A COMPREHENSIVE, MATRIX FREE ALGORITHM FOR ANALYSIS OF VARIANCE

by William J. Hemmerle

Institute of Statistics Mimeo Series #1212

#1212

A COMPREHENSIVE, MATRIX FREE ALGORITHM FOR ANALYSIS OF VARIANCE

SAMPLE PROBLEMS

The following simple examples have been prepared principally to illustrate the different features of the program - options, model specification, computations, output - in many different combinations. They do not necessarily represent the most likely use of the program in analyzing each set of data. The unbalanced data sets used in the first two examples were taken from Searle [19]. The first data set is used in Chapter 7 to illustrate the two-way classification with interaction. The second is used in Chapter 6 to illustrate the two-way nested classification. The third example involving balanced data with 2 crossed factors, 2 nested factors, and interaction terms appears in Appendix B of [12]. The final example, a Latin square, was extracted from Statistical Methods by G. N. Snedecor (5th Edition, page 308). These sample problems have all been run in batch mode.

```
  F(A,B) L(I(3),J(4))
DATA FORMAT AND INPUT DATA-
(1X,2I2,F4.0)
 1 1  8.
 1 1 13.
 1 1  9.
 1 3 12.
 1 4  7.
 1 4 11.
 2 1  6.
 2 1 12.
 2 2 12.
 2 2 14.
 3 2  9.
 3 2  7.
 3 3 14.
 3 3 16.
 3 4 10.
 3 4 14.
 3 4 11.
 3 4 13.
```

```
 0 C  0.
  C(S(8),2,A,C,R)
CELL SUMS, FREQUENCIES, AND MEANS-
   CELL        SUM        FREQ.       MEAN
     1    C.3000C000D 02    3.    0.10000000D 02
     2              (MISSING CELL)
     3    C.1200C000D 02    1.    0.12000000D 02
     4    C.1800C000D 02    2.    0.90000000D 01
     5    C.1800C000D 02    2.    0.90000000D 01
     6    C.2600C000D 02    2.    0.13000000D 02
     7              (MISSING CELL)
     8              (MISSING CELL)
     9              (MISSING CELL)
    10    C.1600C000D 02    2.    0.80000000D 01
    11    C.3000C000D 02    2.    0.15000000D 02
    12    C.4800C000D 02    4.    0.12000000D 02
CLASSIFICATION SUMS, FREQUENCIES, AND MEANS-
I.
     1    C.6000C000D 02    6.    0.10000000D 02
     2    C.4400C000D 02    4.    0.11000000D 02
     3    C.9400C000D 02    8.    0.11750000D 02
•J
     1    C.4800C000D 02    5.    0.96000000D 01
     2    C.4200C000D 02    4.    0.10500000D 02
     3    C.4200C000D 02    3.    0.14000000D 02
     4    C.6600C000D 02    6.    0.11000000D 02
••
     1    0.1980C000D 03   18.    0.11000000D 02
OPTIONS- S= 8, T=0.0500, I=100, R=1, V=0, G=0, P=0
  M+A(I)+B(J)+AB(IJ)
E/R LIST-
    4    3    2    1
THE RANK OF THE M DESIGN MATRIX IS    8
ITERATION   1, SSR(FULL MODEL)=  0.22600000D 04,
               SSE(FULL MODEL)=  0.56000000D 02
  H  AB(IJ)
E/R LIST-
   -4    3    2    1
ITERATION  0, TRACE=      4.000000000
ITERATION  1, TRACE=      4.875000000
ITERATION  2, TRACE=      5.543113426
ITERATION  3, TRACE=      5.886175169
ITERATION  4, TRACE=      5.985797608
THE RANK OF THE H DESIGN MATRIX IS    6
FROM RANK COMPUTATIONS- DF(NUM)=   2, DF(DEN)=   10
ITERATION   1, SSRH=  0.14159687D 04
ITERATION   2, SSRH=  0.18605379D 04
ITERATION   3, SSRH=  0.20375067D 04
ITERATION   4, SSRH=  0.21180574D 04
ITERATION   5, SSRH=  0.21566817D 04
ITERATION   6, SSRH=  0.21811740D 04
ITERATION   7, SSRH=  0.21947132D 04
