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On Properties of Poisson Q-Charts for Attributes

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The sensitivity of four tests on Shewhart type Q-charts and of specially designed EWMA and CUSUM Q-charts to detect one-step permanent shifts of a Poisson parameter λ has been studied. The usual test that signals for one point beyond three standard deviations on a Shewhart chart is found to have poor sensitivity, generally. The test that signals when four out of five consecutive points are beyond one standard deviation in the same direction is found to be a good omnibus test. The EWMA and CUSUM Q-charts are most sensitive and are about comparable in overall performance.

Introduction

Let X denote the number of defects on an inspected standard size unit of product. If the process is stable, so that the rate λ at which defects occur on the standard size unit is constant, then X may be considered a Poisson random variable. Notation for Poisson probability and distribution functions, and other functions required in this work, is defined in Table 1. For $(n_1, y_1), (n_2, y_2), \cdots$ a sequence of relative sample sizes and numbers of defects observed, Quesenberry (1991), Q91, defined Q-statistics that are distributed approximately as standard normal statistics and are also approximately independent. Quesenberry suggested that these statistics be plotted on Shewhart charts with control limits at ± 3, and pointed out that tests such as those of Nelson (1984), that use information from more than one of the plotted points, can be made on these charts. It is also apparent that these Q-statistics can be used as the input data to plot EWMA and CUSUM charts. Some users of these charting methods have expressed an interest in having more guidance in choosing the particular types of charts and tests to be applied on the charts. Since, for a stable Poisson process distribution, the two types of Q-statistics given in Q91 are both approximately independent with standard normal distributions, we know the type of point pattern that represents a stable process on a Shewhart Q-chart of these statistics. If the Poisson defect rate λ is not constant while the data are being taken, then we expect this, in general, to result in anomalous point patterns that are different from those of a stable process. In this work we study the effectiveness of four tests made on Shewhart charts of the Q-statistics and of EWMA and CUSUM charts to detect one-step changes in λ.

TABLE 1: Notation for Probability and Distribution Functions

Normal:		
$\Phi(\cdot)$	–	The standard normal distribution function.
$\Phi^{-1}(\cdot)$	–	The inverse of the standard normal distribution function.

Binomial:

$$b(x;n,p) = \binom{n}{x}p^x(1-p)^{n-x},\ x = 0, 1, 2, \cdots, n$$
$$= 0, \text{ elsewhere}$$

$$B(x;n,p) = 0,\ x < 0$$
$$= 1,\ x \geq n$$
$$= \sum_{x'=0}^{[x]} b(x';n,p),\ 0 \leq x < n$$

Poisson:

$$f(y;\lambda) = \frac{\lambda^y e^{-\lambda}}{y!},\ y = 0, 1, 2, \cdots$$

$$F(y;\lambda) = P(X \leq y) = 0,\ y < 0$$
$$= \sum_{y'=0}^{[y]} f(y';\lambda),\ y \geq 0$$

Tests for Shifts in λ

There are many possible tests that can be made on a Shewhart Q-chart to detect a shift in λ. In this work we consider four tests. Three of these tests were discussed by Nelson (1984). Given a sequence of Q-statistics $Q_{p+1}, Q_{p+2}, \cdots, Q_t, \cdots$; these tests are defined as follows.

The 1-of-1 Test: When Q_r is plotted, the test signals an increase in λ if $Q_r > 3$, and signals a decrease in λ if $Q_r < -3$. This is, of course, the classical Shewhart chart test.

The 9-of-9 Test: When Q_r is plotted, this test signals an increase in λ if $Q_r, Q_{r-1}, \cdots, Q_{r-8}$ all exceed 0, and signals a decrease in λ if $Q_r, Q_{r-1}, \cdots, Q_{r-8}$ are all less than 0. This test can only be made when nine consecutive Q-statistics are available.

The 3-of-3 Test: When Q_r is plotted, this test signals an increase in λ if Q_r, Q_{r-1} and Q_{r-2} all exceed 1, and signals a decrease in λ if Q_r, Q_{r-1} and Q_{r-2} are all less than -1. This test can only be made when three consecutive Q-statistics are available.

The 4-of-5 Test: When Q_r is plotted, this chart signals an increase in λ if at least four of the five values $Q_r, Q_{r-1}, \cdots, Q_{r-4}$, exceed 1, and signals a decrease in λ if at least four of the five values $Q_r, Q_{r-1}, \cdots, Q_{r-4}$ are less than -1. This test can only be made when five consecutive Q-statistics are available.

