UNIVARIATE DATA MODELING WITH THE ANDERSON-DARLING STATISTIC

A unified approach to data modeling is presented which relies on the Anderson-Darling distance $d(F_n,F_\theta)$ between the empirical distribution function F_n and the hypothesized model F_θ . First, parameters are estimated by minimizing $d(F_n,F_\theta)$ while goodness-of-fit is assessed with the minimized distance $d(F_n,F_{\hat\theta})$. Then, confidence regions are constructed by inverting $d(F_n,F_\theta)$ in the manner suggested by Easterling (1976). Primary attention is focused on the null distribution of $d(F_n,F_{\hat\theta})$ and the efficiency of the confidence procedures.

KEY WORDS

Anderson-Darling statistic
Minimum distance estimators
Goodness-of-fit
Confidence regions

1. INTRODUCTION

Recent papers by Parr and Schucany (1978), Parr and DeWet (1979), Millar (1979), and Boos (1980) have shown that the minimization of a weighted Cramér-von Mises distance between the hypothesized model distribution function F_θ and the empirical distribution function F_n results in estimators $\hat{\theta}$ which are consistent, asymptotically normal, and robust and/or efficient if the weight function is chosen appropriately. In this paper we focus on the Anderson-Darling (A-D) distance

$$d_{F_n}(\theta) = \int_{-\infty}^{\infty} [F_n(x) - F_\theta(x)]^2 [F_\theta(x)(1-F_\theta(x))]^{-1} dF_\theta(x) \qquad (1.1)$$

because the weight function $w_\theta = [F_\theta(1-F_\theta)]^{-1}$ allows a nice balance between robustness and efficiency in a variety of models, and $nd_{F_n}(\hat{\theta})$ has a manageable null distribution. We are suggesting a general approach to univariate modeling: first $d_{F_n}(\theta)$ is minimized to obtain $\hat{\theta}$ and $nd_{F_n}(\hat{\theta})$ is used to check the model validity; then, assuming the model is true, $\{\theta: nd_{F_n}(\theta) \le d_\alpha\}$ forms a confidence region for θ, where d_α is the critical value of the A-D statistic when no parameters are estimated. All such computations can be performed easily on the same computer run using the well-known computing formula

$$d_{F_n}(\theta) = \frac{-1}{n^2} \sum_{i=1}^{n} (2i-1)[\ln F_\theta(X_{(i)}) + \ln(1-F_\theta(X_{(i)}))] - 1 ,$$

where $X_{(1)} \le \cdots \le X_{(n)}$ are the ordered sample values.

Sections 2-4 focus on the three different uses of d_{F_n}. Section 2 describes briefly the minimum A-D distance estimates $\hat{\theta}$ for Weibullized models (detailed general treatments are available in the above references). Section 3 provides Monte Carlo estimates of the percentiles of $nd_{F_n}(\hat{\theta})$ and tends to confirm a conjecture of Boos (1980) that the distribution of $nd_{F_n}(\hat{\theta})$ is approximately distribution-free. Thus, only one set of tabled critical values is required for a number of models F_θ. Section 4 investigates the length of confidence intervals obtained by inverting $d_{F_n}(\theta)$ in one parameter models. Section 5 gives a numerical example, and Section 6 is a brief summary.

2. ESTIMATION

Consider a random sample $X_1,\ldots,X_n$ with Weibullized distribution function $F_\theta(x) = F_0((x/\sigma)^c)$, $x \geq 0$, $\sigma > 0$, $c > 0$. Here F_0 may be any suitable distribution function on $[0,\infty)$ with density $f_0(x)$, but $F_0(x) = 1 - \exp(-x)$ is the most important. The minimum A-D distance estimates $\hat{\sigma}$ and $\hat{c}$ obtained by minimizing (1.1) are also *approximate* solutions of $\Sigma\psi(X_i,\sigma,c) = (0,0)^T$, where

