RANDOMIZED-SEQUENTIAL GROUP TESTING PROCEDURES

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ABSTRACT

Dorfman and Dorfman-Sterrett procedures are extended by introducing a random element in decisions on when to revert to group testing after individual testing has commenced. Properties of the proposed procedures, when inspection errors are present, are investigated, and some useful approximations are suggested.

INTRODUCTION

In a series of papers (see, e.g., Johnson et al. (1985), (1986), (1988) and the bibliographies therein for an applications-oriented survey of some of the results) the effects of inspection errors on a number of procedures for detecting nonconforming (NC) items among a set of n items have been studied.

The simplest such scheme is individual testing of each item. This requires n tests. The other procedures have been designed with the aim of reducing the number of tests required. If inspection is perfect then all procedures ultimately identify all the NC items and no others. If errors occur, then this is no longer so. In such cases, in addition to the expected number of tests required, it is necessary to consider the probabilities of correct assessment, PC(NC), PC(C) for nonconforming and conforming (C) items, respectively.

The procedures so far considered include:

1) Dorfman group testing. Dorfman (1943) proposed that (if possible) a single test be applied to all n items as a group to determine if there is at least one NC item among them. Only if the presence of at least one item is indicated, is individual testing undertaken. A set containing at least one NC item will be termed a *NC set*; a set with no NC items is a *C set*.

2) Hierarchical Dorfman group testing. The set of n items is divided into h_1 subsets of n_1 items each; each subset is divided into h_2 sub^2 sets of n_2 items each and so on. ($n = h_1 n_1 = h_2 h_1 n_2 = \ldots$). As in 1) the whole set is tested first, but if the presence of at least one NC item is indicated, each subset is tested (rather than immediate recourse to individual testing). The procedure continues (testing each sub^i set in any sub^{i-1}set found to contain NC items, and so on) until the smallest (sub^k sets) are reached. Only if the latter give NC indication is individual testing undertaken.

3) Dorfman-Sterrett group testing. Sterrett (1957) modified Dorfman's procedure by requiring reversion to group testing of the remaining (untested) items after any item is found NC on an individual test. There are various further modifications of this procedures:

(a) there may be a limit on the number of reversions to group testing;

(b) reversions may occur only after $k(> 1)$ items are found to be NC.

Combinations of 1), 2) and/or 3) are also possible. Combining Dorfman-Sterrett with hierarchal classification we might consider a procedure in which, as in the Dorfman procedure,

(i) the whole set of n items is tested for the presence of at least one NC item, and

(ii) if the presence of an NC item is indicated, all items in the first of h subsets, each of size n_1, are tested individually;

then, *depending on the results obtained in (ii)*, all items in the second subset are tested individually *or* the remaining $(n-n_1)$ items are tested as a group (then proceeding as in (i); and so on).

The dependence on the results in (ii) might be deterministic - for example, proceed to individual testing of the next subset if at least k items (among the n_1 in the subset tested individually) are found to be NC. The Dorfman-Sterrett procedure is a special case of this type with $n_1 = 1$ (so $h_1 = n$) and $k = 1$. (The 'subsets' are, in fact, individual items.)

Since we may find $z = 0,1,\ldots,(n_1-1)$ or n_1 items which appear to be NC among the n_1 tested in the subset it would seem reasonable to try to take into account the actual value of z in a more fine-tuned manner. This can be done by making the decision to revert to group testing a random one, with the probability of such reversion depending on z.

In this paper we will study the effects of errors in inspection on such procedures. We will confine ourselves to