In addition to the four above tests that are applied on Shewhart Q-charts, we also consider here an exponentially weighted moving average, EWMA, and a cumulative sum, CUSUM, chart computed from the sequence of Q-statistics. The EWMA statistic Z_r is given by

$$Z_r = \lambda Q_r + (1-\lambda)\, Z_{r-1} \text{ for } r = 1, 2, \cdots \quad (1)$$

with $Z_0 = 0$. These values are plotted on a chart with control limits at $\pm K\sqrt{\lambda/(2-\lambda)}$. A procedure of Crowder (1989) was used to obtain values of λ and K that give an in-control ARL of 372.6 and an ARL of 5.18 to detect a shift in 1.5 standard deviations in a normal process mean. The values obtained were $(\lambda, K) = (0.25, 2.90)$, which gives control limits at $\pm$ 1.096. This same EWMA chart was used for all of the work reported here.

The EWMA Test: When Q_r and Z_r are computed, if $Z_r > \text{UCL}\ (Z_r) = 1.096$, an increase in p is signaled, and if $Z_r < -1.096$, a decrease in the p is signaled.

The CUSUM statistics S_r^+ and S_r^- are defined as follows:

$$S_r^+ = \text{Max}\left\{0, S_{r-1}^+ + Q_r - k_s\right\}$$
$$S_r^- = \text{Min}\left\{0, S_{r-1}^- + Q_r + k_s\right\} \quad (2)$$

with $S_r^+ = S_r^- = 0$.

The CUSUM Test: If $S_r^+ > h_s$ an increase in p is signaled, and if $S_r^- < -h_s$ a decrease in p is signaled.

The CUSUM test is determined by the reference value k_s and the decision interval h_s. We have used the values of $k_s = 0.75$ and $h_s = 3.34$ for the CUSUM test used in this study. These values were chosen to give a CUSUM test with approximately the same ARL's as the EWMA above for the in-control case (for this CUSUM this is 370.5) and to detect a 1.5 standard deviation shift in a normal process mean (for this CUSUM this is also 5.18, the same as for the EWMA test).

Poisson Q-Charts

Consider next the transformations given in Q91 for transforming observations from binomial distributions to values that are distributed approximately as independent standardized normal Q-statistics. Let y_r denote the number of defects on a sample of size n_r standard inspection units with known defects rate $\lambda = \lambda_0$ per standard inspection unit. The sequence of observed values (n_1, y_1), $(n_2, y_2), \cdots$ are transformed to Q-statistics by the formulas in equations (3).

Case K: $\lambda = \lambda_0$ known

$$u_r = F(y_r; n_r\lambda_0)$$
$$Q_r = \Phi^{-1}(u_r) \quad \text{for } r = 1, 2, \cdots \qquad (3)$$

These values $Q_1, Q_2, \cdots$ are distributed approximately as independent standard normal statistics. It was shown in Q91 that the normal approximation is much improved over that of the usual normal approximation to the Poisson distribution.

Consider next the case when the Poisson parameter λ is not assumed known at the start-up of a process. Some further notation is needed to treat this case. Again, we consider a sequence of values (n_r, y_r) for $r = 1, 2, \cdots$, and when the r^{th} value is obtained, we plot a point on the Q-chart. Let $t_r = y_1 + y_2 + \cdots + y_r$, $N_r = n_1 + n_2 + \cdots + n_r$, and compute the Q-statistics by the equations (4). Note that n_r is the number of standard sampling inspection units and that it needs only be a positive number. In particular, fractional values of n_r are permissible in contexts where they are meaningful.

Case U: λ Unknown

$$u_r = B(y_r; t_r, n_r/N_r)$$
$$Q_r = \Phi^{-1}(u_r) \quad \text{for } r = 2, 3, \cdots \qquad (4)$$

These statistics also are distributed approximately as standard normal statistics, and, in addition, they are approximately independent. These approximations are quite good after the first few observations, generally. We will give results below that reflect upon the goodness of both of these approximations.

