A NEW CHANGE-IN-RATIO PROCEDURE ROBUST TO UNEQUAL CATCHABILITY OF TYPES OF ANIMAL

K. H. Pollock[1], R. A. Lancia[2], M. C. Conner[3] and B. L. Wood[3]

Departments of Statistics[1], Forestry[2] and Zoology[3]
North Carolina State University
Raleigh, North Carolina 27695, U.S.A.

SUMMARY

The change-in-ratio technique is a useful practical procedure for the estimation of game animal population sizes. The major problem with this technique is failure of the assumption that both types of animals are captured or sighted with equal probabilities. Here we extend the change-in-ratio technique to the case where there are two removals with emphasis on the special situation where there are two consecutive single type removals. The advantage of this extension is that it allows an estimation procedure which is robust to unequal capture or sighting probabilities. It is also possible to test the assumption of equal sighting probabilities. Some numerical results on mean squared error of the population size estimator for the new design and the traditional design are given. The procedure is illustrated on some juvenile grass carp data collected in a small pond where the population size is known. We believe this technique is potentially useful to wildlife and fisheries biologists and that more statistical research would be beneficial.

Key Words: Change-in-ratio technique; Robustness; Unequal capture probabilities.

1. Introduction

The change-in-ratio technique (Paulik and Robson, 1969; Seber, 1982) is an attractive practical procedure for the estimation of fish and wildlife population sizes. It requires only pre and post removal "type" ratios (e.g., sex, size) and that the number of removals of each type be known. This is data which is often relatively easy for biologists to collect at a reasonable cost especially for hunted species.

The major problem with use of this technique is the assumption that both "types" are equally likely to be observed (or "captured") in the samples when the ratios are being estimated. For many species the probability of sighting different "types" can be very different. For example with deer it has been documented (Downing et al., 1977; McCullough, 1982; Sage et al., 1983) that antlered deer (adult males) have a much lower probability of being observed than antlerless deer (females and juveniles). This can cause serious bias in population size estimators.

In the special case of a single type removal (for example a bucks only hunt) then it is well known that the estimate of the number of x "types" (bucks) in the population is unbiased (Seber, 1982; p. 358). This is irregardless of whether antlered bucks and antlerless deer are equally sightable.

Here we exploit this idea to develop a new change-in-ratio experiment which is robust to unequal probability of capture. The approach is to have two separate single "type" removals on the opposite "types". There are samples to estimate the ratios at the beginning, between the two removals, and at the end of the study. This design requires the assumption that the bias in the ratio of the two "types" is consistent in all three samples which is much weaker than requiring that the bias is zero in all three samples.

In Section 2 we present a brief description of the traditional change-in-ratio procedure. In Section 3 we present the new two removal change-in-ratio procedure which has two single type removals which allows for unequal catchability. We also present a generalization to two multi "type" removals. In Section 4 we provide some numerical comparisons of the square root of the mean squared error for the various designs. This is followed by an illustrative example on a known population of juvenile grass carp in a small pond (Section 5) and then a general discussion section.

2. Description of the Change-in-Ratio Technique

2.1 Point Estimation

Here we present a brief description of the change-in-ratio technique. For more detail see Seber (1982; p. 353) or Paulik and Robson (1969). We consider a closed population with two types of animals. These are x-type and y-type animals; for example x-type may be antlered and y-type antlerless deer. If there is a differential change in the ratio of x-type to y-type between two times due to a known number of animals being removed (in the deer example by hunting) we can estimate the total population size before the removal (N_1) as follows

$$\hat{N}_1 = \frac{R_x - R\hat{P}_2}{\hat{P}_1 - \hat{P}_2} \tag{2.1}$$

where R_x = the number of x-type removed (known)

$R = R_x + R_y$ = the total number of animals removed (known),

$P_1 = X_1/N_1$ = the proportion of x-type animals before the removal,

$P_2 = X_2/N_2$ = the proportion of x-type animals after the removal.