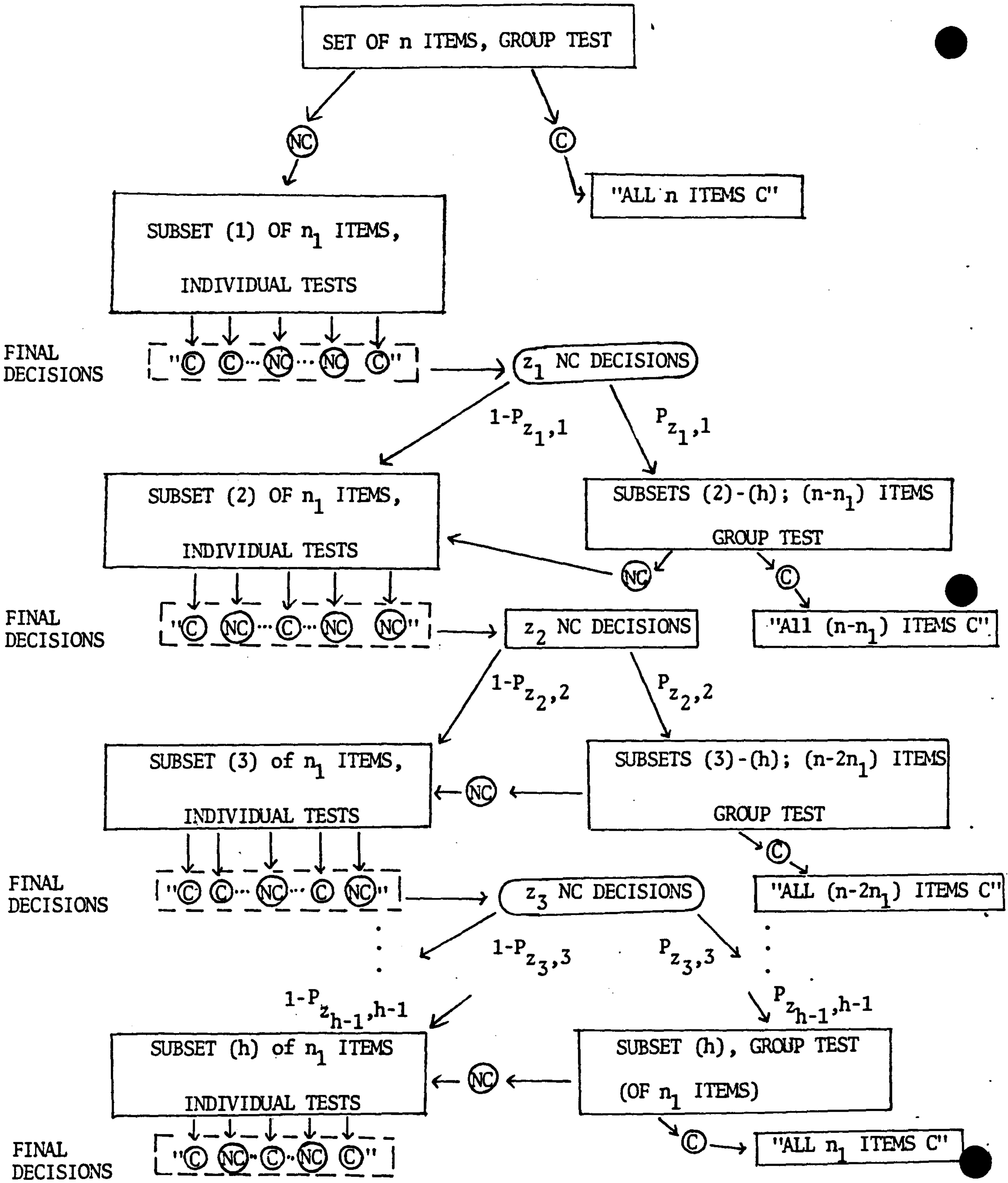

Figure 1. RANDOM-SEQUENTIAL PROCEDURE - FLOW CHART
["FINAL DECISIONS" IN QUOTES]

one-stage hierarchies - that is, with no further subdivision beyond subsets. Some notation will now be introduced.

Notation. In accordance with the practice in earlier papers (see, e.g., Kotz and Johnson (1982)) we will use p to denote the probability that a NC decision is (correctly) obtained when nonconformity is really present, and p' to denote the probability that this decision is (incorrectly) obtained when there is no nonconformity. Subscripts are used to indicate the situations to which these symbols apply.

The absence of a subscript indicates a reference to individual testing; the subscript 0 refers to group (or subgroups) testing. It is possible to allow all these probabilities to depend on the size and/or constitution of the group (or set of subgroups) tested, at the expense of some formal complexity, but this will not be done here. (See also next Section.)