$$\psi(x,\sigma,c) = \begin{pmatrix} \frac{c}{\sigma}\int_0^\infty [I(x \leq \sigma y^{1/c}) - F_0(y)]q_1(y)dy \\ -\frac{1}{c}\int_0^\infty [I(x \leq \sigma y^{1/c}) - F_0(y)]q_2(y)dy \end{pmatrix},$$

and $q_1(y) = f_0^2(y)y[F_0(y)(1-F_0(y))]^{-1}$, $q_2(y) = q_1(y)\,\ell n\, y$. Using integration by parts one can see that $\hat{\sigma}$ and $\hat{c}$ have influence curves which are bounded as long as $\int_0^\infty q_1(y)dy < \infty$ and $\int_0^\infty q_2(y)dy < \infty$. Moreover, the asymptotic covariance matrix of $(\hat{\sigma},\hat{c})$ is given by $n^{-1}\Delta^{-1}C\Delta^{-1}$, where C is the covariance matrix of $\psi(X_1,\sigma,c)$ and

$$\Delta = \begin{pmatrix} \frac{c^2}{\sigma^2}\int_0^\infty y\ q_1(y)f_0(y)dy & \frac{-1}{\sigma}\int_0^\infty y\ q_2(y)f_0(y)dy \\ -\frac{1}{\sigma}\int_0^\infty y\ q_2(y)f_0(y)dy & \frac{1}{c^2}\int_0^\infty y\ \ell n\ y\ q_2(y)f_0(y)dy \end{pmatrix}$$

is the matrix of expected values of the derivative of ψ. For $F_0(x) = 1 - \exp(-x)$ we find by numerical integration that

$$\Delta = \begin{pmatrix} .404\,\frac{c^2}{\sigma^2} & .023\,\frac{1}{\sigma} \\ .023\,\frac{1}{\sigma} & .252\,\frac{1}{c^2} \end{pmatrix}, \quad C = \begin{pmatrix} .192\,\frac{c^2}{\sigma^2} & .039\,\frac{1}{\sigma} \\ .039\,\frac{1}{\sigma} & .049\,\frac{1}{c^2} \end{pmatrix}$$

and thus

$$\Delta^{-1}C\Delta^{-1} = \begin{pmatrix} 1.145\,\frac{\sigma^2}{c^2} & .231\,\sigma \\ .231\,\sigma & .717\,c^2 \end{pmatrix}.$$

We note that the asymptotic relative efficiencies of $\hat{\sigma}$ and $\hat{c}$ compared to the maximum likelihood estimates (m.l.e.'s) are .969 and .848 respectively. However, since the m.l.e.'s are seriously affected by outliers, one might be willing to trade in some efficiency for the robustness achieved by the minimum A-D estimates. We also note that the estimate of σ when c is known has asymptotic variance $.1918/(.4041)^2\ \sigma^2/c^2 = 1.174\ \sigma^2/c^2$ which is larger than the $1.145\ \sigma^2/c^2$ obtained when c is not known. The same phenomenon occurs for estimating c when σ is known. Of course, such results are impossible when using fully efficient estimation schemes.

The minimum A-D estimators do well in other models as well. For normal and logistic location-scale models the efficiencies of $(\hat{\mu}, \hat{\sigma})$ were calculated in Boos (1980) to be (.966, .849) and (1.0, .923) respectively.