Note that P_1 and P_2 need to be estimated by some sampling scheme as $\hat{P}_1$ and $\hat{P}_2$. The number of x-type animals in the population is estimated by

$$\hat{X}_1 = \hat{P}_1 \hat{N}_1 \tag{2.2}$$

Given independent point estimators of P_1 and P_2 the variances of our estimators are given by

$$\text{var}(\hat{N}_1) = \frac{N_1^2 \text{ var}(\hat{P}_1) + N_2^2 \text{ var}(\hat{P}_2)}{(P_1 - P_2)^2} \tag{2.3}$$

and

$$\text{var}(\hat{X}_1) = \frac{N_1^2 P_2^2 \text{ var}(\hat{P}_1) + N_2^2 P_1^2 \text{ var}(\hat{P}_2)}{(P_1 - P_2)^2} \quad . \tag{2.4}$$

We emphasize that these point estimators and their variance equations are valid for a range of probability models for the random variables $\hat{P}_1$ and $\hat{P}_2$.

2.2 Assumptions

The basic assumptions of the change-in-ratio method are:

(1) All animals have the same probability of being included in a particular sample both before and after the removal period.

(2) The population is closed except for the removals.

(3) The number of removals of x-type and y-type is known exactly.

(4) The proportion of x-type animals in the removal is different from that in the pre removal population.

For the deer example assumptions 2, 3 and 4 can be met reasonably if the hunting season is short, if crippling and illegal losses are small and/or can be estimated, and if hunters selectively harvest antlered deer (bucks). However, the estimated proportion of antlered deer in the population is frequently biased with the observed proportion of bucks in the population being less than the true proportion (Downing et al. 1977, McCullough 1982, Sage et al. 1983). If the proportion of antlered deer in the population is underestimated then the size of the population is overestimated. Further discussion of assumption (1) will be given later.

2.3 Probability Models for the "Type" Proportions

In Section 2.1 we presented point estimators for the population size (N_1) and the number of x-type animals (X_1) which depended on the availability of independent estimators of the proportion of x-type removals pre and post removal. Here we discuss several different probability models for these random variables.

2.3.1 Random Sampling with Replacement

If we suppose that a fixed random sample of n_i animals is taken with replacement at time t_i and x_i animals are found to be of x-type then the likelihood is a product of two independent binomial distributions.

$$f(x_1, x_2 | \{X_i, Y_i, n_i\}) = \prod_{i=1}^{2} \binom{n}{x_i} \left(\frac{X_i}{N_i}\right)^{x_i} \left(\frac{Y_i}{N_i}\right)^{y_i} \tag{2.5}$$

The maximum likelihood estimators of N_1 and X_1 are given by 2.1 and 2.2 respectively and the maximum likelihood estimator of p_i is given by

$$\hat{p}_i = \frac{\hat{X}_i}{\hat{N}_i} = \frac{x_i}{n_i}$$

with $\text{var}(\hat{p}_i) = \dfrac{p_i\ (1 - p_i)}{n_i}$

This approach is discussed by Seber (1982; p. 356) and applies at least approximately when animals are just observed and individuals could be counted twice.

2.3.2 Random Sampling without Replacement

If we suppose that a fixed random sample of n_i animals is taken without replacement at time t_i and x_i animals are found to be of x-type then the likelihood is a product of two independent hypergeometric distributions.

$$f(x_1, x_2 | \{X_i, Y_i, n_i\}) = \prod_{i=1}^{2} \frac{\binom{X_i}{x_i}\binom{N_i - X_i}{n_i - x_i}}{\binom{N_i}{n_i}} \tag{2.6}$$

The maximum likelihood estimators are the same as for sampling with replacement but now

$$\mathrm{var}(\hat{p}_i) = \frac{p_i(1 - p_i)}{n_i} \cdot \left(\frac{N_i - n_i}{N_i - 1}\right) .$$

which includes the finite population correction factor (Seber (1982; p. 356). Notice that this will be smaller so that the binomial variance is conservative.