We denote the number of truly NC items in the set of n items by Y, with $Y_1, Y_2, \ldots, Y_h$ denoting the numbers in successive subgroups of size n_1 ($n_1h=n$). Assuming the independence of tests, the number Z_j of items found to be NC when the j-th subgroup is tested individually is distributed as the sum of two independent binomial variables with parameters (y_j, p) and (n_1-y_j, p') - conditional on $Y_j = y_j$ ($j=1,\ldots,h$). We have

$$\pi_{z_j|y_j}=\Pr[Z_j=z_j]=\sum_{i=0}^{z_j}\binom{y_j}{i}\binom{n_1-y_j}{z_j-i}p^i p'^{z_j-i}(1-p)^{y_j-i}(1-p')^{n_1-y_j-z_j+i}$$

$$(z_j = 0,1,\ldots,n_1).$$

We will allow the probability of resorting to group testing for the remaining ($n-n_1j$) untested individuals, given an observed value, z_j, of Z_j to depend on j as well as z_j; we will denote it by $P_{z_j,j}$. Then the probability that the remaining $(n-n_1j)$ items are subjected to a group test, given that the j-th subgroup is tested individually is

$$\tau_{j|y_j} = \sum_{z_j=0}^{n_1} \pi_{z_j|y_j} P_{z_j,j} .$$

(When no confusion can be caused, the symbol τ_j will be used.)

The values $\{P_{z,j}\}$ can be chosen arbitrarily. They constitute the *inspection strategy*. It is to be hoped that, by a suitable choice of inspection strategy, good procedures - (that is, economical in testing and reasonably robust to inspection errors) - can be obtained. Note that if $P_{z,j}=0$ for all z and j we have the original Dorfman procedure.

It will be assumed that the group of n items has been chosen at random from a large (effectively infinite) population. The proportion of NC items in this population will be denoted by ω.

Figure 1 is a flow chart of the procedure. Analytical properties will now be discussed.

ANALYSIS

As already mentioned, in this paper we suppose the probabilities of decision for any group test do not depend on the size of the set tested. The probabilities of NC decision will be denoted by p_0 (for group testing) and p (for individual testing) for truly NC items or (sub)sets; for truly C items or groups the symbols will be p_0' and p' respectively. We also suppose that the total number of items is a multiple of the subset size, so that $n=hn_1$.

A further assumption is that results of tests are mutually independent.

We first make calculations conditioned on fixed values $y_1,\ldots,y_h$ of the actual numbers of NC items in the 1st,..., h-th subsets. The resulting expressions then need to be averaged over the joint distribution of $Y_1,\ldots,Y_h$, with appropriate weights.

We also introduce the symbols

$$t_j = y_{j+1} + \ldots + y_h$$

and

$$\phi_j = \begin{cases} p_0 & \text{if} \quad t_j > 0 \\ p_0' & \text{if} \quad t_j = 0 \end{cases} \qquad (j = 0,1,\ldots,h-1);$$

here t_j is the number of NC items among the last (h-j) subsets, and ϕ_j is the probability that a NC decision will be obtained on testing them as a group. (Note that t_0 corresponds to the total number of NC items.)

Given that the j-th subset is tested individually, the probability that the (j+1)-th subset, also, is tested individually is

θ_j = Pr[last (h-j) subsets not tested as a group]

+ Pr[last (h-j) subsets tested as a group, and NC decision obtained]

$= 1 - \gamma_j + \gamma_j \phi_j \quad (j = 1,\ldots,h-1)$.

The probability that the first subset is tested individually is $\theta_0 = \phi_0$.

The probability that the j-th subset is tested individually is $\prod_{i=0}^{j-1} \theta_i$, $(j = 2,\ldots,h)$.

When the j-th subset is tested individually, there will certainly be n_1 individual tests, plus, with probability γ_j, a group test of the remaining (h-j) subsets. Hence the expected number of tests, given $\underset{\sim}{y}$, is

$$E_{\underset{\sim}{y}} = 1 + \sum_{j=1}^{h} \{ \prod_{i=0}^{j-1} \theta_i \} (n_1 + \gamma_j) \quad \text{with } \gamma_h = 0. \tag{1}$$

(The '1' on the right hand side of (1) corresponds to the initial group test of all n items.)