3. GOODNESS-OF-FIT

Consider the composite goodness-of-fit hypothesis

H_0: distribution function of the data = F_θ, θ unknown but F_θ a member of some specified parametric family. The minimized distance $d_{min} = d_{F_n}(\hat{\theta})$ is a natural statistic for testing H_0 although its null distribution is much smaller than that of $d_{F_n}(\theta)$ when θ is specified. A similar result holds if $\hat{\theta}$ is replaced by other estimators, and Stephens (1974,1976,1977,1979) has published tables of the null distribution of $nd_{F_n}(\theta_M)$ where θ_M is the m.l.e. estimate. Unfortunately, each parametric family requires a different table. In contrast, Boos (1980) considered location-scale models $F_\theta(x) = F_0((x-\mu)/\sigma)$ and conjectured that the null limiting distribution of nd_{min} could be reasonable approximated by the distribution of $A_2^2 = \sum_{i=3}^{\infty} Z_i^2/i(i+1)$ for a range of symmetric distributions F_0, where the Z_i are i.i.d. standard normal random variables. The Monte Carlo results of this section support that conjecture and suggest that the approximation is also valid for more than just symmetric location-scale models. The general use of $A_k^2 = \sum_{i=k+1}^{\infty} Z_i^2/i(i+1)$ for the case of k estimated parameters can be motivated by analogy with the chi-square goodness-of-fit statistic, where typically the degrees of freedom are reduced by the number of estimated parameters. Here, the limiting distribution of $nd_{F_n}(\theta)$ when no parameters are estimated is $A_0^2 = \sum_{i=1}^{\infty} Z_i^2/i(i+1)$

and estimation of parameters results in the approximate loss of unequal degrees of freedom corresponding to $Z_1^2/2$, $Z_2^2/6$, etc.

Table 1 contains Monte Carlo estimates of the upper percentiles of nd_{min} for normal and logistic location-scale models and for the two parameter Weibull. Since smooth transformations of the data such as $Y_i = \ell n\, X_i$ do not affect the distribution of nd_{min}, the results apply as well to the lognormal, log-logistic (see Tadikamalla and Johnson (1979)) or Burr III with $k = 1$, and extreme value distributions respectively. The IMSL minimization routine ZXMIN was used to find d_{min} and 1000 Monte Carlo samples were generated for each situation. If certain convergence criteria were not met, then those samples were not included in the final estimates although their d_{min} values tended to be in the middle of the distribution and would have had very little effect on the percentile estimates. Simple order statistic estimators were used for starting parameter estimates and the random numbers were generated by the McGill Super-Duper random number generator and analyzed by Dickey's (1978) Monte Carlo package. The errors in the estimates are generally in the range $\pm$ .01 to $\pm$ .04. The percentage points of A_2^2 were computed from Pearson curve approximations and should be accurate to the two decimals listed (see Solomon and Stephens (1978) for details).

--- insert Table 1 here ---

The percentiles of A_2^2 tend to be only a little smaller than the estimated percentiles in Table 1 and the usual fast convergence of A-D statistics to asymptotic values appears to hold even with estimated parameters. Thus, unless exact percentiles are required, we expect that the critical values of A_2^2 will be adequate for most applications.

--- insert Table 2 here ---

Table 2 gives results for the one parameter exponential and shows that the percentiles of $A_1^2 = \sum_{i=2}^{\infty} Z_i^2/i(i+1)$ are from .03 to .11 smaller than the estimated percentiles. Since $E A_1^2 = .5$ and $E\, nd_{min} \to .5254$, a possible correction factor would be to multiply the percentiles of A_1^2 by $.5254/.5 = 1.051$ to obtain respectively .65, .91, 1.10, 1.30, and 1.57 for $\alpha = .25$ to .01. A similar correction could be applied to Table 1 where $E A_2^2 = 1/3$ and $E\, nd_{min} \to .3436$, .3371, and .3459 for the normal, logistic, and Weibull.

A further use of nd_{min} would be to compare values of nd_{min} for different models. This comparison would be meaningful but not necessarily the best method for choosing between models. The non-null asymptotic distribution of d_{min} and some examples of asymptotic power were given in Boos (1980).

4. CONFIDENCE REGIONS

If the data $X_1,\ldots,X_n$ belongs to some specified parametric family F_θ, then $C(\theta) = \{\theta: nd_{F_n}(\theta) \le d_\alpha\}$ forms a confidence region for θ, where d_α is the critical value of the A-D statistic with parameters known. (Stephens (1974) notes that the asymptotic critical values can be used for $n \ge 5$.) Littell and Rao (1978) and Salvia (1979, 1980) have described methods for obtaining analogous regions from the Kolmogorov-Smirnov statistic $D_n = \sup_x |F_n(x) - F_\theta(x)|$. Easterling (1976) originally proposed the use of such regions obtained from goodness-of-fit statistics for model fitting: the bigger the region, the more "consonant" the data is with the model. Our approach is to use d_{min} for goodness-of-fit and $C(\theta)$ as a classical confidence region; i.e., such regions should cover the true parameter value $(1 - \alpha) \times 100$ percent of the time in repeated sampling and should be as *small* as possible. This section is intended to shed a little more light on the sampling characteristics of these regions. In particular, it appears that the high efficiency of the minimum distance estimates carries over to confidence procedures for some parameters but not necessarily for others.