Tests for Changes in λ

Consider using the Q-statistics given in equations (3) and (4) to make control chart tests to detect changes in λ. Suppose that immediately after sample c is observed that λ shifts from $\lambda = \lambda_0$ to $\lambda = \delta\lambda_0$, $\delta > 0$. Since the Q-statistics of (3) and (4) are strictly monotone functions of y_r, we can observe the effect of this shift on the distributions of the Q-statistics by studying the effect on the distribution of y_r. Now, the mean and standard deviation of y_r are $E(y_r) = n_r\lambda$ and $SD(y_r) = \sqrt{n_r\lambda}$. Thus when λ changes from $\lambda = \lambda_0$ to $\lambda = \delta\lambda_0$, then $E(y_r)$ shifts from $n_r\lambda_0$ to $n_r\delta\lambda_0$ and $SD(y_r)$ shifts from $SD(y_r) = \sqrt{n_r\lambda_0}$ to $SD(y_r) = \sqrt{n_r\delta\lambda_0}$. Note that both the mean and standard deviation of y_r are strictly increasing functions of λ. Thus, if λ increases, $1 < \delta$, then both the mean and standard deviation of the distribution of y_r, and of Q_r, also increase. On the other hand, if p decreases, $0 < \delta < 1$, then both the mean and standard deviation of the distribution of y_r, and of Q_r, decrease.

This development implies that a Q-chart, or, for that matter, also classical c and u charts, have poor sensitivity to detect *decreases* in λ by having a value fall below the lower control limit. A decrease in λ causes the mean of the plotted statistic to decrease, but at the same time the distribution becomes more concentrated about the mean rather than placing correspondingly more probability below a lower control limit. From this development, we expect both classical c and u charts and the Q-charts to have poor sensitivity to detect decreases in λ by the 1-of-1 test. However, it is apparent that many of the other tests set out above should perform relatively well to detect decreases in λ.

We have conducted a simulation to study the performance of the above six tests to detect changes in λ. The values of $\delta \in \{0.1, 0.5, 0.9, 1, 1.1, 1.5, 2, 3\}$ were used. For values of c in the set $\{1, 3, 5, 10, 20, 30, 50\}$, c samples were first generated from a Poisson $f(y; \lambda_0)$ distribution, and then 30 additional samples were generated from a $f(y; \delta\lambda_0)$ distribution, with $\lambda_o = 10$. The Q-statistics for the sequence $y_1, \cdots, y_c, y_{c+1}, \cdots, y_{c+30}$ were computed for case K (λ Known) of equations (3) and for case U (λ Unknown) of equations (4), using $n_r = 1$. The six test statistics described above were computed for both sequences of Q-statistics. This procedure was replicated 5,000 times and the proportion of times that each statistic signaled either an increase or a decrease in λ on samples $c+1$, $\cdots$, $c+30$, for the first time, was recorded as given in Table A.1 of Appendix A. For each value of c and δ there are four rows of values given in Table A.1. The first row gives the proportions of decreases signaled for the K case and the second row gives the proportions of decreases signaled for the U case. The third row gives the proportion of increases signaled for the K case and the fourth gives the proportion of increases signaled for the U case.

As a particular example to illustrate reading Table A.1, consider the entries for $c = 5$ and $\delta = 1.5$. For this case λ has increased from $\lambda = 10$ to $\lambda = 15$, and $c = 5$ Poisson samples were taken before this shift in λ. The first row shows that none of the tests signaled a (false) decrease in λ. However, the second row gives an estimate of 0.007 of a false signal of a decrease by the 1-of-1 test and an estimate of 0.003 by the 4-of-5 test, when λ is unknown and $c = 5$. The third gives an estimate of 0.968 of a signal by the 1-of-1 test of the increase in λ, and an estimate of 1.000 of a correct signal by the 4-of-5 test. The fourth row estimates these probabilities of a correct signal of this increase, when only $c = 5$ preceding samples were available, as 0.325 for the 1-of-1 test, as 0.630 for the 4-of-5 test, and 0.808 for the EWMA test.

Table A.1 gives useful information of three types. First, for these values of c, it has information that reflects the goodness of the normal approximation, and of the independence of the Q-statistics for the U case of λ unknown. Also, it permits comparison of the performances of the six test statistics, both for the stable case ($\delta = 1$) and when λ changes ($\delta \neq 1$). Finally, it has information that reflects the relative performance of each of the tests for the U case to the K case, for these values of c.