ITERATION   8, SSRH=  0.22034435D 04
ITERATION   9, SSRH=  0.22093714D 04
ITERATION  10, SSRH=  0.22135449D 04
ITERATION  11, SSRH=  0.22165550D 04
ITERATION  11, F=       3.879 , PROB(F) .GT. 0.0559 VS. F LEVEL OF 0.0500
SSR(REDUCED MODEL)=  0.22165550D 04
  H  A(I)
E/R LIST-
```

```
    4   -3    2    1
ITERATION  0, TRACE=      6.666666667
ITERATION  1, TRACE=      7.097222222
ITERATION  2, TRACE=      7.567604784
ITERATION  3, TRACE=      7.886480000
ITERATION  4, TRACE=      7.989797654
THE RANK OF THE H DESIGN MATRIX IS     8
FROM RANK COMPUTATIONS- DF(NUM)=    0, DF(DEN)=    10
  H  B(J)
E/R LIST-
    4    3   -2    1
ITERATION  0, TRACE=      6.000000000
ITERATION  1, TRACE=      6.666666667
ITERATION  2, TRACE=      7.407407407
ITERATION  3, TRACE=      7.882944673
ITERATION  4, TRACE=      7.995432683
THE RANK OF THE H DESIGN MATRIX IS     8
FROM RANK COMPUTATIONS- DF(NUM)=    0, DF(DEN)=    10
  O(P)
OPTIONS- S= 8, T=0.0500, I=100, R=1, V=0, G=0, P=1
  H  AB(IJ)
E/R LIST-
   -4    3    2    1
ITERATION  0, TRACE=      4.000000000
ITERATION  1, TRACE=      4.875000000
ITERATION  2, TRACE=      5.543113426
ITERATION  3, TRACE=      5.886175169
ITERATION  4, TRACE=      5.989797608
THE RANK OF THE H DESIGN MATRIX IS     6
FROM RANK COMPUTATIONS- DF(NUM)=    2, DF(DEN)=    10
ITERATION   1, SSRH=  0.14159687D 04
ITERATION   2, SSRH=  0.18605379D 04
ITERATION   3, SSRH=  0.20375067D 04
ITERATION   4, SSRH=  0.21180574D 04
ITERATION   5, SSRH=  0.21586817D 04
ITERATION   6, SSRH=  0.21811740D 04
ITERATION   7, SSRH=  0.21947132D 04
ITERATION   8, SSRH=  0.22034435D 04
ITERATION   9, SSRH=  0.22093714D 04
ITERATION  10, SSRH=  0.22135449D 04
ITERATION  11, SSRH=  0.22165550D 04
ITERATION  12, SSRH=  0.22187605D 04
ITERATION  13, SSRH=  0.22203929D 04
ITERATION  14, SSRH=  0.22216091D 04
ITERATION  15, SSRH=  0.22225191D 04
ITERATION  16, SSRH=  0.22232018D 04
ITERATION  17, SSRH=  0.22237150D 04
ITERATION  18, SSRH=  0.22241013D 04
ITERATION  19, SSRH=  0.22243922D 04
ITERATION  20, SSRH=  0.22246116D 04
ITERATION  21, SSRH=  0.22247769D 04
ITERATION  22, SSRH=  0.22249017D 04
ITERATION  23, SSRH=  0.22249958D 04
ITERATION  24, SSRH=  0.22250669D 04
ITERATION  25, SSRH=  0.22251205D 04
ITERATION  25, F=        3.114 , PROB(F) .GT. 0.0880 VS. F LEVEL OF 0.0500
SSR(REDUCED MODEL)=  0.22251205D 04
  C(Z,V,G)
OPTIONS- S= 8, T=0.0500, I=100, R=1, V=1, G=1, P=1
  M+A(I)+B(J)
```

```
THE RANK OF THE X DESIGN MATRIX IS     6
ITERATION  64, SSR(FULL MODEL)=  0.22252857D 04,
                SSE(FULL MODEL)=  0.90714289D 02
ESTIMATES OF EXPECTED CELL MEANS-
   CELL   ESTIMATED MEAN
     1    0.87146779D 01
     2    0.88574223D 01 (MISSING CELL)
     3    0.13285000D 02
     4    0.10285360D 02
     5    0.10928171D 02
     6    0.11070915D 02
     7    0.15496573D 02 (MISSING CELL)
     8    0.12498853D 02 (MISSING CELL)
     9    0.97864389D 01 (MISSING CELL)
    10    0.99291833D 01
    11    0.14356841D 02
    12    0.11357121D 02
G-INVERSE SOLUTION-
A
     1  -0.10990847D 01
     2   0.11184084D 01
     3  -0.23323708D-01
B
     1  -0.15709570D 01
     2  -0.14282126D 01
     3   0.29994450D 01
     4  -0.27535484D-03

     1   0.11380720D 02

  F(A,B) L(I(2),J(3))
DATA FORMAT AND INPUT DATA-
(1X,2I2,F4.0)
 1 1  5.
