ANALYSIS OF A MULTI-ITEM INVENTORY PROBLEM USING OPTIMAL POLICY SURFACES

by

Helmut Schneider

University of North Carolina
at Chapel Hill, NC 27514

1. Introduction

In this paper we shall consider a multi-item inventory problem with unspecified single-item unit costs. Rather than examining a single cost function, we shall deal with an approach which incorporates aggregate objectives and constraints. The objectives are low investment, low total costs and high service-level. The constraints are storage room capacity and available workload for handling the orders. These capacities might be increased or decreased in certain fixed quantities; such changes in workload and storage room incur costs which are independent of whether or not these capacities are used to their full extent. The objectives are conflicting and in many real world problems there seems to be no single cost function for determining an optimal decision. This is due to the fact that there is no decision unity; instead, there are different departments, different managers and differing interests involved. How much the company should invest in inventory and what service level should be required cannot be determined simply by evaluating a single cost function, but is rather a result of intensive discussions. What operations research can offer is a description of the relationship between investment and service-level; i.e., for a certain investment we are able to maximize the service-level under the constraints of storage room and workload. Furthermore we might study the effect upon an alteration of the constraints. This enables the management to find an "appropriate" decision weighing the different interests ascertaining that there is no solution with lower total costs and higher service-level.

Despite the enormous number of papers on inventory models there are only a few articles concerning this important problem. Most papers start by assuming that marginal holding, shortage and ordering costs are given. But marginal ordering costs are difficult to measure. Most suggested approaches for determining ordering costs in the accounting literature result in average rather than marginal costs [15]. In practice there is usually a certain workload available which might be increased in fixed quantities. Assigning average order costs therefore does not solve this particular inventory problem. Holding costs should be composed not only of the cost for capital but also of marginal costs taking into consideration a storage room restriction. The use of shortage costs in inventory theory has not been adopted by most practitioners [2] since there is no basis for their measurement in accounting methodology [15].

Only a few authors deal with an aggregate inventory problem as described above.

Starr and Miller [13] have considered an "optimal policy curve" for deterministic demand. Schrady and Choe [11] considered a continuous review inventory system with constraints. Gardner and Dannenbring [3] extended Starr's and Miller's approach by considering a continuous review stochastic model. They presented a method that avoids cost measurement problems and incorporates aggregate objectives and constraints. They describe a procedure for obtaining an optimal policy surface, the axes of which are measured in aggregate terms: the percentage of inventory shortages, as a measure of customer service; the workload in terms of the number of annual stock replenishment orders; and total investment, the sum of cycle and safety stocks. Aggregate inventory decisions are defined as the selection of a combination of the three variables. This model obviously reflects the true decision problem in practice much better than a single-item cost model. In fact data simulation is frequently employed in practice to solve decision problems described above.

Unfortunately the underlying inventory model, considered by Gardner and Dannenbring, has some disadvantages which makes it inapplicable in many situations. Most of the inventory systems installed in practice have periodic review and not continuous review [5]. It has been shown that, given a certain inventory policy, the service-level turns out to be very different in a periodic and in a continuous review system [9]. Furthermore we should be aware that there are different definitions of service-levels [10], which result in very different ordering policies.

In the present paper a similar approach to that of Gardner and Dannenbring is used. But there are some essential differences. First, a periodic review multi-item inventory model is considered. Second, we consider only two objectives, investment and service-level, subject to workload and storage room restrictions. It seems to be more realistic to begin with a given workload and storage room, which can be expanded in fixed quantities, rather than to assume that workload and storage room are continuous variables. Furthermore an overall cost evaluation is considered including costs of investment, storage room and workload.

An interactive algorithm is presented which allows the selection of a combination of service-level, investment, workload and storage room which is "appropriate" for the management. This method produces combinations which lie on an optimal surface.

Since in our method there are some approximations involved, we shall prove the validity of the approximation formulas by means of a Monte Carlo study in the final section.

2. The Model

Consider an inventory model with n items. The stock of every item is inspected at the beginning of a review period and an order is placed for those items for which the stock level has fallen to the reorder point. After a known lead time λ the orders will arrive. The demand of an item in different periods is a random variable with known distribution. Let us define for item k, k=1,2,3,...,n.

X_{kt} - inventory on hand plus on order at the beginning of period t, before an order is placed.

Q_{kt} - order which is placed at the beginning of period t

r_{kt} - stochastic demand in period t. The demand in successive periods is a sequence of independent and identically distribuged random variables with cdf $F_k(r)$, mean μ_k and variance σ_k^2.

p_k - price of item k.

Furthermore, it is assumed that demand which cannot be immediately satisfied is backordered. We discuss a stationary inventory model and thus it is sufficient to consider a single period inventory model where the distribution of X_{kt} is the stationary distribution $\psi(x)$ [6]. We will also assume that a service level γ , which is defined as

$$\gamma = 1- \frac{\text{cumulative backorders per period}}{\text{average demand per period}}$$

is an appropriate measure of customer service for product k=1,...,n. Notice that this definition of a service level is equivalent to assigning shortage costs which are dependent on the amount of items short and the length of time the shortage lasts. A formal proof is given in [9]. This type of shortage cost is considered by most authors [4], [11], [14].

In what follows we consider two objectives:

0_1: minimize the total investment as the sum of cycle and safety stock

0_2: maximize the service level γ and two constraints

and two constraints

C_1: the number of stock replenishment orders is restricted by the workload capacity

C_2: there is a storage room capacity restriction

This two-objective-decision problem is solved by determining the optimal policy surface. For every point on the surface it holds that none of the objectives can be improved without diminishing the other. Before proceeding we shall make some remarks concerning the objectives and constraints. The objectives reflect of course cost considerations and the constraints are associated with costs which become relevant if alterations of the constraints are allowed. With 0_1 we only control the variable holding costs induced by invested capital. If the storage room is fixed the costs for holding this room are fixed too and thus irrelevant for a decision. But often it is the case that storage room is rented in certain quantities and hence the costs for holding a storage capacity becomes relevant for a decision. The same is true for the workload restriction. We therefore consider a second set of objectives without constraints.

0_4: minimize total costs involving cost for workload, cost for invested capital, and cost for holding a storage room.

0_5: maximize service level γ

Both sets of objectives should be available for a decision process for selecting an "appropriate" solution.