Normality and Independence Approximations

For $\delta = 1$ the first and third rows are estimating probabilities that are almost constant functions of c. For the λ known case the Q-statistics have a distribution that is constant in c, however, the value of c does have a slight effect upon the distribution of some of the test statistics. The probability being estimated is, for each statistic and for λ known, the probability that the statistic will give at least one false signal of a decrease (row 1) or of an increase (row 3). From these values we can average the values from the seven values of c to obtain the values in Table 2. These are averages

Table 2: Probabilities of False Signals on Thirty Points with $\lambda = 0.1$ Known

	1-of-1	9-of-9	3-of-3	4-of-5	EWMA	CUSUM
Decrease:	0.0097	0.0124	0.0437	0.0161	0.0094	0.0137
Increase:	0.0560	0.0563	0.1720	0.1073	0.1117	0.0961

from 35,000 samples. The values of the 1-of-1 test can be computed exactly directly, which serves as a check on the computations. The probability of a decrease is actually 0.0096 and of an increase is

0.0577, each accurate to the four digits given. The reason that the probabilities of false signals of increases are larger than those for decreases is the lack of symmetry of the Poisson distribution. By comparing the values in Table 2 with those in rows 2 and 4 of Table A.1 for $\delta = 1$ and varying c, it is observed that both the decreasing and increasing false alarm rates when λ is unknown are reasonably close to those for λ known. This supports the claim that the Q-statistics for the U case are approximately independent standard normal statistics.

Relative Performance of Tests

By studying Table A.1, we see that no one of the tests is best for all cases. From Table 2 and the $\delta = 1$ cases in Table A.1 it is apparent that the false alarm rates are not the same for the six tests. The false alarm rates for decreases and for increases must be considered separately. The 1-of-1 and EWMA have comparable false alarm rates for decreases, as do the 9-of-9, 4-of-5 and CUSUM tests. The 1-of-1 and 9-of-9 tests have comparable false alarm rates for increases and the 4-of-5, EWMA and CUSUM rates are reasonably comparable. The 3-of-3 test has higher false alarm rates than any of the other tests for both decreases and increases.

By keeping in mind these false alarm rates and studying Table A.1, we venture some summary remarks and recommendations.

- The overall performances of the EWMA and CUSUM charts were comparable with each other and better than the Shewhart chart tests to determine one-step permanent parameter shifts. It would be good practice to regularly plot one of these charts. Although there is little difference in these two charts in performance, the personal preference of this author is the EWMA chart. The particular chart used here would work well in charting programs.

- The 4-of-5 test on the Shewhart chart is considered to be the best overall test of the Shewhart chart tests.

- The classical 1-of-1 test is a poor competitor for many of the cases considered. However, it is essentially the only test for detecting a single outlier on a chart for μ.

- If the increased rate of false alarms is considered tolerable in order to improve sensitivity to shifts, then the 3-of-3 test is a reasonable choice of test. This can be a reasonable trade-off in a start-up process before many data are available to improve sensitivity, *i.e.*, while c is small.

Performance of Tests with λ Unknown

By considering the values in Table A.1 for $\delta = 1$ and small values of c, say $c = 1$, we see that the tests based on Q-statistics that do not assume known λ maintain essentially the same error rates as the tests based on Q-statistics with λ having a known value. This permits us to assess the effect on performances of the tests for $\delta \neq 1$ by comparing the U case results for the given values of c with those for the corresponding K case.

The worst case is, of course, for $c = 1$ when the shift in λ occurs after only one Poisson sample has been observed. For $c = 1$ and $\delta = 0.1$, a ten-fold decrease in λ, the estimated probability of detecting this decrease in the next 30 samples is 0.303 for the 1-of-1 test, 0.584 for the CUSUM, 0.586 for the 3-of-3, 0.481 for the EWMA and 0.444 for the 4-of-5. For $\delta = 0.9$, a small decrease, the values for the various tests are small, but are generally larger than the approximate false alarm rates given in Table 2. Similar remarks obtain for increases in λ, $\delta > 1$. With $c = 1$ the tests for λ unknown do not have high probabilities of detecting an increase until δ is 2 or 3. As c increases, the probabilities of detecting shifts increase to the limiting values for the K cases. For example, to detect $\delta = 1.5$ with $c = 1$ the probability of detection by the EWMA test is 0.241, for $c = 3$ it is 0.560, for $c = 5$ it is 0.748, for $c = 10$ it is 0.935, for $c = 20$ it is 0.984, and for c=30 it is 0.990, which is essentially the same as the limiting value for the K case.

This general performance of the Q-charts tests for the U case (λ unknown) is, of course, exactly what we expect. For c small we are essentially making inferences with a small amount of data, and sensitivity is limited. However, the reader should be aware that for these cases there are no known competitors to these Q-charts that maintain known false alarm rates.