 1 2  8.
 1 2 10.
 1 2  9.
 2 1  8.
 2 1 10.
 2 2  6.
 2 2  2.
 2 2  1.
 2 2  3.
 2 3  3.
 2 3  7.
 0 0  0.
  C(Z,C,P,V,G)
CLASSIFICATION SUMS, FREQUENCIES, AND MEANS-
I.
     1   0.32000000D 02   4.   0.80000000D 01
     2   0.40000000D 02   8.   0.50000000D 01
.J
     1   0.23000000D 02   3.   0.76666667D 01
     2   0.39000000D 02   7.   0.55714286D 01
     3   0.10000000D 02   2.   0.50000000D 01
..
     1   0.72000000D 02  12.   0.60000000D 01
OPTIONS- S= 8, T=0.0500, I=100, R=1, V=0, G=0, P=0
  M+A(I)+E(IJ)
E/R LIST-
```

```
    2    3    2    1
THE RANK OF THE M DESIGN MATRIX IS     5
ITERATION    1, SSR(FULL MODEL)=  0.516000000D 03,
                SSE(FULL MODEL)=  0.260000000D 02
  H  B(IJ)
E/R LIST-
   -2    3   -2    1
THE RANK OF THE H DESIGN MATRIX IS     2
FROM RANK COMPUTATIONS- DF(NUM)=    3, DF(DEN)=     7
ITERATION    1, SSRH=  0.456000000D 03
ITERATION    1, F=       5.385*, PROB(F) .GT. 0.0311 VS. F LEVEL OF 0.0500
SSR(REDUCED MODEL)=  0.456000000D 03
  H  A(I)
E/R LIST-
    2   -3    2    1
ITERATION  0, TRACE=      4.166666667
ITERATION  1, TRACE=      4.305555556
ITERATION  2, TRACE=      4.517746914
ITERATION  3, TRACE=      4.767431561
ITERATION  4, TRACE=      4.945912107
THE RANK OF THE H DESIGN MATRIX IS     5
FROM RANK COMPUTATIONS- DF(NUM)=    0, DF(DEN)=     7
  O(Z,V,G)
OPTIONS- S= 8, T=0.0500, I=100, R=1, V=1, G=1, P=0
  M+A(I)+E(IJ)
THE RANK OF THE M DESIGN MATRIX IS     5
ITERATION    1, SSR(FULL MODEL)=  0.516000000D 03,
                SSE(FULL MODEL)=  0.260000000D 02
ESTIMATES OF EXPECTED CELL MEANS-
   CELL  ESTIMATED MEAN
     1   0.500000000D 01
     2   0.900000000D 01
     3   0.000000000D 00 (MISSING CELL)
     4   0.900000000D 01
     5   0.300000000D 01
     6   0.500000000D 01
G-INVERSE SOLUTION-
B
     1   0.333333333D 00
     2   0.433333333D 01
     3  -0.466666667D 01
     4   0.333333333D 01
     5  -0.266666667D 01
     6  -0.666666667D 00
A
     1  -0.500000000D 00
     2   0.500000000D 00

     1   0.516666667D 01
  C(G)
OPTIONS- S= 8, T=0.0500, I=100, R=1, V=1, G=0, P=0
  M+A(I)
THE RANK OF THE M DESIGN MATRIX IS     2
ITERATION    1, SSR(FULL MODEL)=  0.456000000D 03,
                SSE(FULL MODEL)=  0.860000000D 02
ESTIMATES OF EXPECTED CELL MEANS-
   CELL  ESTIMATED MEAN
     1   0.800000000D 01
