF4/ALPHA - EMHLF
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```
      VDIF = VMAXU - VMIN
C                     START ( AND RESTART ) ALGORITHM HERE
  1   CONTINUE
      U = RAN(1)
      V = VDIF*RAN(2) + VMIN
      Z = V / U
C                     DO SOME QUICK REJECT CHECKS FIRST
      IF( Z .LE. - BETA ) GO TO 1
      ZZ = Z * Z
      RNUM = 2.5 - ZZ
      IF( V .LT. 0. ) RNUM = RNUM + ZZ * Z / ( 3. * (Z + BETA ) )
C                     DO QUICK INNER CHECK
      IF( U .LT. RNUM/EQTRT2 ) GO TO 9
      IF( ZZ .GE. C / U + 1.4 ) GO TO 1
C                     ABOVE WAS KNUTH'S NORMAL OUTER CHECK
      W = 2. * ALOG( U )
C                     NOW THE REAL CHECK
      S = - ( ZZ / 2. + Z * BETA )
      IF( BETA .GT. 0. ) S = ALPHM1*ALOG(1.+Z/BETA) + S
      IF( W .GT. S ) GO TO 1
C                     SUCCESS HERE -- TRANSFORM AND GO
  9   GCHIRV = Z + BETA
      RETURN
      END
      SUBROUTINE CHLSKZ(A,N,DET,IDET)
C  CHLSKZ COMPUTES THE CHOLESKY (SQUARE-ROOT)  FACTORIZATION
C   A = L * ( L - TRANSPOSE )   WHERE L IS LOWER TRIANGULAR
C     A STORED IN SYMMETRIC MODE AND OVERWRITTEN BY L
C  SUBROUTINE ADJUST KEEPS DETERMINANT FROM EXPLODING USING
C  1 LE DET LE 16   AND   DETERMINANT OF A IS DET*2**IDET
C
C  J F MONAHAN (DEC,1983) DEPT OF STAT, N C S U, RALEIGH, N C 27650 USA
C
      REAL A(1),DET,T,S
      INTEGER N,IDET,I,J,K,IM1,JM1
C                                       INITIALIZE FOR DETERMINANT
      DET = 1.
      IDET = 0
C                                         DO I-TH ROW
      DO 6 I=1,N
      T = A( (I*(I+1))/2 )
C                                         FIRST ROW IS TRIVIAL
      IF(I.EQ.1) GO TO 5
      IM1 = I-1
      DO 7 J=1,IM1
C                                         WORK ON (I,J)-TH ELEMENT
      S = A( (I*(I-1))/2 + J )
      IF(J.EQ.1) GO TO 9
      JM1 = J-1
      DO 8 K=1,JM1
  8   S = S - A( (J*(J-1))/2 + K ) * A( (I*(I-1))/2 + K )
  9   A( (I*(I-1))/2 + J )= S / A( (J*(J+1))/2 )
C                                   WORK ON DIAGONAL ELEMENT
  7   T = T - A( (I*(I-1))/2 + J ) * A( (I*(I-1))/2 + J )
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```
C                                     FINISHED WITH LOOP ON J
C                                     UPDATE DETERMINANT WITH DIAGONAL
  5   DET = DET * T
C                                     KEEP DET FROM EXPLODING
      CALL ADJUST(DET,IDET)
C                                     ABORT IF DET IS NEGATIVE OR ZERO
      IF(IDET.LT.-2000000000) RETURN
C                                     FINISH A POSITIVE DIAGONAL
      A( (I*(I+1))/2 ) = SQRT(T)
  6   CONTINUE
      RETURN
      END
      SUBROUTINE ADJUST(D,I)
      REAL D
C    ADJUST KEEPS DET FROM EXPLODING
      IF(D.LE.0.) GO TO 6
  3   IF(D.GE.1.) GO TO 4
      D=D*16.
      I=I-4
      GO TO 3
  4   IF(D.LE.16.) RETURN
      D=D/16.
      I=I+4
      GO TO 4
  6   I=-2147483644
      RETURN
      END
      REAL FUNCTION UNSK(IXX)
C   GENERATES NORMAL(0,1) RV'S USING KNUTH'S (V.2,2ND ED P125-7) VERSION
C     OF KINDERMAN-MONAHAN RATIO OF UNIFORMS (ACMTOMS,1977,P257-60)
METHOD
      DATA A/1.7155277/,B/5.136101667/,C/1.036961/
  1   U=RAN(1)
C   A=SQRT(8/E)
      V=RAN(2)
      UNSK=A*(V-0.5)/U
      ZZ=UNSK*UNSK
C   B=4*EXP(1/4)
      IF(ZZ.LE.5.-B*U) RETURN
C   THIS IS KNUTH'S QUICK REJECT TEST,  C=4*EXP(-1.35)
      IF(ZZ.GE.C/U+1.4) GO TO 1
      IF(ZZ.LE.-4.*ALOG(U)) RETURN
      GO TO 1
      END
      REAL FUNCTION RAN(IXX)
C   UNIFORM PSEUDORANDOM NUMBER GENERATOR
C   FORTRAN VERSION OF LEWIS, GOODMAN, MILLER
C   SCHRAGE,  ACM TOMS V.5 (1979) P132
C   FIRST CALL SETS SEED TO IXX, LATER IXX IGNORED
      INTEGER A,P,IX,B15,B16,XHI,XALO,LEFTLO,FHI,K
      DATA A/16807/,B15/32768/,B16/65536/,P/2147483647/
      DATA IX/0/
      IF(IX.EQ.0) IX=IXX
      XHI=IX/B16
```

```
      XALO=(IX-XHI*B16)*A
      LEFTLO=XALO/B16
      FHI=XHI*A+LEFTLO
      K=FHI/B15
      IX=(((XALO-LEFTLO*B16)-P)+(FHI-K*B15)*B16)+K
      IF(IX.LT.0) IX=IX+P
      RAN=FLOAT(IX)*4.656612875E-10
      RETURN
      END
$DATA
         400     23.000000
   4
   1.000000   0.000000
   0.000000   2.000000
   0.000000   0.000000   4.000000
   0.000000   0.000000   0.000000   8.000000
   2
   1.000000
   0.000000   9.000000
$EOF
```

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