The results discussed above were all for the $n = 100$ and the $\lambda_0 = 10$ case. The question arises as to how dependent these inferences are upon this particular value of λ. In Quesenberry (1993b) it was found that for the corresponding binomial charts these types of results did not depend crucially upon the values of the p parameter. Since the Poisson distribution is essentially the limiting case of the binomial distribution, it follows that these results also should hold generally for different values of λ. Finally, it should be observed that the results reported here agree closely with those reported in Quesenberry (1993a) for variables charts, and those in Quesenberry (1993b) for binomial charts.

APPENDIX A, TABLES

Table A.1: Estimated Signal Probabilities for Poisson Q-Charts, $\lambda_0 = 10$

δ	c = 1						c = 3					
	1-of-1	9-of-9	3-of-3	4-of-5	EWMA	CUSUM	1-of-1	9-of-9	3-of-3	4-of-5	EWMA	CUSUM
0.1	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.997	1.000
	0.303	0.216	0.586	0.444	0.481	0.584	0.638	0.899	0.981	0.975	0.977	0.992
	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	0.013	0.062	0.075	0.043	0.024	0.020	0.001	0.001	0.003	0.001	0.000	0.000
0.5	0.712	0.997	1.000	1.000	1.000	1.000	0.710	0.999	1.000	1.000	0.998	1.000
	0.033	0.022	0.109	0.056	0.063	0.068	0.075	0.159	0.341	0.255	0.265	0.284
	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	0.034	0.041	0.118	0.063	0.052	0.049	0.016	0.014	0.040	0.019	0.009	0.011
0.9	0.038	0.082	0.187	0.111	0.076	0.066	0.035	0.090	0.191	0.122	0.086	0.073
	0.022	0.009	0.044	0.015	0.010	0.013	0.025	0.015	0.072	0.031	0.020	0.020
	0.013	0.004	0.052	0.021	0.008	0.010	0.014	0.007	0.058	0.029	0.011	0.013
	0.054	0.058	0.166	0.094	0.087	0.075	0.053	0.054	0.146	0.085	0.073	0.066
1.0	0.014	0.011	0.059	0.021	0.010	0.014	0.017	0.013	0.057	0.023	0.009	0.010
	0.018	0.008	0.049	0.017	0.012	0.013	0.020	0.008	0.044	0.014	0.009	0.011
	0.045	0.042	0.195	0.124	0.100	0.086	0.052	0.047	0.196	0.129	0.105	0.096
	0.063	0.067	0.169	0.107	0.105	0.097	0.065	0.069	0.176	0.106	0.112	0.093
1.1	0.005	0.001	0.011	0.003	0.000	0.002	0.005	0.001	0.013	0.004	0.001	0.001
	0.017	0.006	0.037	0.016	0.007	0.009	0.014	0.005	0.030	0.011	0.004	0.006
	0.124	0.193	0.490	0.389	0.471	0.404	0.125	0.205	0.474	0.385	0.461	0.397
	0.068	0.071	0.188	0.115	0.123	0.104	0.074	0.103	0.228	0.146	0.167	0.143
1.5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.000	0.000	0.000
	0.012	0.005	0.034	0.016	0.005	0.009	0.009	0.002	0.015	0.004	0.001	0.004
	0.923	0.976	0.999	1.000	1.000	1.000	0.924	0.975	0.999	1.000	1.000	1.000
	0.127	0.100	0.277	0.188	0.241	0.219	0.208	0.247	0.520	0.419	0.560	0.532
2.0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	0.012	0.003	0.023	0.009	0.002	0.005	0.004	0.001	0.008	0.002	0.000	0.001
	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	0.284	0.143	0.425	0.312	0.461	0.480	0.585	0.481	0.828	0.779	0.935	0.949
3.0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	0.011	0.001	0.017	0.005	0.000	0.002	0.006	0.000	0.002	0.000	0.000	0.001
	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.999	1.000
	0.781	0.224	0.662	0.547	0.896	0.935	0.991	0.798	0.988	0.983	1.000	1.000