For any given NC item (and given $\underset{\sim}{y} = (y_1,\ldots,y_h)$):

$$PC(NC|\underset{\sim}{y}) = \sum_{j=1}^{h} \Pr[\text{NC is in j-th subset}] \Pr[\text{j-th subset is tested individually}]\, p$$

$$= \sum_{j=1}^{h} \{(y_j t_0^{-1}) (\prod_{i=0}^{j-1} \theta_i)\} p = (p\, t_0^{-1}) \sum_{j=1}^{h} (y_j \prod_{i=0}^{j-1} \theta_i). \tag{2}$$

For any given C item:

$$PC(C|\underset{\sim}{y}) = \sum_{j=1}^{h} \Pr[\text{C is in j-th subset}] \{\Pr[\text{j-th subset } \textit{not} \text{ tested individually}]$$

$$+ \Pr[\text{j-th subset tested individually}] (1-p')\}$$

$$= \sum_{j=1}^{h} \left[\frac{n_1 - y_j}{n - t_0} \{1 - \prod_{i=0}^{j-1} \theta_i + (\prod_{i=0}^{j-1} \theta_i)(1-p')\} \right]$$

$$= 1 - p'(n-t_0)^{-1} \sum_{j=1}^{h} \{(n_1 - y_j)(\prod_{i=0}^{j-1} \theta_i)\} . \tag{3}$$

In order to obtain the unconditioned values, (1), (2) and (3) must be averaged over all $\underset{\sim}{y}$ with appropriate weights.

The probability that $\underset{\sim}{Y} = \underset{\sim}{y}$ (i.e. $Y_j = y_j$; for j=1,...,h) is

$$P_{\chi} = \prod_{j=1}^{h} \binom{n_1}{y_j} \omega^{y_j}(1-\omega)^{n_1-y_j}$$

$$= \{ \prod_{j=1}^{h} \binom{n_1}{y_j}\} \omega^{t_0}(1-\omega)^{n-t_0}. \qquad (4)$$

For the expected number of tests, we have

$$E = \sum_{\chi} P_{\chi} E_{\chi}. \qquad (5)$$

The expected percentage reduction (as compared with simple individual inspection) in number of tests is

$$EPR=100(1-E/n) \qquad (5')$$

For probability of correct classification of NC item, (since $PC(NC|\chi)$ applies to each of t_0 NC items):

$$PC(NC) = \sum_{\chi} t_0 P_{\chi} PC(NC|\chi)/(\sum_{\chi} t_0 P_{\chi})$$

$$= \sum_{\chi} t_0 P_{\chi} PC(NC|\chi)/E[T_0] = \sum_{\chi} t_0 P_{\chi} PC(NC|\chi)/(n\omega). \qquad (6)$$

For probability of correct classification of a C item (since $PC(C|\chi)$ applies to each of $(n-t_0)$ C items)

$$PC(C) = \sum_{\chi} (n-t_0)P_{\chi} PC(C|\chi)/\{\sum_{\chi}(n-t_0)P_{\chi}\}$$

$$= \sum_{\chi} (n-t_0)P_{\chi} PC(C|\chi)/\{n(1-\omega)\}. \qquad (7)$$

Tables I and II (extracted from a considerably more extensive set, available from the second author) provide some representative values of EPR, PC(C) and PC(NC) for n = 12, a few combinations of values of the parameters n_1, p_0, p_0', p, p' and $P_{z,j}$. Although it is necessary, in any specific case, to take into account details specific thereto, it is possible to make some useful general comments.

Even without studying specific numerical values, it can be seen that

(i) $PC(C) \geq 1 - p'$

and (ii) $PC(NC) \leq p_0 p$.

This is because (i) an item can be declared C either on individual testing <u>or</u> as a result of a group test, and (ii) to be declared NC an item <u>must</u> be tested individually.

Conversely, any item which is not tested individually <u>must</u> have been declared C. We would expect, therefore, that any procedure that increases the amount of group testing will tend to increase PC(C), but, unfortunately, to decrease PC(NC).

Of course, numerical values assist in assessing the relative importance of these two effects, though it is also desirable to have some idea of relative costs (and also sampling costs, when taking EPR into consideration).