We shall now formally define our objectives and constraints. It is well known that when given a single item inventory model of the type described above an (s,S) policy is optimal [6]. We will thus consider (s_k,S_k) policies in our approach. Let

$E[I_k|s_k,S_k]$ - expected inventory at the end of a period for product k

$E[Q_k|s_k,S_k]$ - expected order quantity for product k

$E[NO_k|s_k,S_k]$ - expected number of orders per period for product k

$E[CI|\{s_k,S_k\}]$ - expected invested capital in inventory

$E[NO|\{s_k,S_k\}]$ - expected number of orders per period

$E[SR|\{s_k,S_k\}]$ - expected storage room

Note that

$$E[CI|\{s_k,S_k\}] = \sum_{k=1}^{n} P_k E[I_k|s_k,S_k]$$

$$E[SR|\{s_k,S_k\}] = \sum_{k=1}^{n} a_k E[I_k|s_k,S_k]$$

where a_k is the unit storage room for product k. We finally formulate the two-objective-decision problem as

$$\min_{\{s_k,S_k\}} \quad E[SI|\{s_k,S_k\}] \qquad (1)$$

$$\max_{\{s_k,S_k\}} \quad E[\gamma(\{s_k,S_K\})] \qquad (2)$$

subject to

$$E[SR|\{s_k,S_k\}] \leq \text{Storage room capacity} \quad \text{(SRR)} \tag{3}$$

$$E[NO|\{s_k,S_k\}] \leq \text{Workload capacity} \quad \text{(WLR)} \tag{4}$$

Considering a single item the expected values can be derived by [9].

$$E[I_k|(s_k,S_k)] = \frac{L^+(S_k)+\int_0^{D_k} L^+(S_k-x)m(x)dx}{1+M(D_k)} \tag{5}$$

$$E[NO_k|(s_k,S_k)] = \frac{1}{1+M(D_k)} \tag{6}$$

$$E[\gamma(s_k,S_k)] = 1 - \frac{L^-(S_k)+\int_0^{D_k} L^-(S_k-x)m(x)dx}{[1+M(D_k)]\mu_k} \tag{7}$$

where (see [9]) $M(\cdot)$ and $m(\cdot)$ are solutions of the renewal equations

$$M(z) = F(z)+\int_0^z M(z-t)dF_k(t) \qquad m(z) = f(z)+\int_0^z m(z-t)f_k(t)dt$$

and L^+ and L^- are defined by

$$L^+(x) = \int_0^x (t-x)f_k^{\lambda+1}(t)dt$$

$$L^-(x) = \begin{cases} \int_x^\infty (t-x)f_k^{\lambda+1}(t)dt & x \geq 0 \\ \int_0^\infty (t-x)f_k^{\lambda+1}(t)dt & x \leq 0 \end{cases}$$

where $f_k^{\lambda+1}(t)$ is the pdf of demand in lead time plus review time. Note that $D_k = S_k-s_k$. Although an exact solution, i.e. the determination of the optimal policy surface, is in principle possible, we will not recommend it since the approximation, provided below, will give such excellent results that there is no incentive to carry out the extraordinary high computations. Our approximation is based on empirical results [14] that there is only little loss of optimality if the optimization is separated in determining first $D_k = S_k - s_k$ and afterwards s_k. Furthermore, we consider the expected values under the realistic assumption that D_k is large and we are thus able to simplify the expressions (5) to (7), an approach which was introduced by Roberts [8]. Following these principles we derive the simplified expected values

$$E[I_k|(s_k,S_k)] = D_k(1-\tfrac{1}{2}\frac{D_k}{Q_k}) + s_k - (\lambda_k+1)\mu_k + (1-\gamma)\mu_k \tag{8}$$

$$E[NO_k|(s_k,S_k)] = \frac{\mu_k}{D_k+\mu_{2k}/2\mu_k} \tag{9}$$

$$\gamma(s_k,S_k) = 1 - \frac{\int_{s_k}^{\infty}(t-s_k)^2 f_k^{\lambda+1}(t/dt}{2[D_k+\mu_{2k}/2\mu_k]} \tag{10}$$

where $Q_k = D_k + \dfrac{\mu_k^2+\sigma_k^2}{\mu_k^2}$.

The expression (8) is derived in the appendix; (9) is a well-known limiting theorem of the renewal function M(D) [12] and (10) was derived in [9].

Since the constraint (4) will always be active we can solve this optimization problem using the Lagrange method. Let

$$L(Q_1,\ldots,Q_n,\rho) = \sum_{k=1}^{n} P_k E[I_k|Q_k] + \rho[\sum_{k=1}^{n} E[NO_k|Q_k] - WLR] \tag{11}$$

A straightforward application of the Lagrange method yields

$$Q_k^* = \sqrt{\frac{2\rho\mu_k}{P_k}} \qquad k=1,\ldots,n \tag{12}$$

and

$$\rho \frac{(\Sigma\sqrt{\mu_k p_k})^2}{WLR^2} \quad . \tag{13}$$

The reorder points s_k can now be calculated in a second step for various service levels γ. We determine s_k by equation (10).

The cycle plus safety stock for product k and fixed service-level is then

$$E[I_k|s_k,D_k] = D_k(1-0.5\,\frac{D_k}{Q_k}) + s_k-(\lambda_k+1)\mu_k+(1-\gamma)\mu_k \tag{14}$$