Table A.1: Estimated Signal Probabilities for Poisson Q-Charts, $\lambda_0 = 10$

δ	c = 5						c = 10					
	1-of-1	9-of-9	3-of-3	4-of-5	EWMA	CUSUM	1-of-1	9-of-9	3-of-3	4-of-5	EWMA	CUSUM
0.1	1.000	1.000	1.000	1.000	0.991	1.000	1.000	1.000	1.000	1.000	0.977	1.000
	0.795	0.992	0.999	0.999	0.993	1.000	0.946	1.000	1.000	1.000	0.975	1.000
	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.5	0.712	0.998	1.000	1.000	0.993	1.000	0.699	0.998	1.000	1.000	0.978	1.000
	0.120	0.314	0.508	0.429	0.472	0.502	0.207	0.578	0.738	0.685	0.765	0.789
	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	0.008	0.004	0.022	0.008	0.001	0.004	0.002	0.001	0.003	0.001	0.000	0.001
0.9	0.039	0.087	0.198	0.116	0.078	0.070	0.045	0.089	0.175	0.123	0.083	0.075
	0.029	0.018	0.068	0.033	0.022	0.025	0.036	0.027	0.082	0.043	0.028	0.031
	0.012	0.008	0.051	0.027	0.011	0.013	0.011	0.010	0.053	0.024	0.011	0.015
	0.040	0.046	0.129	0.077	0.062	0.056	0.037	0.039	0.094	0.053	0.037	0.038
1.0	0.013	0.012	0.056	0.026	0.010	0.011	0.012	0.017	0.059	0.023	0.009	0.011
	0.017	0.007	0.044	0.019	0.009	0.011	0.015	0.011	0.048	0.018	0.009	0.013
	0.046	0.057	0.212	0.128	0.115	0.101	0.045	0.056	0.198	0.124	0.112	0.096
	0.064	0.081	0.190	0.119	0.119	0.103	0.056	0.090	0.183	0.109	0.114	0.098
1.1	0.005	0.002	0.015	0.005	0.001	0.001	0.006	0.002	0.015	0.006	0.000	0.002
	0.014	0.005	0.031	0.014	0.003	0.007	0.012	0.004	0.021	0.008	0.002	0.006
	0.133	0.205	0.471	0.401	0.475	0.418	0.126	0.205	0.481	0.401	0.472	0.413
	0.089	0.122	0.246	0.174	0.207	0.177	0.096	0.142	0.286	0.204	0.253	0.212
1.5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	0.006	0.001	0.009	0.004	0.000	0.002	0.003	0.000	0.004	0.000	0.000	0.000
	0.924	0.978	0.999	0.999	0.999	1.000	0.935	0.978	0.999	1.000	0.997	1.000
	0.304	0.393	0.639	0.566	0.748	0.730	0.444	0.596	0.824	0.787	0.935	0.918
2.0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	0.003	0.000	0.004	0.001	0.000	0.000	0.002	0.000	0.001	0.000	0.000	0.000
	1.000	1.000	1.000	1.000	0.999	1.000	1.000	1.000	1.000	1.000	0.997	1.000
	0.759	0.726	0.940	0.932	0.994	0.995	0.923	0.915	0.993	0.997	0.997	1.000
3.0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	0.002	0.000	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	1.000	1.000	1.000	1.000	0.999	1.000	1.000	1.000	1.000	1.000	0.997	1.000
	1.000	0.957	1.000	1.000	1.000	1.000	1.000	0.999	1.000	1.000	0.998	1.000