From Tables I and II, we see that

(a) EPR is not greatly affected by inspection strategy

(b) PC(NC) is affected more substantially by inspection strategy. There is an adverse effect when strategies with $P_{z,j}$ decreasing with z are used - more specifically when $P_{0,j}$ is high.

(c) EPR increases as ω decreases.

If it is desired to optimize only one of the values PC(C) and PC(NC) a deterministic strategy, with each $P_{z,j}$ equal to either 0 or 1, will be nearly optimal, but when some compromise is needed a probabilistic strategy may be advantageous.

We now present studies of a few specific cases. We take n = 12, $n_1 = 2$ and inspection strategy

$$P_{0,j} = 0;\ P_{1,j} = 1-(0.5)^j;\ P_{2,j} = 1 \quad (j=1,\ldots,h-1)$$

Table I: $n = 12$; $n_1 = 2$; $h = 6$

$z = 0, 1, 2$				$P_{z,j} = \begin{cases} 1.00 \\ (0.50)^j \\ 0.00 \end{cases}$			1.00, 0.50, 0.00			0.00, $1 - (0.50)^j$, 1.00			0.00, 0.50, 1.00		
P_0	P'_0	P	P'	EPR	PC(C)	PC(NC)	EPR	PC(C)	PC(NC)	EPR	PC(C)	PC(NC)	EPR	PC(C)	PC(NC)
$\omega=0.01$															
.99	.01	.99	.01	80.7	.9993	.9566	81.1	.9993	.9564	82.3	.9992	.9794	81.4	.9991	.9796
.95	.05	.95	.05	80.8	.9964	.8053	81.1	.9966	.8028	79.4	.9947	.8930	78.6	.9942	.8959
.95	.10	.95	.05	79.3	.9959	.8053	79.7	.9961	.8028	75.3	.9927	.8930	74.4	.9921	.8959
.90	.10	.90	.10	80.4	.9924	.6563	80.7	.9928	.6495	76.7	.9871	.7798	75.7	.9857	.7884
.80	.20	.80	.20	78.2	.9807	.4484	78.6	.9818	.4337	72.6	.9677	.5569	71.1	.9637	.5776
$\omega=0.05$															
.99	.01	.99	.01	46.4	.9970	.9579	47.7	.9972	.9573	54.8	.9970	.9780	52.1	.9966	.9787
.95	.05	.95	.05	50.3	.9861	.8103	51.4	.9870	.8064	54.0	.9846	.8874	51.6	.9827	.8918
.95	.10	.95	.05	49.2	.9856	.8103	50.2	.9865	.8064	51.0	.9831	.8874	48.7	.9812	.8918
.90	.10	.90	.10	54.2	.9742	.6632	51.1	.9758	.6546	53.7	.9692	.7715	51.5	.9656	.7822
.80	.20	.80	.20	58.8	.9527	.4549	59.8	.9562	.4387	54.3	.9410	.5487	52.7	.9341	.5708
$\omega=0.10$															
.99	.01	.99	.01	18.1	.9949	.9596	19.5	.9953	.9585	31.5	.9951	.9764	28.0	.9945	.9775
.95	.05	.95	.05	25.1	.9768	.8164	26.3	.9783	.8109	32.4	.9762	.8807	29.5	.9733	.8868
.95	.10	.95	.05	24.2	.9765	.8164	25.4	.9779	.8109	30.3	.9750	.8807	27.7	.9723	.8868
.90	.10	.90	.10	32.3	.9579	.6718	33.6	.9606	.6610	34.5	.9542	.7615	32.0	.9491	.7744
.80	.20	.80	.20	42.9	.9282	.4631	44.3	.9335	.4450	40.4	.9189	.5389	38.1	.9100	.5623