2..3.3 Poisson Model (Equal "Sightability")

In practice sample size may not be fixed in which case we need to model the counts x_i and y_i. It is a characterization of the Poisson distribution that x_i conditional on $n_i = x_i + y_i$ will be binomial with parameters n_i and p_i if x_i and y_i are independent poisson random variables with the same mean (Johnson and Kotz 1969; p. 93). Therefore if x_i and y_i are Poisson then the now conditional likelihood 2.5 results. Seber (1982; p. 356) also discusses this model very briefly.

This binomial likelihood will only be an approximation for any other distributions for x_i and y_i with the same means. In terms of point estimation there will not be a problem but in terms of variances the binomial variance will only be an approximation. Alternatively one could obtain a variance for the p_i's based on interpenetrating subsamples similar to the approach of Cochran (1963; p. 383). See also Seber (1982; p. 117).

2.3.4 Poisson Model (Unequal "Sightability)

Now suppose (following Seber (1982; p. 358) that the ratio of the probability of sampling y-type animals to the probability of sampling x-type animals is given by λ and $d = 1 - \lambda$ and that the distributions of x_i and y_i are again Poisson. It follows that the conditional distribution of x_i given $n_i = x_i + y_i$ is binomial with parameters n_i and $P_{i\lambda} = X_i/(X_i + \lambda Y_i)$. This is just a generalization of the result presented in the previous section.

It is again important to emphasize that $\hat{p}_i = x_i/(x_i + y_i)$ will have asymptotic expectation $p_{i\lambda} = X_i/(X_i + \lambda Y_i)$ much more generally but that the exact distribution will only be binomial if x_i and y_i are Poisson random variables. The large sample expectations of $\hat{N}_1$ and $\hat{X}_1$ are given by

$$E\,(\hat{N}_1) \simeq (X_1 + \lambda Y_1)\left[\frac{N_1R_x - X_1R - dY_1R_x + dR_xR_y}{N_1R_x - X_1R - dY_1R_x + dX_1R_y}\right] \tag{2.7}$$

$$E\,(\hat{X}_1) \simeq X_1\left(\frac{N_1R_x - X_1R - dY_1R_x + dR_xR_y}{N_1R_x - X_1R - dY_1R_x + dX_1R_y}\right) \tag{2.8}$$

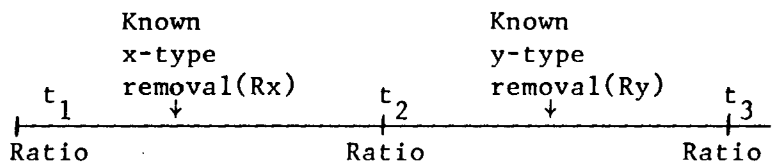
It is interesting to note, that $E\,(\hat{X}_1) = X_1$, i.e, that the number of x-type animals can still be estimated without bias if $R_y = 0$ (x-type only removed). This will be very important to the design developed in the next section.

3. The Two Removals Procedures

3.1 Two Single Type Removals

In this section we extend some of the ideas in the previous section to develop a new change-in-ratio experiment which is robust to unequal probability of capture of the two types of animals. The approach is to have two separate single type removals on the opposite types. There are samples to estimate the ratios at the beginning, between the two removals and at the end of the study. Following the previous section (2.3.4) we denote λ as the relative probability of sampling y-type to x-type and assume that the catches are either Poisson distributed, or if they are not that we can still assume that the large sample expectations of the $\hat{p}_i$'s are still valid.