Table A.1: Estimated Signal Probabilities for Poisson Q-Charts, $\lambda_0 = 10$

δ	c = 20						c = 30					
	1-of-1	9-of-9	3-of-3	4-of-5	EWMA	CUSUM	1-of-1	9-of-9	3-of-3	4-of-5	EWMA	CUSUM
0.1	1.000	1.000	1.000	1.000	0.942	1.000	1.000	1.000	1.000	1.000	0.904	1.000
	0.997	1.000	1.000	1.000	0.926	1.000	1.000	1.000	1.000	1.000	0.897	1.000
	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.5	0.719	0.998	1.000	1.000	0.939	1.000	0.710	0.998	1.000	1.000	0.903	1.000
	0.320	0.821	0.914	0.895	0.898	0.959	0.383	0.907	0.958	0.958	0.886	0.990
	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	0.000	0.000	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.9	0.037	0.088	0.193	0.121	0.081	0.070	0.036	0.081	0.187	0.115	0.077	0.073
	0.035	0.029	0.108	0.058	0.034	0.037	0.035	0.036	0.117	0.058	0.040	0.042
	0.012	0.012	0.053	0.027	0.010	0.013	0.015	0.008	0.055	0.025	0.012	0.015
	0.029	0.032	0.081	0.042	0.033	0.030	0.029	0.024	0.073	0.033	0.022	0.028
1.0	0.015	0.016	0.057	0.024	0.008	0.013	0.014	0.018	0.063	0.027	0.007	0.009
	0.017	0.013	0.047	0.019	0.009	0.014	0.015	0.016	0.053	0.021	0.009	0.013
	0.045	0.051	0.202	0.133	0.110	0.098	0.043	0.057	0.198	0.126	0.112	0.098
	0.057	0.074	0.187	0.118	0.109	0.100	0.057	0.086	0.180	0.110	0.116	0.101
1.1	0.006	0.002	0.017	0.005	0.000	0.002	0.006	0.002	0.014	0.004	0.001	0.001
	0.012	0.002	0.024	0.008	0.001	0.004	0.008	0.002	0.017	0.007	0.001	0.004
	0.132	0.205	0.498	0.407	0.480	0.403	0.123	0.220	0.476	0.391	0.463	0.404
	0.112	0.175	0.337	0.248	0.305	0.254	0.116	0.203	0.346	0.262	0.332	0.280
1.5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	0.002	0.000	0.001	0.000	0.000	0.000	0.002	0.000	0.001	0.000	0.000	0.000
	0.925	0.980	1.000	1.000	0.994	1.000	0.921	0.976	1.000	0.999	0.990	1.000
	0.602	0.770	0.937	0.929	0.984	0.990	0.699	0.858	0.968	0.964	0.990	0.997
2.0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.000	0.000	0.000	0.000	0.000
	1.000	1.000	1.000	1.000	0.997	1.000	1.000	1.000	1.000	1.000	0.990	1.000
	0.993	0.987	1.000	1.000	0.994	1.000	0.999	0.998	1.000	1.000	0.991	1.000
3.0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	1.000	1.000	1.000	1.000	0.995	1.000	1.000	1.000	1.000	1.000	0.992	1.000
	1.000	1.000	1.000	1.000	0.995	1.000	1.000	1.000	1.000	1.000	0.991	1.000

Table A.1: Estimated Signal Probabilities for Poisson Q-Charts, $\lambda_0 = 10$

δ	c = 50					
	1-of-1	9-of-9	3-of-3	4-of-5	EWMA	CUSUM
0.1	1.000	1.000	1.000	1.000	0.826	1.000
	1.000	1.000	1.000	1.000	0.829	1.000
	0.000	0.000	0.000	0.000	0.000	0.000
	0.000	0.000	0.000	0.000	0.000	0.000
0.5	0.710	0.998	1.000	1.000	0.836	1.000
	0.494	0.958	0.987	0.985	0.818	0.999
	0.000	0.001	0.000	0.000	0.000	0.000
	0.000	0.001	0.000	0.000	0.000	0.000
0.9	0.035	0.090	0.186	0.117	0.065	0.071
	0.035	0.050	0.132	0.076	0.042	0.052
	0.012	0.009	0.063	0.025	0.012	0.011
	0.021	0.024	0.073	0.030	0.021	0.019
1.0	0.013	0.015	0.060	0.025	0.007	0.012
	0.014	0.011	0.050	0.021	0.008	0.012
	0.046	0.052	0.202	0.127	0.111	0.089
	0.062	0.077	0.174	0.110	0.107	0.094
1.1	0.007	0.002	0.014	0.006	0.000	0.002
	0.008	0.002	0.016	0.006	0.001	0.003
	0.143	0.205	0.480	0.395	0.478	0.418
	0.146	0.206	0.373	0.292	0.380	0.333
1.5	0.000	0.000	0.000	0.000	0.000	0.000
	0.001	0.000	0.000	0.000	0.000	0.000
	0.931	0.978	0.999	0.999	0.986	1.000
	0.793	0.910	0.981	0.986	0.980	0.999
2.0	0.000	0.000	0.000	0.000	0.000	0.000
	0.000	0.000	0.000	0.000	0.000	0.000
	1.000	1.000	1.000	1.000	0.983	1.000
	1.000	1.000	1.000	1.000	0.982	1.000
3.0	0.000	0.000	0.000	0.000	0.000	0.000
	0.000	0.000	0.000	0.000	0.000	0.000
	1.000	1.000	1.000	1.000	0.982	1.000
	1.000	1.000	1.000	1.000	0.982	1.000

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