     2   0.800000000D 01
     3   0.800000000D 01 (MISSING CELL)
```

```
      4    0.5000000D 01
      5    0.5000000D 01
      6    0.5000000D 01
  F   A(I)
THE RANK OF THE F DESIGN MATRIX IS     1
FROM RANK COMPUTATIONS- DF(NUM)=    1, DF(DEN)=    10
ITERATION    1, F=        2.791 , PROB(F) .GT. 0.1231 VS. F LEVEL OF 0.0500
SSR(REDUCED MODEL)=  0.4320000D 03

  F(A,B,C,D) L(I(2),J(2),K(2),L(2))
DATA FORMAT AND INPUT DATA-
(1X,4I2,F5.2)
 1 1 1 1 6.01
 1 1 1 1 5.95
 1 1 1 1 5.85
 1 1 1 1 6.43
 1 1 1 2 6.35
 1 1 1 2 7.00
 1 1 1 2 5.90
 1 1 1 2 6.00
 1 1 2 1 6.00
 1 1 2 1 7.15
 1 1 2 1 7.35
 1 1 2 1 7.20
 1 1 2 2 7.10
 1 1 2 2 6.15
 1 1 2 2 7.50
 1 1 2 2 7.15
 1 2 1 1 5.87
 1 2 1 1 6.18
 1 2 1 1 5.90
 1 2 1 1 6.33
 1 2 1 2 5.84
 1 2 1 2 5.55
 1 2 1 2 6.03
 1 2 1 2 6.64
 1 2 2 1 6.08
 1 2 2 1 6.17
 1 2 2 1 6.13
 1 2 2 1 7.62
 1 2 2 2 6.55
 1 2 2 2 5.40
 1 2 2 2 7.20
 1 2 2 2 6.66
 2 1 1 1 6.00
 2 1 1 1 5.80
 2 1 1 1 6.60
 2 1 1 1 5.54
 2 1 1 2 5.50
 2 1 1 2 5.45
 2 1 1 2 6.00
 2 1 1 2 6.05
 2 1 2 1 6.25
 2 1 2 1 6.75
 2 1 2 1 5.60
 2 1 2 1 6.40
 2 1 2 2 6.17
 2 1 2 2 6.33
 2 1 2 2 6.32
 2 1 2 2 5.97
```

```
 2 2 1 1 6.30
 2 2 1 1 6.35
 2 2 1 1 7.00
 2 2 1 1 9.05
 2 2 1 2 6.55
 2 2 1 2 5.90
 2 2 1 2 5.67
 2 2 1 2 6.30
 2 2 2 1 5.30
 2 2 2 1 6.10
 2 2 2 1 6.50
 2 2 2 1 6.95
 2 2 2 2 6.10
 2 2 2 2 6.10
 2 2 2 2 6.30
 2 2 2 2 6.75
 0 0 0 0 0.00
  C(Z,A,C,R,V)
CELL SUMS, FREQUENCIES, AND MEANS-
   CELL        SUM          FREQ.       MEAN
     1   0.24240000D 02    4.    0.60600000D 01
     2   0.25250000D 02    4.    0.63125000D 01
     3   0.27700000D 02    4.    0.69250000D 01
     4   0.27900000D 02    4.    0.69750000D 01
     5   0.24280000D 02    4.    0.60700000D 01
     6   0.24060000D 02    4.    0.60150000D 01
     7   0.26000000D 02    4.    0.65000000D 01
     8   0.29810000D 02    4.    0.74525000D 01
     9   0.23940000D 02    4.    0.59850000D 01
    10   0.23000000D 02    4.    0.57500000D 01
    11   0.24000000D 02    4.    0.60000000D 01
    12   0.24790000D 02    4.    0.61975000D 01
    13   0.28700000D 02    4.    0.71750000D 01
    14   0.24420000D 02    4.    0.61050000D 01
    15   0.24850000D 02    4.    0.62125000D 01
    16   0.25250000D 02    4.    0.63125000D 01
CLASSIFICATION SUMS, FREQUENCIES, AND MEANS-
IJK.