Consider a one parameter family $F_\theta(x)$ for which a random sample $X_1,\ldots,X_n$ is available. Let $\theta_{L,\alpha}$ and $\theta_{R,\alpha}$ be the left and right endpoints of the $(1 - \alpha) \times 100$ percent confidence interval constructed from $d_{F_n}(\theta)$, and let $\hat{\theta}$ be the minimum A-D estimate. A heuristic derivation of the asymptotic length of $n^{\frac{1}{2}}(\theta_{R,\alpha} - \theta_{L,\alpha})$ is as follows. By Taylor expansion

$$\frac{d_\alpha}{n} = d_{F_n}(\hat{\theta}) + d_{F_n}'(\hat{\theta})(\theta_{R,\alpha} - \hat{\theta}) + \tfrac{1}{2}d_{F_n}''(\hat{\theta} + \epsilon_{1,n})(\theta_{R,\alpha} - \hat{\theta})^2$$

$$= d_{F_n}(\hat{\theta}) + d_{F_n}'(\hat{\theta})(\theta_{L,\alpha} - \hat{\theta}) + \tfrac{1}{2}d_{F_n}''(\hat{\theta} - \epsilon_{2,n})(\theta_{L,\alpha} - \hat{\theta})^2$$

Since $d_{F_n}'(\hat{\theta}) = 0$ and $\theta_{L,\alpha} \leq \hat{\theta} \leq \theta_{R,\alpha}$ (if $d_\alpha \geq nd_{F_n}(\hat{\theta})$), then

$$n^{\frac{1}{2}}(\theta_{R,} - \theta_{L,\alpha}) = 2\left(\frac{d_\alpha - nd_{F_n}(\hat{\theta})}{\tfrac{1}{2}d_{F_\theta}''(\theta)}\right)^{\frac{1}{2}}\left[\left(\frac{d_{F_\theta}''(\theta)}{d_{F_n}''(\hat{\theta}+\epsilon_{1,n})}\right)^{\frac{1}{2}} + \left(\frac{d_{F_\theta}''(\theta)}{d_{F_n}''(\hat{\theta}-\epsilon_{2,n})}\right)^{\frac{1}{2}}\right]$$

$$\xrightarrow{d} 2\left(\frac{d_\alpha - T_1(U)}{\tfrac{1}{2}d_{F_\theta}''(\theta)}\right)^{\frac{1}{2}} I(d_\alpha \geq T_1(U)) , \qquad (2.1)$$

where $T_1(U)$ is the limiting distribution of $nd_{F_n}(\hat{\theta})$. One interesting feature is that (2.1) is a random variable instead of a constant. In contrast, the typical confidence interval for the scale parameter of the exponential distribution $F_\theta(x) = 1 - \exp(-x/\theta)$ has length when multiplied by $n^{\frac{1}{2}}$ which converges to $2z_{\alpha/2}$, where z_α is the upper $(1-\alpha) \times 100$ percentile of the standard normal (see Bain (1978), p. 129). For comparison purposes we might use an approximate bound on the expectation of (2.1),

$$E(2.1) \mathrel{\dot{\leq}} 2\left(\frac{d_\alpha - ET_1(U)}{\frac{1}{2}d_{F_\theta}''(\theta)}\right)^{\frac{1}{2}} . \qquad (2.2)$$

In Table 3 we compare $2z_{\alpha/2}$ with (2.2) for the exponential, where $\frac{1}{2}d_{F_\theta}''(\theta) = .40411$ and $ET_1(U) = .525$.