and thus the total investment is

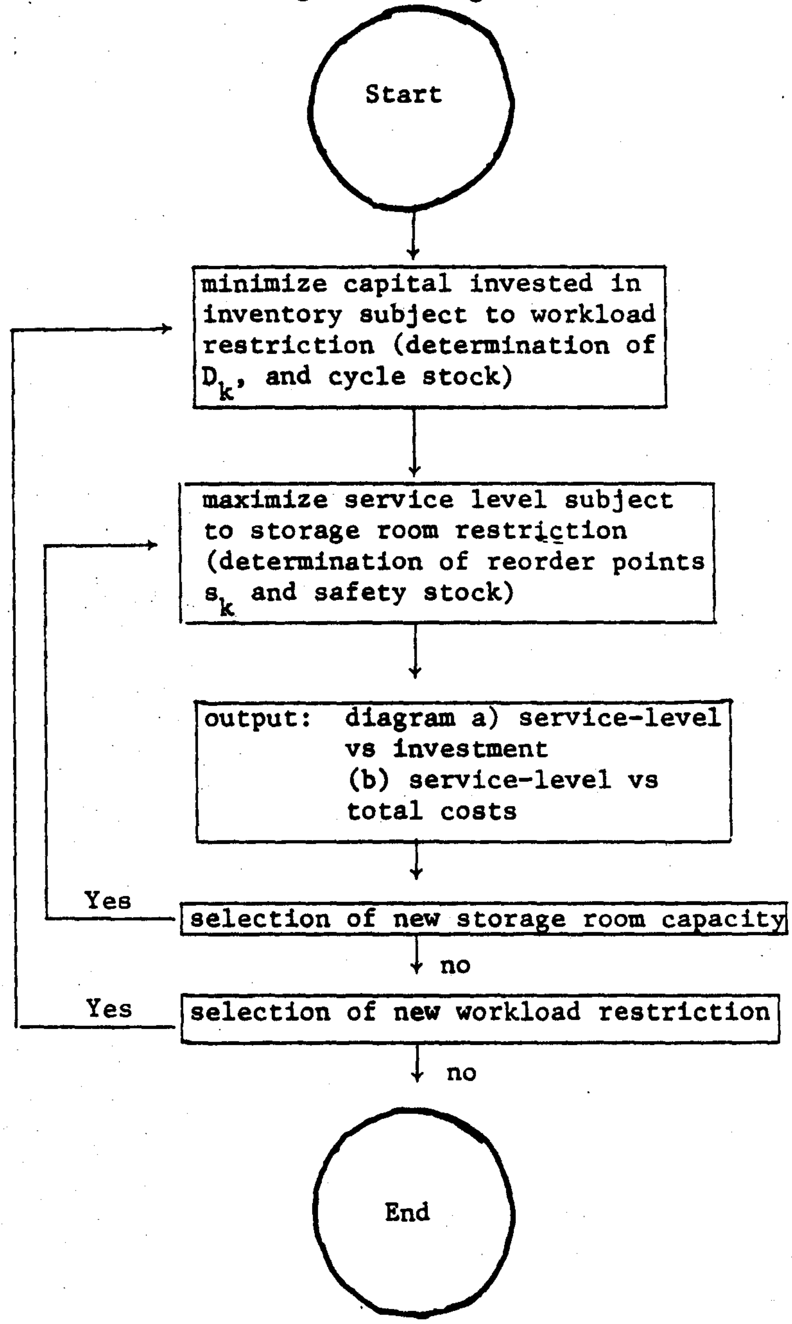
$$E[CI|\gamma] = \Sigma p_k E[I_k|(s_k,S_k)]$$

and the expected total storage room is

$$E[SR|\gamma] = \Sigma a_k E[I_k|(s_k,S_k)]$$

which are both functions of the service level γ. The latter value is now compared with the storage room capacity. The service level can be increased as long as the storage room constraint is not active. We obtain a diagram which shows the investment versus service-level γ; this diagram will end at γ' which marks the maximum service-level at which the storage room constraint becomes active. In addition a diagram which gives the total costs (for investment plus fixed costs for the storage room capacity plus fixed costs for the workload capacity) versus service-level can be presented (see figures 2 and 3). In contrast to the unit single item costs these fixed costs are easy to determine. These diagrams are the basics for selecting an "appropriate" service-level and investment by the management. The steps for obtaining the diagrams are summarized in the following algorithm.

Figure 1: Algorithm



Example

In this subsection we shall consider an example with only 100 items to describe the algorithm. In practice the service-level of items varies from product to product. Some products, for example, are more vital than others. Usually, however, one is able to classify the items and put them into certain groups which have the same service-level requirements.

We thus define m groups such that

$$\gamma_j = g_j \gamma, \; j=1,2,3,\ldots,m$$

let n_j be the number of products in subgroup j then

$$\gamma = \frac{1}{n} \sum_{k=1}^{n} \gamma_k = \frac{1}{n} \sum_{j=1}^{m} n_j g_j \cdot \gamma$$

which yields to the restriction

$$\frac{1}{n} \sum_{j=1}^{m} n_j g_j = 1 \tag{15}$$

Furthermore

$$\gamma \leq 1/\max_j \{g_j\} \tag{16}$$

The problem of assigning γ_k is thus reduced to determining m numbers g_j which fulfill (15) and (16). This will usually be possible in a reasonable way.

We shall now consider a situation in which the management has to decide how much to invest during the year to come and what service-level should be required. There is a certain workload available which allows 22 orders per week. The cost of this workload is $115,000 per year. Free handling capacity cannot be used for other purposes. An increase or decrease of workload is only possible in quantities which are equivalent to 4 orders per week. The cost for such a workload quantum is $15,000 per year. There is also storage room available which allows for the storage of 60,000 units. The cost for renting the storage room is

$130,000 per year. Increase or decrease is only possible in quantities of 10,000 units which will cause costs of $20,000 per year. The following data are assumed to be known

mean demand: $\mu_k = 20+9.89(k-1) \quad k = 1,2,...,100$

variance: $\sigma_k^2 = c^2\mu_k^2 \quad k = 1,2,...,100$

$c = 0.2,\ 0.4,\ 0.6,\ 0.8,\ 1.2,\ 1.6$

unit storage room of product k: is randomly chosen to be in the interval [0.5,1.5]

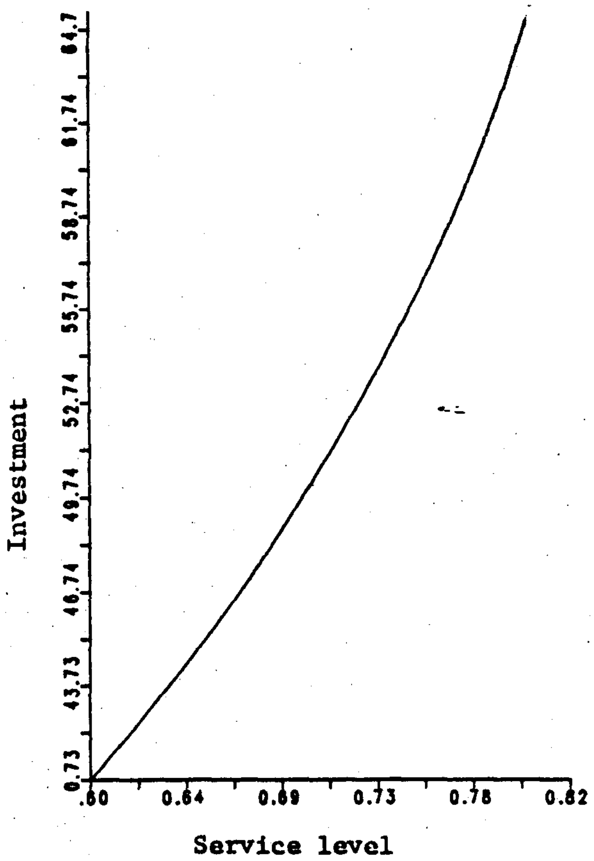
lead time: is randomly chosen to be {2,3,4,5,6,7,8}

price: $p_k = 500/\mu_k \quad k = 1,2,...,100$

service level: 5 groups of 20 products with the service levels 0.8γ, 0.9γ, γ, 1.1γ, 1.2γ