Table I (continued): $n = 12;\ n_1 = 2;\ h = 6$

z = 0, 1, 2				$P_{z,j} = \begin{cases}1.00 \\ (0.90)^j \\ (0.80)^j\end{cases}$			1.00, 0.90, 0.80			0.00, $1-(0.75)^j$, 1.00			0.00, 0.25, 1.00		
P_0	P_0'	P	P'	EPR	PC(C)	PC(NC)	EPR	PC(C)	PC(NC)	EPR	PC(C)	PC(NC)	EPR	PC(C)	PC(NC)
				ω=0.01											
.99	.01	.99	.01	81.4	.9994	.9561	81.6	.9994	.9560	81.2	.9991	.9797	80.5	.9990	.9798
.95	.05	.95	.05	81.4	.9968	.7993	81.6	.9969	.7981	78.4	.9941	.8964	77.7	.9936	.8990
.95	.10	.95	.05	80.0	.9964	.7993	80.2	.9965	.7981	74.2	.9920	.8964	73.3	.9914	.8990
.90	.10	.90	.10	81.2	.9934	.6394	81.2	.9936	.6354	75.4	.9855	.7901	74.4	.9841	.7980
.80	.20	.80	.20	79.7	.9843	.4086	79.7	.9847	.4012	70.5	.9627	.5819	68.9	.9582	.6015
				ω=0.05											
.99	.01	.99	.01	48.7	.9973	.9565	49.3	.9975	.9562	51.6	.9966	.9788	49.1	.9962	.9793
.95	.05	.95	.05	52.3	.9877	.8008	52.8	.9882	.7986	51.3	.9826	.8927	48.9	.9808	.8966
.95	.10	.95	.05	51.2	.9873	.8008	51.7	.9878	.7986	48.3	.9811	.8927	46.1	.9793	.8966
.90	.10	.90	.10	56.2	.9776	.6414	56.6	.9784	.6369	51.2	.9654	.7842	49.0	.9617	.7940
.80	.20	.80	.20	61.7	.9612	.4104	62.2	.9627	.4021	52.2	.9333	.6459	49.9	.9257	.5964
				ω=0.10											
.99	.01	.99	.01	20.3	.9955	.9570	21.2	.9957	.9564	27.8	.9945	.9977	24.7	.9939	.9786
.95	.05	.95	.05	27.2	.9794	.8026	27.9	.9803	.7997	29.4	.9734	.8880	26.6	.9706	.8935
.95	.10	.95	.05	26.3	.9791	.8026	27.0	.9799	.7997	27.5	.9724	.8880	24.8	.9697	.8935
.90	.10	.90	.10	34.8	.9633	.6439	35.4	.9648	.6381	31.7	.9454	.7769	29.2	.9440	.7888
.80	.20	.80	.20	46.7	.9409	.4126	47.3	.9433	.4032	37.6	.9095	.5675	35.2	.8999	.5898

TABLE II $n = 12$; $\omega = 0.05$

$n_1=3$; $h = 4$

z	$P_{z\|j}$ =			
0	1.00	1.00	0.00	0.00
1	$(2/3)^j$	$(0.9)^j$	$1 - (2/3)^j$	$1 - (0.9)^j$
2	$(1/3)^j$	$(0.7)^j$	$1 - (1/3)^j$	$1 - (0.7)^j$
3	0.00	$(0.5)^j$	1.00	$1 - (0.5)^j$

P_0	P'_0	P	P'	EPR	PC(C)	PC(NC)	EPR	PC(C)	PC(NC)	EPR	PC(C)	PC(NC)	EPR	PC(C)	PC(NC)
0.95	0.05	0.95	0.05	51.2	.9848	.8452	53.0	.9862	.8401	50.7	.9822	.8938	47.4	.9797	.8993
0.95	0.10	0.95	0.05	49.7	.9842	.8452	51.5	.9856	.8401	47.8	.9807	.8938	44.7	.9783	.8993
0.90	0.10	0.90	0.10	53.3	.9708	.7164	55.2	.9726	.7040	50.7	.9644	.7874	47.3	.9593	.8012
0.80	0.20	0.80	0.20	55.7	.9431	.5171	58.2	.9502	.4902	51.3	.9305	.5848	47.4	.9187	.6164

$n_1=4$; $h = 3$

z	$P_{z\|j}$ =			
0	1.00	1.00	0.00	0.00
1	$(0.75)^j$	$(0.9)^j$	$1 - (0.75)^j$	$1 - (0.9)^j$
2	$(0.50)^j$	$(0.8)^j$	$1 - (0.50)^j$	$1 - (0.8)^j$
3	$(0.25)^j$	$(0.7)^j$	$1 - (0.25)^j$	$1 - (0.7)^j$
4	0.00	$(0.6)^j$	1.00	$1 - (0.6)^j$