This procedure can best be illustrated diagrammatically



$$P_{1\lambda} = \frac{X_1}{X_1+\lambda Y_1} \qquad P_{2\lambda} = \frac{X_1 - R_x}{X_1-R_x+\lambda Y_1} \qquad P_{3\lambda} = \frac{X_1 - R_x}{X_1-R_x+\lambda(Y_1-R_y)}$$

Given that we have independent asymptotically unbiased estimates of $P_{1\lambda}$, $P_{2\lambda}$ and $P_{3\lambda}$ and R_x and R_y are assumed known we have three equations in three unknowns which can easily be solved to give

$$\hat{X}_1 = \frac{R_x\hat{P}_1(1-\hat{P}_2)}{(\hat{P}_1-\hat{P}_2)} \tag{3.1}$$

$$\hat{Y}_1 = \frac{R_y\hat{P}_3(1-\hat{P}_2)}{(\hat{P}_3-\hat{P}_2)} \tag{3.2}$$

$$\hat{\lambda} = \frac{R_x(1-\hat{P}_1)(\hat{P}_3-\hat{P}_2)}{R_y\hat{P}_3(\hat{P}_1-\hat{P}_2)} \tag{3.3}$$

The variance and covariance expressions based on Taylor Series arguments are

$$\mathrm{var}(\hat{X}_1) = \frac{(X_1+\lambda Y_1)^2 P_2^2\,\mathrm{var}(\hat{P}_1) + (X_1-R_x+\lambda Y_1)^2\,P_1^2\,\mathrm{var}(\hat{P}_2)}{(P_1-P_2)^2} \tag{3.4}$$

$$\mathrm{var}(\hat{Y}_1) = \frac{(X_1 + R_x + \lambda Y_1)^2 (1-P_3)^2 \mathrm{var}(\hat{P}_2) + [X_1 - R_x + \lambda(Y_1 - R_y)]^2 (1-P_2)^2 \mathrm{var}(\hat{P}_3)}{(P_2 - P_3)^2} \tag{3.5}$$

$$\mathrm{cov}(\hat{X}_1, \hat{Y}_1) = \frac{R_x R_y \quad P_1 P_3}{(P_1 - P_2)(P_3 - P_2)} \quad \mathrm{var}(\hat{P}_2) \tag{3.6}$$

$$\mathrm{var}(\hat{N}_1) = \mathrm{var}(\hat{X}_1) + \mathrm{var}(\hat{Y}_1) + 2\ \mathrm{cov}(\hat{X}_1, \hat{Y}_1) \tag{3.7}$$

$$\mathrm{var}(\hat{\lambda}) = \left(\frac{R_x}{R_y}\right)^2 \left[\mathrm{var}(\hat{P}_1) \left\{ \frac{(P_3 - P_2)^2 (1-P_2)^2}{P_3^{\,2}\ (P_1 - P_2)^4} \right\} + \mathrm{var}(\hat{P}_2) \left\{ \frac{(1-P_1)^2 (P_3 - P_1)^2}{P_3^{\,2} (P_1 - P_2)^4} \right\} + \mathrm{var}(\hat{P}_3) \left\{ \frac{(1-P_1)^2\ P_2^{\,2}}{(P_1 - P_2)^2 P_3^{\,4}} \right\} \right] \tag{3.8}$$

To use these variance equations in practice either the binomial variances will have to be used as an approximation or the method of interpenetrating subsamples described in Section 2.3.3.

The assumptions of this modified change-in-ratio procedure are:

(1) The relative probabilities of capture of y-type to x-type animals (λ) is constant from sample to sample.

(2) The population is closed except for the removals.

(3) The number of removals of x-type and y-type animals is known exactly.

Assumptions (2) and (3) were required in the traditional method and were discussed in Section 2. Assumption 1 may not always be valid in practice but is a weaker assumption than assuming that both types of animals have equal capture probabilities (λ = 1) which was assumed by the traditional method.

A large sample test for equal capture probabilities (H_0 $\lambda = 1$ versus H_1 $\lambda \neq 1$ would involve using a normal test statistic $Z = (\hat{\lambda} - 1)/\sqrt{\widehat{\text{var}}(\hat{\lambda})}$ based on equations 3.3 and 3.8.