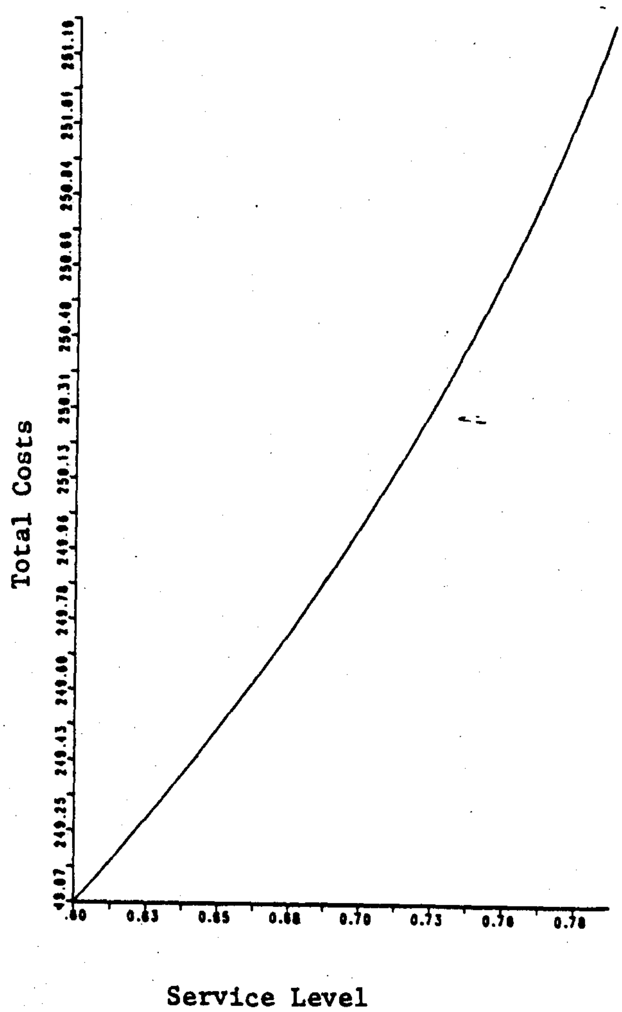
We assume a 10% interest for the invested capital. The algorithm is demonstrated for $c = 0.2$. The management assigns an interval for the service-level $60\% \leq \gamma \leq 100\%$. According to the algorithm the following diagram is plotted. Due to the workload restriction for handling and the storage room restriction the maximum average service level is 79.5%, thus the diagram ends at that point.

Figure 2: Service-level versus investment



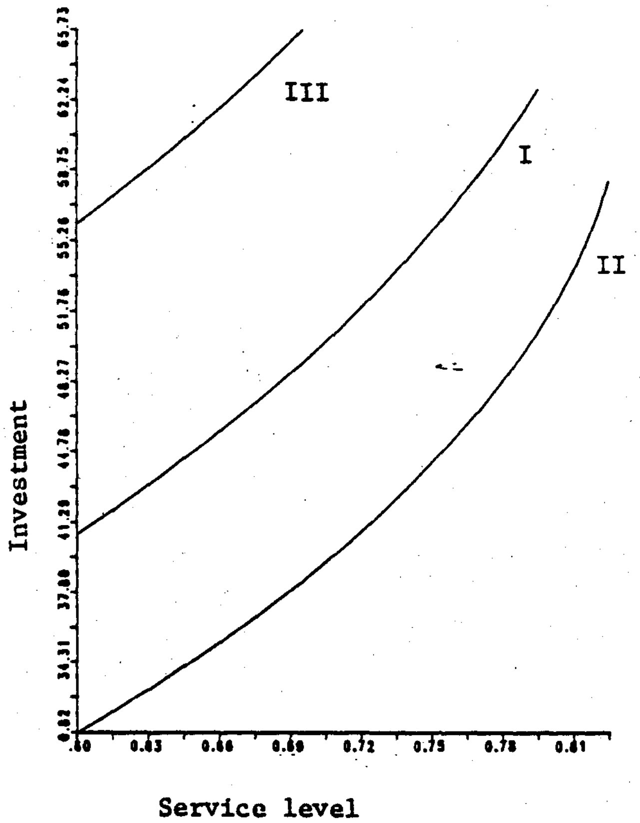
The total costs are presented in the next diagram. The total costs are the sum of the costs for storage room, workload and invested capital.

Figure 3: Service-level versus total costs



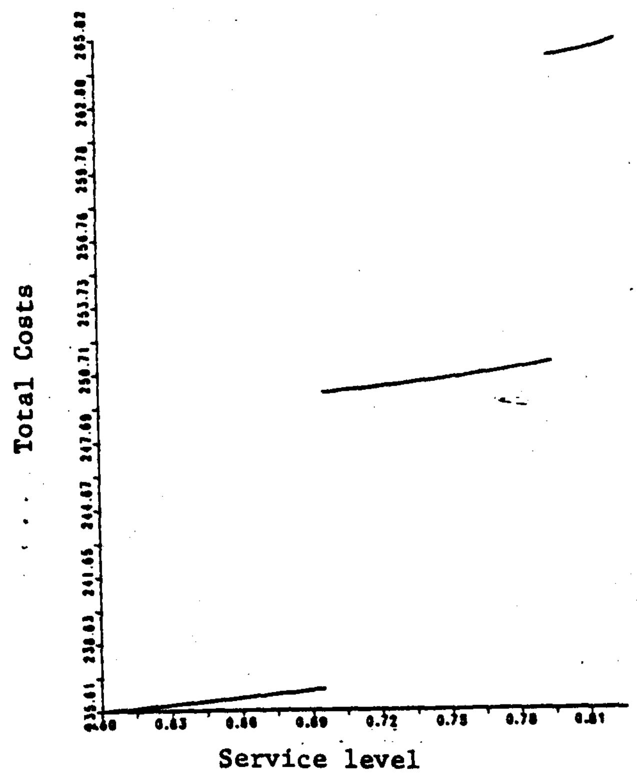
We assume that the management likes to study the effect of an alteration of the restrictions on investment and service-level. First the workload restriction is increased (II) and decreased (III) by 4 orders. Again the invested capital versus service-level is plotted for the three possibilities.

Figure 4: The effect of an alteration of the workload restriction. I) initial situation, II) increase, III) decrease of workload restriction.



We see that a reduction of workload (curve III) results in a lower available service-level while an increase allows a higher service-level than 79.5%. This is caused by a change of the cycle stock, which decreases (curve III) or increases (curve II) the available storage room for safety stock. Diagram 5 gives the total costs for the alternatives.