--- insert Table 3 here ---

These results are consistent with Table 7 of Easterling (1976) and help verify that the asymptotic efficiency .85 of $\hat{\theta}$ carries over to confidence interval construction as well. We now show that such efficiency need not carry over to confidence intervals in every situation.

Let $F_c(x) = 1 - \exp(-x^c)$, Weibull with scale equal to one. In Table 4 we compare (2.2) with the asymptotically efficient method based on the m.l.e. of c which has asymptotic length when multiplied by $n^{\frac{1}{2}}/c$ converging to $2z_{\alpha/2}(.608)^{\frac{1}{2}}$. Here $\frac{1}{2}d_{F_c}''(c) = .2519/c^2$ and $ET_1(U) = .8062$.

--- insert Table 4 here ---

Now the intervals derived from $d_{F_n}(\theta)$ are just not competitive with the intervals derived from the m.l.e. although the asymptotic relative efficiency of $\hat{c}$ to the m.l.e. is .71. The approximation (2.2) might not be as accurate for this situation, but a 1000 sample Monte Carlo estimate of the expected length for $\alpha = .05$, $n = 20$, yielded 4.96 which is reasonably close to the tabled value 5.17 of (2.2).

--- insert Tables 5 and 6 here ---

Tables 5 and 6 give analogous results for the normal location (scale known) and normal scale (location known) respectively where the minimum A-D estimates have asymptotic relative efficiencies of .97 and .85. We infer the following basic principle: inversion of a Cramér-von Mises goodness-of-fit statistic will yield a competitive confidence interval (with regard to length) only for the parameter which corresponds to the *first* component of the statistic involved. Moreover, we expect the general principle to hold for confidence regions as well. For example, in the two parameter Weibull $F_\theta(x) = 1 - \exp(-(x/\sigma)^c)$, we expect the confidence region to be relatively short in the σ direction but quite longer than necessary in the c direction. The procedure is still useful since exact confidence regions are obtained for a variety of models and the total area of such regions could easily be competitive with say a Bonferroni rectangular region obtained from individual confidence intervals derived from the m.l.e.'s. Lastly, the goodness-of-fit approach is adaptable to censored samples by use of a recent result of Michael and Schucany (1980).

5. NUMERICAL EXAMPLE

Salvia (1979) used the data in Table 7, originally found in Visscher and Goldman (1978), and constructed 80 percent confidence regions for the two parameter Weibull and exponential distributions. The conclusion

--- insert Table 7 here ---

was that the data was more "consonant" with a Weibull than with an exponential. In Table 8

--- insert Table 8 here ---

we have computed the m.l.e.'s, minimum A-D estimates, and our goodness-of-fit statistic for a number of models. Recall from Tables 1 and 2 that the $\alpha = .05$ critical value for nd_{min} is approximately 1.05 and .63 for one and two parameter models respectively. The data speaks strongly against a one or two parameter exponential, but either the Weibull, normal, or logistic fit the data fairly well. In Figure 1

--- insert Figure 1 here ---

we have drawn the 75 and 90 percent confidence regions for (c,σ) under the Weibull model. In addition, the dotted lines are individual 95 percent confidence intervals for c and σ computed from the m.l.e.'s using Bain (1978), Ch. 4. The dotted region thus forms a 90 percent Bonferroni rectangular region for (c,σ) and illustrates the low efficiency of the A-D region in the c direction. However, we note that the 90 percent A-D region is shorter in the c direction than is the 80 percent Kolmogorov-Smirnov region calculated by Salvia (1979).

Finally, we show how Table 8 would change if $X_{(28)} = 5885$ were miscoded to be 9885. Table 9

--- insert Table 9 here ---

presents those altered calculations. As expected, the minimum distance estimates are more stable (robust!) than the m.l.e.'s. The normal model is no longer acceptable at the .05 level, although the Weibull and logistic remain acceptable.

6. SUMMARY AND CONCLUSIONS

A comprehensive approach to univariate data modeling has been suggested which includes estimation of parameters, testing goodness-of-fit, and construction of confidence regions, all based on the Anderson-Darling goodness-of-fit statistic. Results for the normal (lognormal), logistic (log-logistic), and Weibull (extreme value) indicate that the approach will be useful for a variety of possible model distributions.