     1   0.49490000D 02    8.    0.61862500D 01
     2   0.55600000D 02    8.    0.69500000D 01
     3   0.48340000D 02    8.    0.60425000D 01
     4   0.55810000D 02    8.    0.69762500D 01
     5   0.46940000D 02    8.    0.58675000D 01
     6   0.48750000D 02    8.    0.60987500D 01
     7   0.53120000D 02    8.    0.66400000D 01
     8   0.50100000D 02    8.    0.62625000D 01
IJ.L
     1   0.51940000D 02    8.    0.64925000D 01
     2   0.53150000D 02    8.    0.66437500D 01
     3   0.50280000D 02    8.    0.62850000D 01
     4   0.53870000D 02    8.    0.67337500D 01
     5   0.47940000D 02    8.    0.59925000D 01
     6   0.47790000D 02    8.    0.59737500D 01
     7   0.53550000D 02    8.    0.66937500D 01
     8   0.49670000D 02    8.    0.62087500D 01
IJ..
     1   0.10509000D 03   16.    0.65681250D 01
     2   0.10415000D 03   16.    0.65093750D 01
     3   0.95730000D 02   16.    0.59831250D 01
     4   0.10322000D 03   16.    0.64512500D 01
```

```
I.KL
      1    0.48520000D 02    8.    0.60650000D 01
      2    0.49310000D 02    8.    0.61637500D 01
      3    0.53700000D 02    8.    0.67125000D 01
      4    0.57710000D 02    8.    0.72137500D 01
      5    0.52640000D 02    8.    0.65800000D 01
      6    0.47420000D 02    8.    0.59275000D 01
      7    0.48850000D 02    8.    0.61062500D 01
      8    0.50040000D 02    8.    0.62550000D 01
I.K.
      1    0.97830000D 02   16.    0.61143750D 01
      2    0.11141000D 03   16.    0.69631250D 01
      3    0.10000600D 03   16.    0.62537500D 01
      4    0.98890000D 02   16.    0.61806250D 01
I..L
      1    0.10222000D 03   16.    0.63887500D 01
      2    0.10702000D 03   16.    0.66887500D 01
      3    0.10149000D 03   16.    0.63431250D 01
      4    0.97460000D 02   16.    0.60912500D 01
I...
      1    0.20924000D 03   32.    0.65387500D 01
      2    0.19895000D 03   32.    0.62171875D 01
.JKL
      1    0.48180000D 02    8.    0.60225000D 01
      2    0.48250000D 02    8.    0.60312500D 01
      3    0.51700000D 02    8.    0.64625000D 01
      4    0.52690000D 02    8.    0.65862500D 01
      5    0.52980000D 02    8.    0.66225000D 01
      6    0.48480000D 02    8.    0.60600000D 01
      7    0.50850000D 02    8.    0.63562500D 01
      8    0.55060000D 02    8.    0.68825000D 01
.JK.
      1    0.96430000D 02   16.    0.60268750D 01
      2    0.10439000D 03   16.    0.65243750D 01
      3    0.10146000D 03   16.    0.63412500D 01
      4    0.10591000D 03   16.    0.66193750D 01
.J.L
      1    0.99880000D 02   16.    0.62425000D 01
      2    0.10094000D 03   16.    0.63087500D 01
      3    0.10383000D 03   16.    0.64893750D 01
      4    0.10354000D 03   16.    0.64712500D 01
.J..
      1    0.20082000D 03   32.    0.62756250D 01
      2    0.20737000D 03   32.    0.64803125D 01
..KL
      1    0.10116000D 03   16.    0.63225000D 01
      2    0.96730000D 02   16.    0.60456250D 01
      3    0.10255000D 03   16.    0.64093750D 01
      4    0.10775000D 03   16.    0.67343750D 01
..K.
      1    0.19789000D 03   32.    0.61840625D 01
      2    0.21030000D 03   32.    0.65718750D 01
...L
      1    0.20371000D 03   32.    0.63659375D 01
      2    0.20448000D 03   32.    0.63900000D 01
....