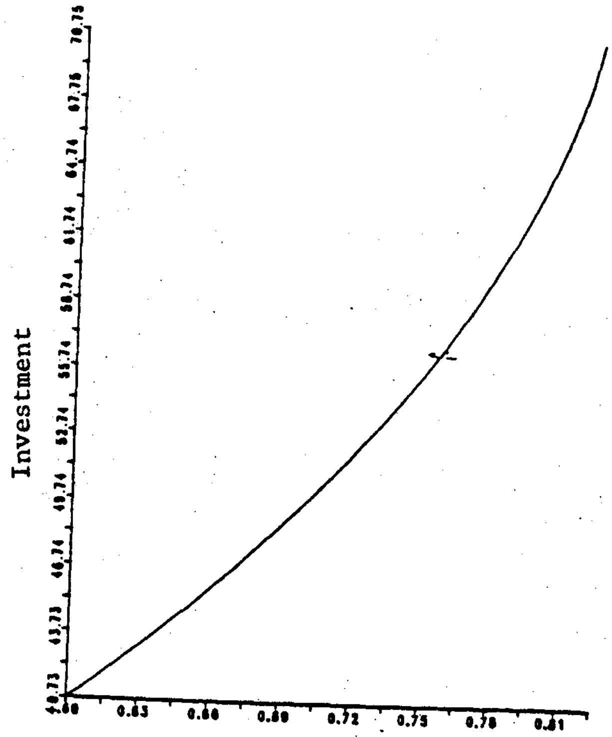
Figure 5: Service-level versus total costs for different workload restrictions.



We notice that service-levels lower than 70% can be reached by a lower workload than available at the beginning and thus by lower total costs. The opposite holds if a higher service-level than 79.5% is desired. Such service-levels can only be reached with higher total costs.

Secondly, the storage room capacity is altered. For the initial situation, i.e. a workload which allows 22 orders, we increased and decreased the storage room by 10,000 units. We obtain the invested capital versus service-level as shown in figure 6.

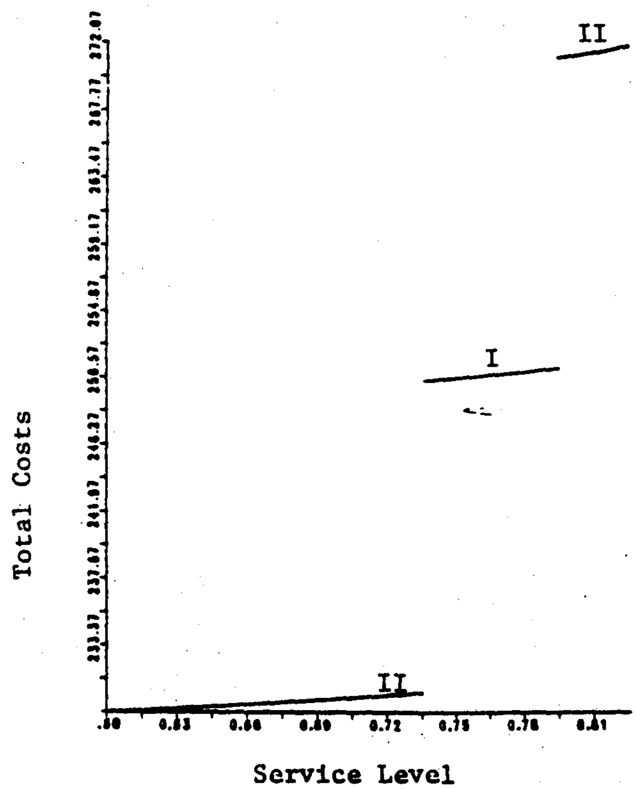
Figure 6: Service-level versus investment for different storage room restrictions



Since the workload restriction is fixed, the cycle stock is constant and only the safety stock increases, which results in a curve ending at a service-level of 82.5%.

Figure 7 gives the total costs of the three alternatives.

Figure 7: Service-level versus total costs I) initial situation II) increase, III) decrease of storage room restriction



Altogether we have 9 alternatives; the maximum service-levels and total costs of these service levels are presented in Table 1.

Table 1: Maximal service-level and total costs for various combinations of storage room and workload capacity

Storage Room Capacity / Workload Capacity	50,000		60,000		70,000	
	Max. Serv.	Total Costs	Max. Serv.	Total Costs	Max. Serv.	Total Costs
18	60%	216	69.5%	237	76.5%	258
22	73.5%	230	79.5%	251	82.5%	272
26	79.5%	245	82.5%	266	83%	286

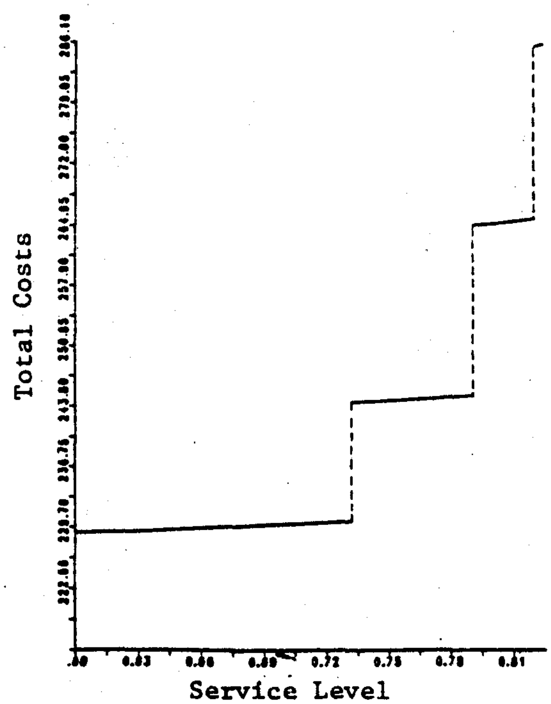
The four minimal cost combinations are presented in Table 2; figure 8 gives the minimal total costs versus service-level. Table 2 shows that for our example a maximum service-level of 73.5% can be reached by the combination of workload (22) and storage room (50,000). A service-level of 79.2% can be reached by increasing the workload capacity to (26). If a higher service-level than 79.5% is required we increase the storage room capacity.

Table 2: Optimal Combinations of Workload and Storage Room

Maximal Obtainable Service-Level	Workload	Storage Room
-73.5%	22	50,000
-79.5%	26	50,000
-82.5%	26	60,000
-83%	26	70,000

The total costs can be obtained from figure 8.

Figure 8: Service-level versus Total Costs of Optimal Combinations



The management now has all the necessary information to decide what service-level and corresponding investment should be selected, and whether or not the workload and the storage room restrictions should be altered.

We notice that with the selection of a certain point on one of the curves presented above we determined the optimal inventory policy as well, i.e. the set $\{s_k, S_k\}$.