TABLE 1-*Monte Carlo estimates of the percentage points* d_α, *such that* $P(nd_{min} \leq d_\alpha) = 1 - \alpha$.

α	.25	.10	.05	.025	.01
Normal					
n = 20	.42	.55	.69	.79	.98
n = 50	.41	.53	.63	.76	.90
Logistic					
n = 20	.40	.53	.62	.72	.82
n = 50	.42	.54	.63	.71	.84
Weibull					
n = 20	.41	.54	.63	.75	.87
n = 50	.42	.56	.66	.76	.87
$A_2^2 = \sum_3^\infty Z_i^2/i(i+1)$*					
	.41	.54	.63	.73	.86

*Percentage points of A_2^2 calculated from Pearson curves, see Solomon and Stephens (1978).

TABLE 2-*Monte Carlo estimates of the percentage points* d_α, *such that* $P(nd_{min} \leq d_\alpha) = 1 - \alpha$.

α	.25	.10	.05	.025	.01
Exponential					
n = 20	.64	.91	1.13	1.35	1.56
n = 50	.65	.91	1.11	1.33	1.61
$A_1^2 = \sum_2^\infty Z_1^2/i(i+1)$*					
	.62	.86	1.05	1.24	1.50

*Percentage points of A_2^2 calculated from Pearson curves, see Solomon and Stephens (1978).

TABLE 3–*Asymptotic lengths of confidence intervals for the exponential scale parameter.*

α	$2z_{\alpha/2}$	(2.2)	Ratio
.01	5.15	5.74	.90
.05	3.92	4.41	.89
.10	3.29	3.73	.88
.25	2.30	2.67	.86

TABLE 4–*Asymptotic lengths of confidence intervals for the Weibull shape parameter (scale known).*

α	$2z_{\alpha/2}(.608)^{\frac{1}{2}}$	(2.2)/c	Ratio
.01	4.02	6.96	.58
.05	3.06	5.17	.59
.10	2.57	4.23	.61
.25	1.79	2.65	.68

TABLE 5–*Asymptotic lengths of confidence intervals for a normal location parameter (scale known).*

α	$2z_{\alpha/2}$	(2.2)	Ratio
.01	5.15	5.28	.97
.05	3.92	4.07	.96
.10	3.29	3.45	.95
.25	2.30	2.49	.92

TABLE 6–*Asymptotic lengths of confidence intervals for a normal scale parameter (location known).*

α	$(2)^{\frac{1}{2}}z_{\alpha/2}$	(2.2)	Ratio
.01	3.64	6.68	.55
.05	2.77	4.95	.56
.10	2.33	4.02	.58
.25	1.63	2.46	.66

TABLE 7-*Casino earnings.*[1]

416	1555	2595	3162	3516	5395
594	2065	2845	3251	3729	5520
1192	2070	2967	3283	3963	5885
1269	2438	2999	3414	4006	7059
1453	2497	3130	3467	4338	

[1]Data first appeared in Visscher and Goldman (1978).

TABLE 8-*Analysis of Table 7.*

	Model	m.l.e.	Min. A-D Est.	nd_{min}
Exponential	$1 - \exp(-x/\sigma)$	$\hat{\sigma} = \bar{X} = 3106$	3872	2.75
Exponential	$1 - \exp(-(x-\mu)/\sigma)$	$(\hat{\mu},\hat{\sigma}) = (416,2690)$	(401,3280)	1.95
Weibull	$1 - \exp(-(x/\sigma)^c)$	$(\hat{c},\hat{\sigma}) = (2.13,3501)$	(2.16,3515)	.34
Normal	$\Phi((x-\mu)/\sigma)$	$(\bar{X},S) = (3106,1551)$	(3045,1509)	.36
Logistic	$(1+\exp(-(x-\mu)/\sigma))^{-1}$	$(\hat{\mu},\hat{\sigma}) = (3033,846)$	(3034,861)	.27

TABLE 9-*Analysis of Table 7 with* 5885 *changed to* 9885.