      1    0.40819000D 03   64.    0.63779688D 01
OPTIONS- S= 8, T=0.0500, I=100, R=0, V=0, G=0, P=0
  M+A(I)+B(IJ)+C(K)+AC(IK)+BC(IJK)/
  +D(IJL)+CD(IJKL)
```

```
E/R LIST-
    4    7    2    5    4   11    2    9    4    7    2    5    4    3    2    1
ITERATION    1, SSR(FULL MODEL)=  0.26179143D 04,
                SSE(FULL MODEL)=  0.195237750 02
  H A(I)
E/R LIST-
    4    7    2    5    4   11    2   -9    4    7    2    5    4    3    2    1
FROM RANK COMPUTATIONS- DF(NUM)=    1, DF(DEN)=   48
ITERATION    1, SSRH=  0.261625990 04
ITERATION    1, F=        4.068*, PROB(F) .GT. 0.0466 VS. F LEVEL OF 0.0500
SSR(REDUCED MODEL)=  0.261625990 04
  H B(IJ)
E/R LIST-
    4    7    2   -5    4   11    2    9    4    7    2   -5    4    3    2    1
FROM RANK COMPUTATIONS- DF(NUM)=    2, DF(DEN)=   48
ITERATION    1, SSRH=  0.261613360 04
ITERATION    1, F=        2.189 , PROB(F) .GT. 0.1212 VS. F LEVEL OF 0.0500
SSR(REDUCED MODEL)=  0.261613360 04
  C(Z)
OPTIONS- S= 8, T=0.0500, I=100, R=0, V=0, G=0, P=0
  H C(K)
FROM RANK COMPUTATIONS- DF(NUM)=    1, DF(DEN)=   48
ITERATION    1, F=        5.916*, PROB(F) .GT. 0.0178 VS. F LEVEL OF 0.0500
SSR(REDUCED MODEL)=  0.261550790 04
  H AC(IK)
FROM RANK COMPUTATIONS- DF(NUM)=    1, DF(DEN)=   48
ITERATION    1, F=        8.358*, PROB(F) .GT. 0.0059 VS. F LEVEL OF 0.0500
SSR(REDUCED MODEL)=  0.261451490 04
  H BC(IJK)
FROM RANK COMPUTATIONS- DF(NUM)=    2, DF(DEN)=   48
ITERATION    1, F=        0.982 , PROB(F) .GT. 0.3837 VS. F LEVEL OF 0.0500
SSR(REDUCED MODEL)=  0.261711540 04
  H D(IJL)
FROM RANK COMPUTATIONS- DF(NUM)=    4, DF(DEN)=   48
ITERATION    1, F=        1.131 , PROB(F) .GT. 0.3534 VS. F LEVEL OF 0.0500
SSR(REDUCED MODEL)=  0.261607500 04
  H CD(IJKL)
FROM RANK COMPUTATIONS- DF(NUM)=    4, DF(DEN)=   48
ITERATION    1, F=        1.605 , PROB(F) .GT. 0.1873 VS. F LEVEL OF 0.0500
SSR(REDUCED MODEL)=  0.261530230 04
  O(V,G)
OPTIONS- S= 8, T=0.0500, I=100, R=0, V=1, G=1, P=0
  M+A(I)+C(K)+AC(IK)
ITERATION    1, SSR(FULL MODEL)=  0.261088330 04,
                SSE(FULL MODEL)=  0.265548C60 02
ESTIMATES OF EXPECTED CELL MEANS-
   CELL   ESTIMATED MEAN
     1    0.611437500 01
     2    0.611437500 01
     3    0.696312500 01
     4    0.696312500 01
     5    0.611437500 01
     6    0.611437500 01
     7    0.696312500 01
     8    0.696312500 01
     9    0.625375000 01
    10    0.625375000 01
    11    0.618062500 01
    12    0.618062500 01
    13    0.625375000 01
```

```
    14    0.62537500D 01
    15    0.61806250D 01
    16    0.61806250D 01
G-INVERSE SOLUTION-
AC
     1  -0.23046875D 00
     2   0.23046875D 00
     3   0.23046875D 00
     4  -0.23046875D 00
A
     1   0.16078125D 00
     2  -0.16078125D 00
C
     1  -0.19390625D 00
     2   0.19390625D 00

     1   0.63779688D 01

  F(R,C,T) L(I(3),J(3),K(3))
DATA FORMAT AND INPUT DATA-
(1X,3I2,F6.0)
 1 1 1  606.
 1 2 2  665.
 1 3 3  940.
 2 1 2  715.
 2 2 3 1087.