3. Validation of the Model by Monte Carlo Simulation

The purpose of the simulation provided in this section is twofold. First, we want to use the simulation results to test the accuracy of the approximation formulas derived in section 2. Second, we investigate the variance of the various measures of system performance such as handling used, inventory on hand, invested capital over the periods. Since the restrictions are met by expected values, we have to be careful that the variance is not too large.

We have performed 6 simulations for different demand structures, i.e. $c = 0.2, 0.4, 0.6, 0.8, 1.2, 1.6$. For $c \leq 0.5$ a normal distribution was used; otherwise the gamma distribution was found to be appropriate [9]. Notice that for increasing c the demand becomes very erratic. The simulations were run for workload WL = 22 and service level $\gamma = 0.82$. The theoretic results for inventory on hand and invested capital are given in Table 3.

Table 3: Theoretical results ($\gamma = 0.82$ required) for mean storage room and invested capital.

$c = \sigma/\mu$	Expected Storage Room Used	Expected Invested Capital In Inventory In $
0.2	67.312	68,855
0.4	86,185	87,167
0.6	120,282	120,445
0.8	162,687	163,212
1.2	279,741	283,268
1.6	440,667	451,172

Table 4 gives the simulation results with 1000 periods and 50 repetitions.

Table 4: Simulation results for mean storage room, mean invested capital, mean handling and mean service-level.

$c = \sigma/\mu$	Storage Room Used			Handling		
	Mean	Std	Δ%	Mean	Std	Δ%
0.2	68,322	60	+1.5	22	0.01	+0.1
0.4	86,761	47	+0.6	22	0.01	+0.1
0.6	122,373	61	+1.7	21.8	0.01	-0.7
0.8	166,104	187	+2.1	21.9	0.02	-0.6
1.2	286,735	226	+2.5	21.6	0.01	-1.9
1.6	445,082	261	+1.0	22	0.01	-0.2

	Invested Capital			Service-Level	
	Mean	Std		Mean	Mean - γ
0.2	69,938	51	+1.6	82%	0
0.4	87,830	43	+0.8	82%	0
0.6	122,855	53	+2.0	82.5%	0.5
0.8	167,129	115	+2.4	84%	2
1.2	289,783	183	+2.3	83.5%	1.5
1.6	452,373	222	+0.2	86%	4

Δ% is percentage of deviation from theoretic value

The mean values of inventory and invested capital are about 2% higher than the expected values. The mean workload meets the constraint for normal demand, while for sporadic demand c > 0.5 the actual mean workload tends to be lower than the constraint. The service-level γ is as expected if the demand is normal. For sporadic demand the actual service-level is higher than the theoretical value. Tables 5 and 6 give the service-levels in the 5 groups for c = 0.2 and c = 1.6, respectively.

Table 5: Service-level γ for c = 0.2

Group N	Required γ%	Mean of Simulation $\hat{\gamma}$%
1	65.6	65.8
2	73.8	73.8
3	82.0	82.0
4	90.2	90.2
5	98.4	98.4

Table 6: Service-level γ for c = 1.6

Group N	Required γ%	Mean of Simulation $\hat{\gamma}$%
1	65.6	73.4
2	73.8	80.4
3	82.0	86.8
4	90.2	92.9
5	98.4	98.7

The expected value as a measure of performance might not be satisfactory alone. In some periods the actual values fall below the restrictions; in other periods above. But if the variance is not too high this is acceptable. We therefore will present the standard deviation of the actual inventory on hand, invested capital and handling in a single period.

We would expect that as demand becomes more erratic the fluctuation of the mentioned variables increases and thus results in a higher standard deviation. But as we see from Table 7, while the standard deviation of demand increases from 0.2μ to 1.6μ, the standard deviation of the inventory on hand in a single period increases only from 0.1 × inventory on hand to 0.75 × inventory on hand.

Table 7: The effect of demand structure on the performance of storage room and invested capital.

	Storage Room Used in 1000 Units			Invested Capital			Workload		
$c = \sigma/\mu$	Mean	Std	Std/Mean	Mean	Std	Std/Mean	Mean	Std	Std/Mean
0.2	68	7	0.1	70	6	0.08	22	4.0	0.18
0.4	87	8	0.09	88	7	0.08	22	4.0	0.18
0.6	122	11	0.09	123	10	0.08	22	4.1	0.19
0.8	166	17	0.1	167	15	0.09	22	4.1	0.19
1.2	287	43	0.15	290	39	0.13	22	4.3	0.2
1.6	445	64	0.15	452	57	0.13	22	4.4	0.2

To obtain a more precise picture of the actual performance of the inventory system under consideration we studied the frequencies of inventory and handling. Table 8 gives the frequency of inventory on hand in a single period measured in terms of deviation from the theoretical expected value.

Table 8: Frequency of inventory on hand

σ/μ	80%	90%	100%	110%	120%	130%	140%	150%
0.2	0.04	0.21	0.39	0.28	0.07	0.01	0	0
0.4	0.04	0.23	0.42	0.26	0.05	0	0	0
0.6	0.02	0.18	0.47	0.29	0.04	0	0	0
0.8	0.07	0.14	0.51	0.31	0.03	0.0	0	0
1.2	0.003	0.14	0.62	0.27	0.006	0.002	0.002	0.01
1.6	0.001	0.18	0.71	0.09	0.004	0.002	0.002	0.01

The conclusions we draw from Table 8 are not what we expected. For c = 0.2, normal demand, in 39% of the periods the actual inventory is as predicted (100% of expected value). In 21% of the periods the inventory is 10% below the expected value, while in 28% of the periods the inventory is 10% above the expected value. As the variance of the demand increases, the concentration around the expected values becomes stronger. For c = 1.6, which is a very erratic demand, for 71% of the periods the actual inventory is as predicted. We notice that the modal value of the simulation corresponds with the theoretical expected value. The distribution of the inventory turns out to be skewed to the right. When demand becomes erratic it happens that in a very small number of cases the actual inventory is 50% above the expected value. The outliers cause the increasing variance.

Table 9 gives the frequencies of the orders per period. These frequencies are almost independent of demand structure given by the value c.