	Model	m.l.e.	Min. A-D Est.	nd_{min}
Exponential	$1 - \exp(-x/\sigma)$	$\hat{\sigma} = \bar{X} = 3244$	3941	2.48
Exponential	$1 - \exp(-(x-\mu)/\sigma)$	$(\hat{\mu},\hat{\sigma}) = (416,2828)$	(400,3344)	1.72
Weibull	$1 - \exp(-(x/\sigma)^c)$	$(\hat{c},\hat{\sigma}) = (1.79,3653)$	(1.98,3575)	.51
Normal	$\Phi((x-\mu)/\sigma)$	$(\bar{X},S) = (3244,1937)$	(3066,1632)	.72
Logistic	$(1+\exp(-(x-\mu)/\sigma))^{-1}$	$(\hat{\mu},\hat{\sigma}) = (3047,952)$	(3043,900)	.45

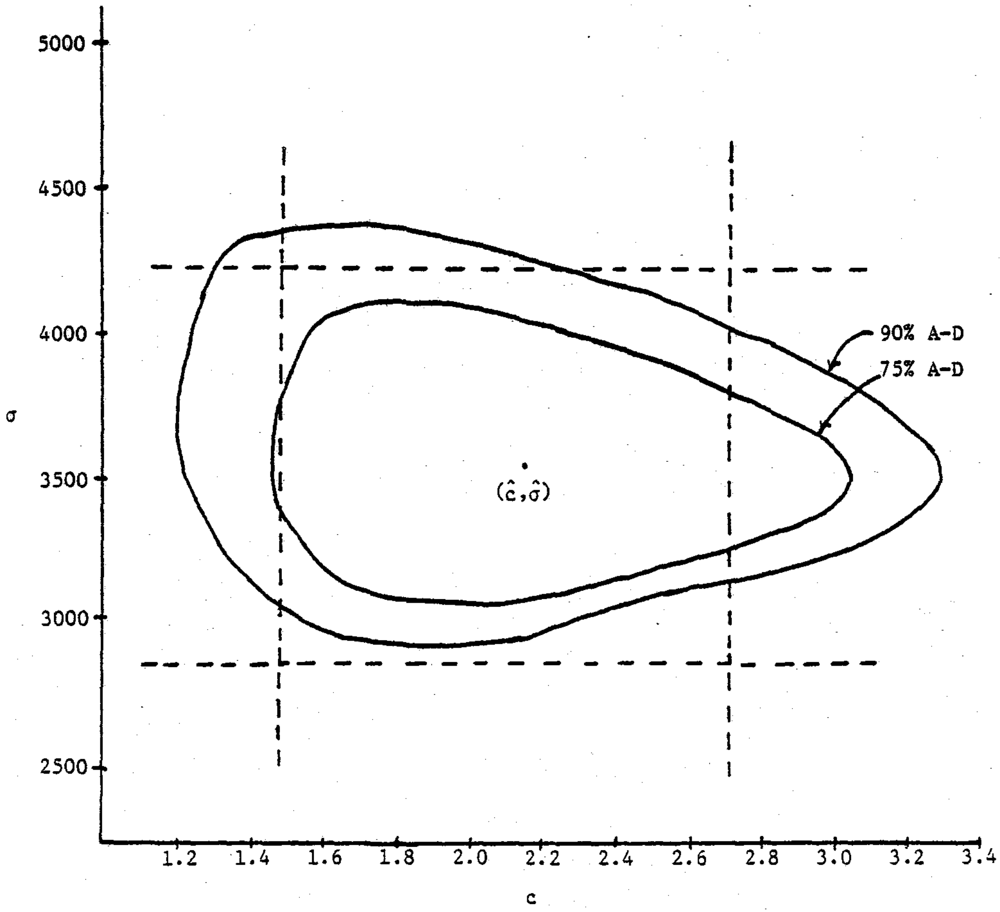

FIGURE 1. Confidence regions for (c,σ).

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