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A REPEATED-MEASUREMENTS MODEL FOR OVER-STRESSED DEGRADATION DATA

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Abstract

A repeated-measurements model with a functional structure in variance-covariance is employed to analyze the degradation data collected from an accelerated life-test experiment. A transformation is utilized to permit the calculation of the generalized least-squares estimators and their covariance matrices by ordinary least-squares regression. The mean and percentile of the distribution of the time to failure at normal use condition are calibrated with a construction of their interval estimates. The techniques of the analysis of variance and the analysis of means are applied to compare several different types of materials. The model and estimation procedures are illustrated using life-test data from Takeda's and Stinebaugh's studies on the MOSFET's.

KEY WORDS: Accelerated life test; Degradation data; Repeated measurements model; Transformation; Variance function estimation; Calibration;

1. INTRODUCTION

Most of the statistical literature on the assessment of device reliability is based on failure-time observations. With the increasing emphasis on quality and reliability, life length of the device is getting longer. In experiments of device life-tests, few failures are being observed, even in accelerated failure-time studies. In some industrial applications, only measurements of progressive degradation of device can be obtained. Although the degradation mechanism of solid-state semiconductor devices is widely studied in engineering literature (see Howes and Morgan, 1981 for examples and references), there are not many papers in statistical journals that discuss the use of degradation measurements to improve product quality or reliability. The objectives of this article are to draw attention to this area and to discuss some of the statistical aspects. Specifically, we consider a repeated-measurements model to analyze over-stressed degradation data. The empirical model then is used to predict mean and percentile of the degradation distribution and to compare the lifetime of different types of components at normal use condition.

There are many different types of degradation data. For example, Bogdanoff and Kozin (1985) analyzed fatigue crack growth data sets. The crack lengthes of tension test specimens are measured at several successive time points. A degradation mechanism, known as electrical treeing, of epoxy electrical cable insulation specimens is reported by Stone (1978) (see Lawless 1982, p. 476). In this article, we consider the following degradation mechanism of semiconductors. When a semiconductor device is fabricated, it goes through a number of processes after which it is metallized, passivated, encapsulated, packed or any combination of these (c.f. Olson, 1977). The different interfaces in the device may change their characteristics through materials transfer by various means (c.f. Poate, *et. al.*, 1978) at the different stages mentioned above. The interfacial changes invariably alter the electrical and/or mechanical performances of the device and the device is said to have degraded. Hot-Carrier-Induced degradation is of a concern in the long-term reliability of very large scale integration (VLSI) device/circuit operations as the feature sizes of individual devices are scaled down continuously (c.f. Meyer and Fair, 1983; Takeda and Suzuki, 1983; Stinebaugh, *et. al.*, 1989). Once the surface of a transistor is inverted, carriers will be accelerated from the source to the drain by the applied drain voltage. Most of these carriers traverse the gate oxide and can be measured as gate current. A small percentage of these carriers, however, will be trapped in the gate oxide and cause measurable device-performance degradation (c.f. Chen, *et. al.*, 1985).

With technological advances leading to enhancement of product life, accelerated life tests (ALT) are assuming an ever increasing role in engineering experimentation. In an ALT experiment, a number of larger than normal stress settings x_j, $j = 1, \ldots, m_j$ are chosen. A sample of n_j units is subjected to the constant setting x_j, and their failure-times are observed. Due to possible difficulties in observing time to failure, we propose to use a supplementary sampling of device degradations (if it is possible). We monitor the deteriorating process of tested device over time to collect the degree of degradation. The stress levels can be set lower than the usual settings of ALT to collect degradation data before any failure of the device. The plausibility of the model can be checked near the operating condition and the long range extraplolation used in ALT might be avoided.

Device aging because of hot-carrier effects is usually measured in terms of threshold voltage shift, transconductance degradation, or loss of transistor gain (c.f. Stinebaugh *et. al.*). Chen *et. al.* presented an experimental model of the mean time to failure (when TVS reaches 10-mV) for nitride-metal-oxide-silicon (NMOS) devices fabricated with two different source-drain diffusions. Stinebaugh *et. al.* evaluated several different plasma-deposited silicon nitride films for use as an encapsulation material for polysilico-gate metal-oxide-silicon-field-effect transistors (MOSFET's). In both papers, the amount of device degradation over time was reported, which serves as the supplementary data in the ALT.