P_0	P'_0	P	P'	EPR	PC(C)	PC(NC)	EPR	PC(C)	PC(NC)	EPR	PC(C)	PC(NC)	EPR	PC(C)	PC(NC)
0.95	0.05	0.95	0.05	51.4	.9840	.8632	52.8	.9851	.8603	48.8	.9807	.8973	46.8	.9792	.9003
0.95	0.10	0.95	0.05	49.7	.9833	.8632	51.0	.9843	.8603	45.9	.9792	.8973	44.0	.9778	.9003
0.90	0.10	0.90	0.10	52.6	.9685	.7446	54.1	.9708	.7371	48.7	.9612	.7963	46.5	.9580	.8042
0.80	0.20	0.80	0.20	53.7	.9367	.5513	55.7	.9427	.5335	49.0	.9231	.6056	46.2	.9150	.6251

with two sets of decision probabilities

(a) $p_o = p = 0.98$; $p_o' = p' = 0.05$;

(b) $p_o = 0.9$; $p = 0.95$; $p_o' = p' = 0.05$.

Values for the Dorfman procedure are also shown the following table

ω	Random-Sequential			Dorfman		
Set (a)	EPR	PC(C)	PC(NC)	EPR	PC(C)	PC(NC)
0.05	52.8	.9843	.9539	43.9	.9774	.9604
0.10	30.25	.9754	.9509	20.9	.9656	.9604
0.20	4.25	.9658	.9452	0.0	.9556	.9604
Set (b)						
0.05	56.2	.9855	.8268	47.8	.9792	.8550
0.10	36.4	.9777	.8143	25.7	.9683	.8550
0.20	15.1	.9697	.7913	7.5	.9587	.8550

In this particular comparison it can be seen that the random-sequential procedure requires substantially less inspection, on average, and increases PC(C) as compared with the Dorfman procedure. However, it leads to a decrease in PC(NC) which might be of greater importance if the nonconformity is of serious nature. Note that the lower values of p_o and p in (b) result, as is to be expected, in lower probabilities of detecting NC items by either procedure. This effect is of greater magnitude then that resulting from differences between the procedures. For large values of p_0 and p, (such as $p_0, p \geq 0.98$, say), however, the decrease in PC(NC) is slight.

ADDENDA

(I) We plan to study the effects of departures from our assumptions on the properties of randomized-sequential procedures. In an initial inquiry, we have supposed that the probability (p_0) of a NC decision for group test of the last (h-j) subsets containing at least one truly NC item is

$$p_{00} - k \times \text{(proportion of C items)}$$

$$= p_{00} - k[1 - t_j/\{(h-j)n_1\}] \quad (t_j > 0) \qquad (8)$$

$$(j=0,1,\ldots,h-1)$$

with k=0.1 for illustration. This reflects a dilution effect (see Hwang (1976)). [If $t_j=0$ we still have arbitrary p_0' for probability of a NC decision.]

Table III shows values obtained when n=12, $n_1=2$, h=6; $p_{00}=0.98$, $p_0'=0.05$, p=0.98, p'=0.05, for four inspection strategies and three values (0.01, 0.05, 0.10) of ω.

Values obtained with fixed values, 0.88 and 0.90, of p_0 are also included for purposes of comparison, to assess the accuracy of certain rough approximations. It seems reasonable that a fixed "equivalent value" for p_0, equal to a rough average of (8) might give useful approximations. The expected value of

$$p_{00} - 0.1[1 - T_j/\{(h-j)n_1\}]$$

depends on j (as well as ω). Since the initial test (j=0) will *always* be used we consider using the corresponding expected value of T_0. Even with this restriction, the appropriate values of $E[T_0|T_0 > 0]$ depend on which of EPR, PC(C) and PC(NC) is being approximated.

For EPR, $E[T_0|T_0 > 0] = n\omega\{1 - (1-\omega)^n\}^{-1}$; (9.1)

for PC(C), $E[T_0|T_0 > 0] = (n-1)\omega\{1-(1-\omega)^{n-1}\}^{-1}$; (9.2)

for PC(NC), $E[T_0] = 1 + (n-1)\omega$. (9.3)

(T_0 is necessarily greater than zero in the last case.)