3.2 Multi-Type Removals

We wish to emphasize that an important practical generalization to the approach in Section 3.1 is to allow for both types of removals at each stage. We do not wish to go into detail but note that it is still possible to estimate X_1, Y_1 and λ from the following set of three equations. However, now these equations cannot be solved explicitly and an iterative approach will be needed.

$$\hat{P}_1 = \frac{X_1}{X_1 + \lambda Y_1}$$

$$\hat{P}_2 = \frac{X_1 - R_{x1}}{X_1 - R_{x1} + \lambda(Y_1 - R_{y1})}$$

$$\hat{P}_3 = \frac{X_1 - R_{x1} - R_{x2}}{X_1 - R_{x1} - R_{x2} + \lambda(Y_1 - R_{y1} - R_{y2})}$$

4. Numerical Considerations

4.1 Comparison of Square Root Mean Squared Errors

Here we compare the square root of the mean squared error of the estimators of N_1 for the traditional design and the new two-stage design for a constant amount of sampling effort. For the new design the large sample mean squared error of N_1 is just the variance of N_1 (Equation 3.7) as there is no large sample bias. For the traditional design the mean squared error is the sum of the variance (Equation 2.3) and the square of the large sample bias. The large sample bias is derived from Equation 2.7. For simplicity we considered the proportions to be from a binomial distribution.

In Table 1 we consider a population of N_1 = 1000 with X_1 = 100 and Y_1 = 900 over a range of values of R_x, R_y and λ. In Table 2 we again consider a population of N_1 = 1000 over the same range of R_x, R_y, and λ but now with X_1 = 500 and Y = 500. In both cases we considered total sightings to be 1500 animals. For the traditional design we used two samples of 750 each while for the new two-stage design we used three samples of 500 each.

(Tables 1 and 2 to appear here)

We now summarize some general results based on the numerical work:

(i) The traditional change-in-ratio procedure does not work unless the ratio changes. Notice that for this design the square root of the mean squared error decreases as the ratio changes by a greater amount between the two samples.

(ii) The new two-stage design does not work unless there are both kinds of removals, R_x at the first stage, R_y at the second stage.

(iii) For a fixed proportion of X "type" removals the square root of the mean squared error is monotone decreasing with increase in the proportion of Y "type" removals for the new design.

(iv) For the new design there is no need for the ratio of X "type" to Y "type" to change in the population from time 1 to time 3 (however it will obviously change at time 2 because R_x removals are between times 1 and 2, R_y removals between times 2 and 3).

(v) When there is equal catchability (or sightability) of both types of animals ($\lambda = 1$) the traditional design performs at its best. However, even then it may not beat the new design if the ratio of X "type" to Y "type" does not change much.

(vi) When there is unequal catchability ($\lambda \neq 1$) the traditional design sometimes beats the new design when the proportion of Y "type" removals is small and the change in ratio is large.

(vii) For both designs the square root of the mean squared error is smaller for the case where $X_1 = 500$ and $Y_1 = 500$ given the same relative proportions of X and Y type removals.

4.2 Efficiency of the New Design

As there is no large sample bias for the new estimator the values given in Tables 1 and 2 are approximate standard errors based on equations (3.4) - (3.7). Examination of the equations and the values in the tables shows that these standard errors are dependent on:

(1) The degree of change in the proportions between the three sampling times ($P_1 - P_2$, $P_2 - P_3$). The larger these quantities the smaller the standard errors will be.

(2) The size of λ with smaller values giving the smaller standard errors.

(3) The sample sizes n_1, n_2 and n_3 are also important. If we assume $n_1 = n_2 = n_3 = n$ for simplicity and consider Equations (3.4) - (3.7) we note that the effect of sample size on our standard errors is the usual multiple $1/\sqrt{n}$. Therefore the results in Tables 1 and 2 can be multiplied by the appropriate factor to give approximate standard errors for other sample sizes. (If the sample sizes were increased by a factor of 4 the standard errors would be decreased by a factor of 2.)

If we consider a proportional standard error of 20% as reasonable in wildlife experiments this gives actual standard errors of 200 as reasonable in our Tables 1 and 2. In this perspective removals of X and Y need to be substantial unless λ is small and sample sizes large.