Section 2 is devoted to model the degradation data collected at several higher than normal use stress levels. A repeated-measurements model is proposed to analyze the ALT results. Generalized least squares unbiased estimators of the model parameters are obtained by using the algorithm for simple linear regression. Section 3 employs the techniques of analysis of variance and analysis of means (c.f. Nelson, 1983) to compare several different types of devices. In Section 4 we consider the model with a functional structure in the variance-covariance in order to accommodate the possible trend of increasing variances and/or covariances. Based on the data collected by Takeda *et. al.* (1983 and their recent study) and Stinebaugh *et. al.* (1989), several examples are provided in Section 5 to illustrate the use of the proposed model and procedures.

2. THE REPEATED-MEASUREMENTS MODEL

The fabricated MOSFET's were test devices in MOS VLSI electronic circuits in the studies of hot-carrier-induced degradation by Takeda and Suzuki, Chen *et. al.* and Stinebaugh *et. al.*. All devices were aged under several stresses such as drain voltage, temperature and gate voltage. The degradation of transconductance g_m (percent of $\Delta g_m/g_{m0}$) and the threshold voltage shift (ΔV_T) are monitored at periodic time intervals, where g_{m0} is the maximum transconductance of the tested device and $\Delta g_m = g_{m0} - g_m$. At a fixed stress level, the log-log plots of percent of $\Delta g_m/g_{m0}$ and ΔV_T

versus time (see Figure 1 in Section 5 or Figure 2 of Takeda and Suzuki, Figure 9 of Chen *et. al.* and Figure 4 of Stinebaugh *et. al.*) reveal that degradation of g_m and $\triangle V_T$ both have linear relationship with time, *i.e.*, $\log \triangle g_m/g_{m0}$ (or $\log \triangle V_T) = \log A + \gamma \log t + \text{error}$. Instead of using a deterministic model to describe this relationship (c.f. Takeda and Suzuki), we assume that random errors come from production noises. For example, in manufacturing an integrated circuit transistor, unavoidable variation in (radiation) mask dimensions cause transistor performance to vary. Coupled with this is the fact that the chemical and physical properties of the masked layers are subject to random variation which contribute also to transistor variations (c.f. Ilumoka and Spence, 1988).

The slope γ, is strongly dependent on the gate voltage V_G but has little dependence on drain voltage V_D. On the other hand, the magnitude of degradation A is strongly dependent on V_D but has little dependence on V_G. Based on the plots (see Figure 7 in Section 5 or Figure 3, 11 and 5 of Takeda and Suzuki, Chen *et. al.* and Stinebaugh *et. al.*, respectively) of the time to 10-percent g_m degradation (or the degradation factor A) versus inverse of the drain voltage $(1/V_D)$, we conclude that $\log A = \alpha + \beta(1/V_D) + \text{error}$, experimentally. Since the primary interest of stress is the drain voltage, we propose the following model to analyze this type of degradation data

$$y_{ijk} = \alpha + \beta(1/V_{Di}) + \gamma \log t_{ik} + u_{ijk}, \tag{2.1}$$
$$i = 1, \ldots, a; \; j = 1, \ldots, b; \; k = 1, \ldots, c,$$

and

$$u_{ijk} = \tau_{ij} + e_{ijk}, \tag{2.2}$$

where

y_{ijk} denotes the log percent of g_m degradation (or log threshold voltage shift) of the kth successive measurement of the jth individual (replication) at ith stress level;

V_{Di} denotes ith level of drain voltage;

t_{ik} denotes the time at which kth measurement of the ith stress level is observed;

u_{ijk} is the random error associated with y_{ijk}, which is assumed to be the sum of the random effect associated with jth sample individual at ith stress level (τ_{ij}) and the random effect associated with the kth measurement for the jth individual at the ith stress level (e_{ijk});

The random errors τ_{ij} and e_{ijk} are assumed independently distributed with zero means and variances σ_τ^2 and σ_e^2, respectively, where $\sigma_\tau^2 \geq 0$ and $\sigma_e^2 > 0$. The covariance structure for the random errors u_{ijk}, defined by (2.2), is thus expressed by

$$\begin{aligned} E(u_{ijk}\, u_{i'j'k'}) &= \sigma_\tau^2 + \sigma_e^2, && \text{if } i = i',\ j = j',\ k = k', \\ &= \sigma_\tau^2, && \text{if } i = i',\ j = j',\ k \neq k', \\ &= 0, && \text{if } i \neq i' \text{ or } j \neq j'. \end{aligned} \qquad (2.3)$$

Remark: For simplicity we take the number of replications, b, the number of time measurements, c, to be the same across all stress levels. And, we treat the covariances of the successive observations y_{ijk} measured at different time points t_{ik} to be the same regardless the distance of the time points. In Section 4 we relax this assumption and consider a variance-covariance funcion in model (2.1). One may also use a sophisticated time series model to handle the repeated measurements data. For example, Pantula and Pollock (1985) and Chi and Reinsel (1989) both use autoregressive models (with order 1) to describe the time-induced correlation of the repeated measurements.