Table 9: Frequencies of orders per period

$c=\sigma/\mu$	60%	70%	80%	90%	100%	110%	120%	130%	140%	150%
0.2	0.02	0.06	0.12	0.17	0.28	0.16	0.11	0.05	0.02	0.01
0.4	0.02	0.06	0.12	0.16	0.28	0.15	0.11	0.06	0.03	0.01
0.6	0.03	0.06	0.12	0.17	0.28	0.15	0.10	0.05	0.03	0.01
0.8	0.03	0.06	0.12	0.17	0.28	0.15	0.10	0.05	0.03	0.01
1.2	0.03	0.06	0.12	0.17	0.28	0.15	0.10	0.05	0.03	0.01
1.6	0.03	0.06	0.11	0.16	0.28	0.16	0.11	0.06	0.02	0.01
Cumulative	0.03	0.09	0.20	0.36	0.64	0.80	0.91	0.97	0.99	1.00

The reason for the unexpected result, namely that fluctuation decreases as demand variation increases, is due to the correlation of the investigated variables. The following figures show the autocorrelation function of inventory and the number of orders for c = 0.2 and c = 1.6 respectively. We see that for normal demand we obtain cycles of high inventory and a high number of orders. This is due to the relatively high deterministic part of normal demand. It is obvious that during every 4th and 5th period a high number of orders arrives. If demand is sporadic there is no significant correlation between the number of orders in different periods. The inventory is highly autocorrelated as one would expect for sporadic demand since there is no demand in most periods. But we also notice that no cycles appear if demand is sporadic.

We might conclude from our simulation study that the performance of the inventory system was close to that predicted by the theoretical model. The modal values are the same as the expected values of the model. But since the distribution of the variables under consideration tend to be skewed to the right the average of actual inventory and invested capital is 2% higher than the expected value. The service-levels are as required. The variation of the variables are higher when demand has a high deterministic part, i.e. $c < 0.5$. In this case we notice the appearance of cycles in total inventory and orders. It seems that there has been very little attention given to this problem up till now in inventory literature.