These three values do not vary much if ω is small and n is not too small (≥ 3, say). In the last case the suggested "equivalent p_0" is

$$p_{00} - 0.1[1-n^{-1}\{1 + (n-1)\omega\}] = p_{00} - 0.1(1-n^{-1})(1-\omega) \ . \qquad (10)$$

For the parameter values used in Table III, the "equivalent p_0" values from (10) are

0.889, 0.893 and 0.8975

for $\omega = 0.01$, 0.05 and 0.10 respectively.

The approximations are quite good, but it appears that somewhat greater (by about 0.007) values for p_0 would give even better results.

For given ω, the smaller the size of group ('n' or $(h-j)n_1$) the larger the "equivalent p_0". Since smaller groups are used in the reversions to group testing (in the course of individual testing) one would expect values greater than those given by (10) to produce some improvement in approximation. Taking an average value (between 12 and the minimum possible, 2) of 7 for 'n' in (10) produces "equivalent p_0" values of

0.895, 0.899 and 0.902

for $\omega = 0.01$, 0.05 and 0.10 respectively.

Very nearly the same values for approximating values of EPR and PC(C) are obtained using (9.1) and (9.2), namely:

EPR: 0.895, 0.897 and 0.899

PC(C): 0.895, 0.896 and 0.898

for $\omega = 0.01$, 0.05 and 0.10 respectively.

(II) The analysis of this paper may be extended to situations in which there are more than one kind (g, say) of nonconformities. Special consideration has to be given to the inspection strategy. In general, the probability of proceeding to a group test of the remaining subsets (not yet tested individually) might depend on the vector $\underset{\sim}{z}$ of numbers of items with each of the possible 2^g combinations of nonconformities.

It might also be necessary to allow for possible differences in probabilities of correct classification for

Table III. $n = 12$; $n_1 = 2$; $h = 6$; $p_0' = 0.05$; $p = 0.98$; $p = 0.05$

(*'Variable" denotes p_0 value is 0.98-0.1×(proportion of C items) in group testing)

	$z = 0$	$P_{z,j} =$	0			0			0			1.0	
	1		0			$1-(0.5)^j$			1.0			1.0	
	2		1.0			1.0			1.0			1.0	
ω	p_0	EPR	PC(C)	PC(NC)	EPR	PC(C)	PC(NC)	EPR	PC(C)	PC(NC)	EPR	PC(C)	PC(NC)
0.01	0.88	77.4	.9933	.8615	80.1	.9951	.8407	81.1	.9956	.8332	83.2	.9975	.6415
	0.90	77.2	.9932	.8812	79.9	.9950	.8635	80.9	.9955	.8570	82.8	.9973	.6888
	*Variable	77.3	.9932	.8707	80.0	.9950	.8530	81.0	.9956	.8461	83.0	.9974	.6754
0.05	0.88	49.4	.9802	.8600	57.3	.9860	.8278	59.7	.9876	.8161	59.9	.9904	.6415
	0.90	48.5	.9798	.8799	56.4	.9857	.8524	58.9	.9873	.8425	58.1	.9899	.6888
	*Variable	48.9	.9800	.8730	56.7	.9858	.8462	59.2	.9875	.8359	58.6	.9901	.6840
0.10	0.88	28.6	.9702	.8571	38.2	.9785	.8125	41.0	.9807	.7965	39.7	.9841	.6415
	0.90	27.1	.9695	.8775	36.7	.9779	.8392	39.4	.9801	.8255	36.7	.9832	.6888
	*Variable	27.5	.9697	.8752	37.0	.9781	.8393	39.7	.9802	.8255	37.0	.9834	.6947

different combinations of nonconformities. Also, one would need formulas for correct detection of each possible combination of nonconformities. The analysis would follow the same lines as in this paper, but $\underset{\sim}{y}$ would now be a $2^g \times h$ matrix showing numbers of each possible combination of nonconformities in each of the h subsets. The weights for combining the conditional probabilities of correct assignment would be proportional to the product of $P_{\underset{\sim}{y}}$ and the number of items with the appropriate combination according to $\underset{\sim}{y}$. The weights for the expected number of tests would still be $P_{\underset{\sim}{y}}$.

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