5. Example

Brian Wood decided to carry out a field test of the two-stage removal method on a population of juvenile grass carp in a small pond. The artificial pond was 90 feet in diameter and 3 feet deep and could be drained. He divided the fish into two size classes with x-type being greater than 5" and y-type being less than 5". A forty-foot bag seine was used to catch the fish at each of the three times to estimate the ratios. Each sample was a mixture of sampling with and without replacement. Between the first and second samples 274 (R_x) x-type fish were "removed" by clipping their dorsal fin. Between the second and third samples 159 (R_y) y-type fish were "removed". The data can best be summarized in the following diagram.

$$
\begin{array}{ccccc}
 & R_x = 274 & & R_y = 159 & \\
t_1 & \text{———————} & t_2 & \text{———————} & t_3 \\
\hat{P}_1 = 0.3821 & & \hat{P}_2 = 0.3142 & & \hat{P}_3 = 0.3914 \\
(n_1 = 2586) & & (n_2 = 2018) & & (n_3 = 912)
\end{array}
$$

In Table 3 we present the results of estimation using the equations given in Section 3. We emphasize that the binomial variance was used as an approximation here. We also present the true values obtained after the pond was drained. Notice that the total population estimate of 1609 is very close to the true value of 1689 but that this is somewhat fortuitous as the individual components $\hat{X}_1, \hat{Y}_1$ are not nearly as close. On the other hand the traditional removal estimate just based on the first two samples assuming equal catchability of the two sizes is 2768 which is much worse. Also if we consider the traditional method just based on time 1 and time 3 with $R_x = 274$ and $R_y = 159$ there is such a small change in the ratio 0.3821 to 0.3914 that the method basically fails because we get an estimate of 11,141.

(Table 3 to appear here)

The precision of our estimates is not very good especially on $\hat{Y}_1$. We really needed to have larger changes in the ratios by "removing" more fish.

We also decided to test if the two groups of fish were equally catchable. We estimated $\lambda = 3.10$ with standard error 0.71 using equations (3.3) and (3.8). The Z test gives a value of $(3.10-1)/0.71 = 2.95$ which is clearly highly significant. This gives very strong evidence that the smaller fish are much more catchable than the larger fish. Our two-stage removal procedure is therefore justified over the traditional procedure which assumes equal catchability ($\lambda = 1$).

6. Discussion

Examination of our numerical comparisons and the fish example lead us to the conclusion that the new two-stage removal procedure can be more efficient than the traditional procedure. Obviously this is true if there is unequal catchability (or sightability) of both "types" ($\lambda \neq 1$) which violates an assumption of the traditional design. We also note that even if there is equal catchability ($\lambda = 1$) the new design may still do better if the change-in-ratio between time 1 and 3 is small or non existent.

We believe that this new procedure may be potentially useful to wildlife and fisheries biologists. A small albeit rather artificial fisheries example was presented earlier. Another example would be to apply this procedure to a deer population where the removals are obtained by hunting. In many states there are altlered only deer hunts (R_x) followed by antlerless hunts (R_y). The wildlife biologist would need to obtain estimates of the proportion of antlerless deer before the antlered hunt (time 1); between the antlered and antlerless hunt (time 2); and then finally at the end of the antlerless hunt (time 3). This approach would allow for the fact that for deer there is a large difference in sightability between antlered and antlerless deer as we discussed earlier.

We wish to emphasize that modelling counts in the change-in-ratio procedure needs further attention for both the new and the traditional designs. In the literature we have found the poisson assumption used purely for convenience because it results in an exact binomial likelihood. In many applications the poisson assumption will not be reasonable. We wish to emphasize that while our point estimates are robust to this assumption being violated binomial variance expressions may not be. We suggest the sensible approach is to consider the use of subsamples as we discussed in Section 2.3.3.