 2 3 1  766.
 3 1 3  844.
 3 2 1  711.
 3 3 2  832.
 0 0 0    0.
  C(Z,R,V,G)
OPTIONS- S= 8, T=0.0500, I=100, R=1, V=0, G=0, P=0
  M+R(I)+C(J)+T(K)
E/R LIST-
    0    0    0    5    0    3    2    1
VECTOR   1, ITERATIONS   6, TRACE=     0.771783175
VECTOR   5, ITERATIONS   6, TRACE=     1.543566350
VECTOR   9, ITERATIONS   6, TRACE=     2.315349525
VECTOR  11, ITERATIONS   6, TRACE=     3.087132699
VECTOR  15, ITERATIONS   6, TRACE=     3.858915874
VECTOR  16, ITERATIONS   6, TRACE=     4.630699049
VECTOR  21, ITERATIONS   6, TRACE=     5.402482224
VECTOR  22, ITERATIONS   6, TRACE=     6.174265399
VECTOR  26, ITERATIONS   6, TRACE=     6.946048574
THE RANK OF THE M DESIGN MATRIX IS    7
ITERATION   1, SSRM=  0.34562651D 07
ITERATION   2, SSRM=  0.49923829D 07
ITERATION   3, SSRM=  0.56751019D 07
ITERATION   4, SSRM=  0.59785326D 07
ITERATION   5, SSRM=  0.61133906D 07
ITERATION   6, SSRM=  0.61733276D 07
ITERATION   7, SSRM=  0.61999662D 07
ITERATION   8, SSRM=  0.62118056D 07
ITERATION   9, SSRM=  0.62170675D 07
ITERATION  10, SSRM=  0.62194062D 07
ITERATION  11, SSRM=  0.62204456D 07
ITERATION  12, SSRM=  0.62209075D 07
ITERATION  13, SSRM=  0.62211129D 07
ITERATION  14, SSRM=  0.62212041D 07
```

```
ITERATION  15, SSRM=  0.622124470 07
ITERATION  16, SSRM=  0.622126270 07
ITERATION  17, SSRM=  0.622127070 07
ITERATION  18, SSRM=  0.622127430 07
ITERATION  19, SSRM=  0.622127580 07
ITERATION  20, SSRM=  0.622127650 07
ITERATION  21, SSRM=  0.622127690 07
ITERATION  22, SSRM=  0.622127700 07
ITERATION  23, SSRM=  0.622127710 07
ITERATION  24, SSRM=  0.622127710 07
ITERATION  25, SSRM=  0.622127710 07
ITERATION  26, SSRM=  0.622127710 07
ITERATION  27, SSRM=  0.622127710 07
ITERATION  27, SSR(FULL MODEL)=  0.622127710 07,
               SSE(FULL MODEL)=  0.484289080 04
  H  R(I)
E/R LIST-
    0    0    0   -5    0    3    2    1
THE RANK OF THE H DESIGN MATRIX IS    5
FROM RANK COMPUTATIONS- DF(NUM)=   2, DF(DEN)=    2
ITERATION   1, SSRH=  0.621537690 07
ITERATION   1, F=       1.218 , PROB(F) .GT. 0.4509 VS. F LEVEL OF 0.0500
SSR(REDUCED MODEL)=  0.621537690 07
  H  C(J)
E/R LIST-
    0    0    0    5    0   -3    2    1
THE RANK OF THE H DESIGN MATRIX IS    5
FROM RANK COMPUTATIONS- DF(NUM)=   2, DF(DEN)=    2
ITERATION   1, SSRH=  0.617406360 07
ITERATION   1, F=       9.749 , PROB(F) .GT. 0.0996 VS. F LEVEL OF 0.0500
SSR(REDUCED MODEL)=  0.617406360 07
  H  T(K)
E/R LIST-
    0    0    0    5    0    3   -2    1
THE RANK OF THE H DESIGN MATRIX IS    5
FROM RANK COMPUTATIONS- DF(NUM)=   2, DF(DEN)=    2
ITERATION   1, SSRH=  0.611784090 07
ITERATION   1, F=      21.358 , PROB(F) .GT. 0.0541 VS. F LEVEL OF 0.0500
SSR(REDUCED MODEL)=  0.611784090 07
  E
```