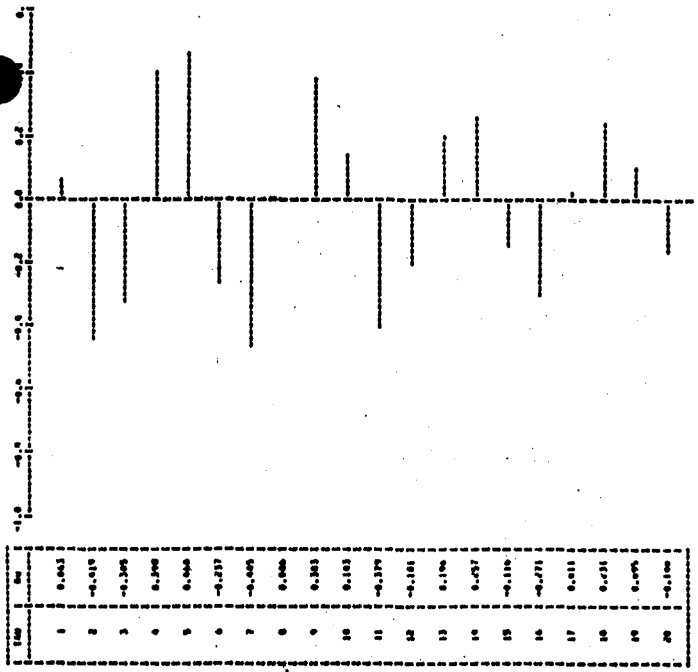


Fig. 9a Correlation coefficient RO of average inventory on hand. c = 0.2

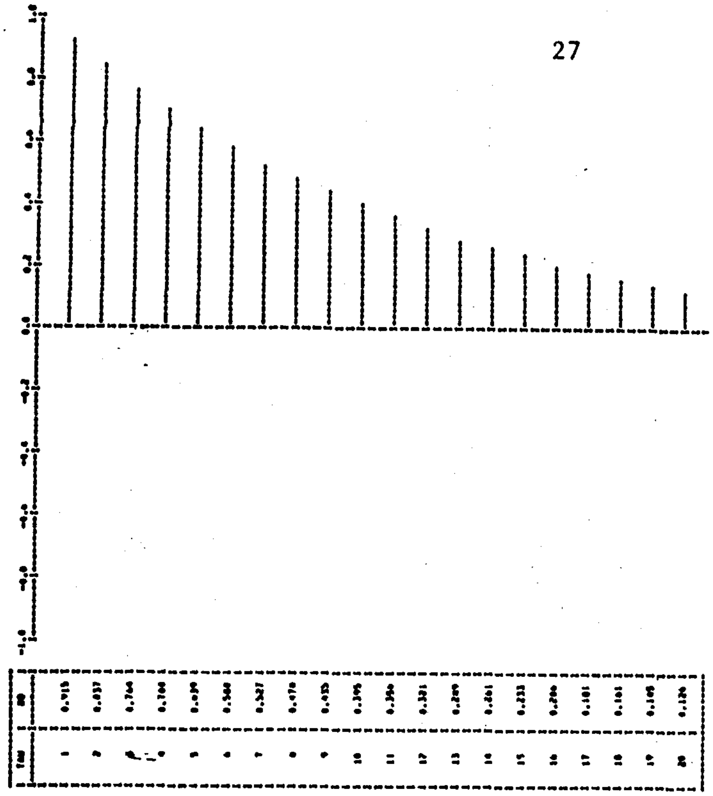


Fig. 9b Correlation coefficient RO of average inventory on hand. c = 1.6

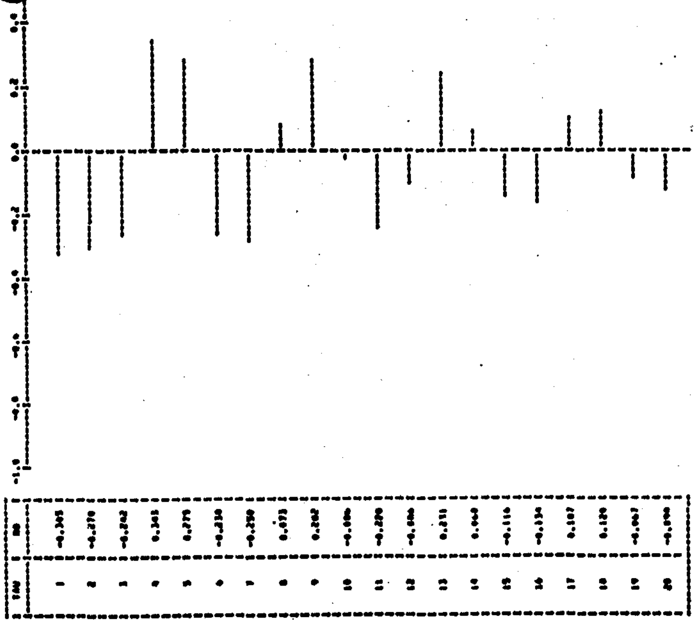


Fig. 9c Correlation coefficient RO of handling. c = 0.2

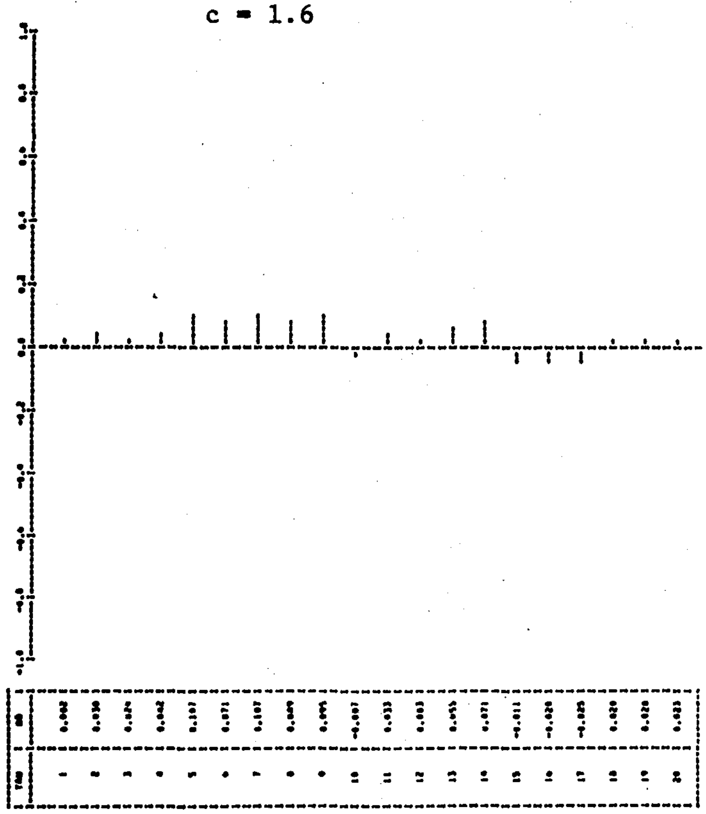


Fig. 9d Correlation coefficient RO of handling. c = 1.6

References

1. Alscher, T. and H. Schneider. Resolving a multi-item inventory problem with unknown costs. Engineering Costs and Production Economics, 6, 9-15 (1982).

2. Brown, R.G. Decision Rules for Inventory Management. Holt, New York, 1967.

3. Gardner, E.S. and D.G. Dannenbring. Using optimal policy surfaces to analyze aggregate inventory tradeoffs. Management Science, Vol. 25, No. 8 (August 1979).

4. Hadley, G. and T.M. Whitin. Analysis of Inventory Systems. Prentice-Hall, Englewood Cliffs, 1963.

5. IBM System 1360 inventory control (360 A-MF-04X). Program description manual GH20-555-1, New York.

6. Karlin, S. Steady state solutions. In: Studies in the Mathematical Theory of Inventory and Production (K.T. Arrow, S. Karlin and H. Scarf, Eds.) Chapter 14. Stanford University Press, Stanford.

7. Peterson, R. and Silver, E.A. Decision Systems for Inventory Management and Production Planning, New York: John Wiley, 1979.

8. Roberts, D.M. Approximations to optimal policies in dynamic inventory model. In: Studies in Applied Probability and Management Science (K.T. Arrow, S. Karlin and H. Scarf, Eds.) Stanford University Press, Stanford, 1962.

9. Schneider, H. Methods for determining the re-order point of an (s,S) ordering policy when a service-level is specified. J. Oper. Res. Soc. Vol. 29, 12, 1181-1193, (1978).

10. Schneider, H. "Effect of service-levels on order-points or order-levels in inventory models," Int. J. Prod. Res., Vol. 19, No. 6, 615-631 (1981).

11. Schrady, D.A. and Choe, V.E. "Models for Multi-Item Continuous Review Inventory Policies Subject to Constraints", Naval Res. Logist. Quart. Vol. 8, No. 4 (December 1977).

12. Smith, W.L. Asymptotic renewal theorems. Proc. R. Soc., Edinb-64, 9-48, (1954).

13. Starr, M.K. and Miller, D.W. Inventory control: Theory and practice, Prentice-Hall, Englewood Cliffs, N.J. (1962).

14. Wagner, H.M. and Hagan, M.O. and Lundh, B. An empirical study of exactly and approximately optimal inventory policies. Management Science, Vol 11, 690-723 (1965).

15. Ziegler, R.E. Criteria for measurement of the cost parameters of an economic order quantity inventory model, Unpublished Ph.D. Dissertation, University of North Carolina, 1973.

APPENDIX

We will show that the expected inventory on hand has the asymptotic value

$$E[I|s,S] \to D(1 - \frac{D}{2Q}) + s-\mu_{\lambda+1} + (1-\gamma)\mu \text{ as } D \to \infty \qquad \text{(A1)}$$

Note that we omit the index k for convenience.

Proof: Let

$$E[I|s,S] = \frac{\int_0^S (S-x)\phi(x)^{\lambda+1}dx + \int_0^D \int_0^{D+s} (D+s-y-x)\phi(x)^{\lambda+1}m(y)dxdy}{1+M(D)}$$

(i) It is easily seen that

$$\int_0^S (S-x)\phi(x)^{\lambda+1}dx \to D+s - \mu_{\lambda+1} \quad \text{as } D \to \infty$$

(ii) Note that (see Smith [12])

$$1+M(D) \to \frac{D}{\mu} + \frac{\mu_2}{2\mu^2} \quad \text{as } D \to \infty$$

where $\mu_2 = \mu^2 + \sigma^2$

(iii) First notice that

$$\int_0^D \int_0^{D+s-y} (D+s-y-x)\phi(x)^{\lambda+1}m(y)dxdy = \int_0^D (D+s-\mu_{\lambda+1}-y)m(y)dy - \int_0^D \int_{D+s-y}^{\infty} (D+s-y-x) \times \phi(x)^{\lambda+1}m(y)dxdy \qquad \text{(A2)}$$

then the first term at the right hand side of (A2) is

$$(D+s-\mu_{\lambda+1})M(D) - \int_0^D ym(y)dy \to (D+s-\mu_{\lambda+1})(\frac{D}{\mu} + \frac{\mu_2}{2\mu^2} - 1) + D(\frac{D}{\mu} + \frac{\mu_2}{2\mu^2} - 1) - \frac{D^2}{2\mu} + D(\frac{\mu_2}{2\mu^2} - 1) \quad \text{as } D \to \infty$$

The second term of (A2) is asymptotically given (see [9]) by

$$\int_0^D \int_{D+s-y}^{\infty} (D+s-y-x)\phi^{\lambda+1}(x)m(y)dxdy \to \frac{1}{2\mu} \int_s^{\infty}(x-s)^2\phi(x)dx \quad \text{as } D \to \infty$$

But $\frac{1}{2\mu} \int_s^{\infty}(x-s)^2\phi(x) = (1-\gamma)\mu \cdot [1+M(D)]$ and with (i), (ii) and (iii) we obtain (A1).