Finally we emphasize that this paper is only a beginning to the research in this area. We suggest future research could concentrate on generalization to more than two stages of removal with both types being removed at each stage. It may also be possible to generalize the results here to more than two types of animals (see Otis (1980) for such an extension which assumes equal sightability). There is also the need for a large scale simulation study to evaluate the asymptotic variances used. Comparison of efficiencies of estimators based on capture-recapture versus change-in-ratio would also be fruitful.

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Table 1. Comparison of square root mean squared error of $\hat{N}_1$ for the traditional and new designs when $X_1 = 100$, $Y_1 = 900$ for a total sample size of 1500 ($n_1 = n_2 = 750$ or $n_1 = n_2 = n_3 = 500$) over a range of values of R_x, R_y, and λ.

		R_x/X_1					
		0.1		0.5		0.9	
λ	R_y/Y_1	Trad	New	Trad	New	Trad	New
0.5	0	836.1	-	464.7	-	452.9	-
	0.1	-	692.2	469.5	836.8	453.0	1713.9
	0.3	529.6	226.1	502.0	208.5	453.3	421.5
	0.5	464.1	165.5	-	86.8	454.1	170.3
	0.7	453.6	152.7	467.5	39.0	456.8	68.3
	0.9	450.7	150.1	451.3	19.8	-	16.6
1.0	0	1661.0	-	283.2	-	129.1	-
	0.1	-	1755.9	325.7	2242.5	131.5	4792.4
	0.3	648.6	474.3	536.9	554.0	138.7	1176.9
	0.5	260.5	262.4	-	226.0	153.1	474.6
	0.7	127.4	206.8	296.8	94.0	195.6	189.8
	0.9	52.8	194.7	77.4	33.0	-	45.7
2.0	0	4354.2	-	1165.8	-	963.8	-
	0.1	-	4708.4	1238.4	6167.1	966.1	13476.5
	0.3	1879.6	1188.8	1660.7	1517.8	973.1	3307.2
	0.5	1114.7	538.0	-	614.5	987.8	1332.5
	0.7	954.5	322.7	1177.3	248.5	1038.2	532.3
	0.9	909.1	265.2	920.6	68.3	-	127.3

Table 2. Comparison of square root mean squared error of $\hat{N}_1$ for the traditional and new designs when X_1 = 500, Y_1 = 500 for a total sample size of 1500 ($n_1 = n_2 = 750$ or $n_1 = n_2 = n_3 = 500$) over a range of values of R_x, R_y, and λ .

		R_x/X_1					
		0.1		0.5		0.9	
λ	R_y/Y_1	Trad	New	Trad	New	Trad	New
0.5	0	788.1	-	267.4	-	251.5	-
	0.1	-	668.2	274.2	290.5	251.6	375.3
	0.3	409.1	602.7	320.5	93.7	251.9	93.3
	0.5	288.4	598.4	-	65.7	252.6	37.2
	0.7	263.3	597.6	289.8	58.9	255.4	11.5
	0.9	255.0	597.4	256.5	57.2	-	-
1.0	0	956.3	-	134.2	-	46.8	-
	0.1	-	805.0	157.4	602.0	48.1	969.6
	0.3	401.4	587.8	272.5	164.9	51.8	240.5
	0.5	172.8	572.5	-	89.4	59.2	98.2
	0.7	94.6	569.6	180.5	67.5	80.9	40.0
	0.9	52.5	568.9	62.8	61.8	-	9.9
2.0	0	1626.9	-	553.4	-	508.9	-
	0.1	-	1362.7	570.9	1403.5	509.3	2611.8
	0.3	804.4	683.8	683.7	359.6	510.6	644.4
	0.5	565.1	623.2	-	163.2	513.2	261.6
	0.7	518.5	611.8	574.9	94.4	522.8	105.9
	0.9	504.6	609.4	507.5	73.5	-	27.7

Table 3. Comparison of the two-stage removal estimates (standard errors) and the true counts obtained by draining the pond.

Type	Two-Stage-Estimate	True Values
X-type (X_1)	1057 (190)	604
Y-type (Y_1)	552 (352)	1085
Total Population (N_1)	1609 (400)	1689