We write the linear model (2.1) as

$$y_{ijk} = \sum_{\ell=1}^{p} x_{ijk\ell}\, \theta_\ell + u_{ijk},$$

where the parameters are $p = 3$, $\theta_1 = \alpha$, $\theta_2 = \beta$, $\theta_3 = \gamma$ and $x_{.1} = 1$, $x_{.2} = 1/V_{Di}$,

$x_{.3} = \log t_{ik}$. In the matrix notation

$$Y = X\,\theta + u,$$

where $\theta = (\theta_1, \theta_2, \theta_3)'$, $Y = (Y_1', \ldots, Y_a')'$, $Y_i' = (Y_{i1}', \ldots, Y_{ib}')'$, $Y_{ij}' = (Y_{ij1}, \ldots, Y_{ijc}')'$, and X and u are constructed similarly. Let V denote the variance-covariance matrix of u. Then V is a block diagonal of matrix $\{V_1, \ldots, V_a\}$, where V_i is a block diagonal matrix $\{V_{i1}, \ldots, V_{ib}\}$;

$$V_{ij} = \text{Block diagonal } \{\sigma_e^2\,\mathrm{I} + \sigma_T^2\,\mathrm{J}, \ldots, \sigma_e^2\,\mathrm{I} + \sigma_T^2\,\mathrm{J}\},$$

and I is a $c \times c$ identity matrix and J is a $c \times c$ matrix with all elements equal to one.

The variance component V depends on σ_e^2 and σ_T^2, and is a square matrix of order $N = abc$. When σ_e^2 and σ_T^2 are known, the generalized least squares (GLS) estimator of θ is given by

$$\tilde{\theta}(\sigma_e^2, \sigma_T^2) = (X'\,V^{-1}X)^{-1}\,X'\,V^{-1}\,Y. \tag{2.4}$$

The computation of the GLS estimator, using equation (2.4), involves inverting the matrix V, which typically is of large dimension. Fuller and Battese (1973) showed that the estimates $\tilde{\theta}(\sigma_e^2, \sigma_T^2)$ can be computed by the following simple procedure, which transforms the generalized least squares problem into an ordinary least squares problem. Let y_{ijk}^* be defined by

$$y_{ijk}^* = y_{ijk} - \delta\,\bar{y}_{ij.} = \sum_{\ell=1}^{p} (x_{ijk\ell} - \delta\bar{x}_{ij.\ell})\,\theta_\ell + u_{ijk}^*, \tag{2.5}$$

where

$$\delta = 1 - [\sigma_e^2/(\sigma_e^2 + c\sigma_T^2)]^{1/2}, \tag{2.6}$$

and $\bar{y}_{ij.}$, $\bar{x}_{ij.\ell}$, $\ell = 1, \ldots, p$, denote the averages of the c $y-$ and $x-$measurements on the jth individual (at ith stress level). The transformed errors, $u_{ijk}^* \equiv u_{ijk} - \delta\,\bar{u}_{ij.}$, where

$$\bar{u}_{ij.} = \sum_{k=1}^{c} u_{ijk}/c,$$

are uncorrelated and have variance σ_e^2.

When the variance components σ_e^2 and σ_T^2 are unknown, the value of the transformation factor δ defined in (2.6) must be estimated using some estimates of σ_ϵ^2 and σ_τ^2. There are different techniques available for the estimation of variance components [see Swallow and Monahan (1984) for a comparison of different estimation methods]. To estimate the variance components in our model, we use the procedure of Fuller and Battese, who used the well known "fitting-of-constants" method suggested by Henderson (1953) and discussed by Searle (1971). By regressing the y-deviations, $y_{ijk} - \bar{y}_{ij.}$, on the x-deviations, $x_{ijk} - \bar{x}_{ij.\ell}$, $\ell = 1, 2, \cdots, p$, that are not identically zero, we obtain an unbiased estimator for σ_ϵ^2

$$\hat{\sigma}_\epsilon^2 = \hat{\epsilon}'\hat{\epsilon} / (N - ab - 1), \tag{2.7}$$

where $\hat{\epsilon}'\hat{\epsilon}$ denotes the residual sum of squares obtained from the regression. The variance component σ_τ^2 may be estimated by

$$\tilde{\sigma}_\tau^2 = \frac{\hat{u}'\hat{u} - (N-p)\hat{\sigma}_\epsilon^2}{N - \text{tr}\,[(X'X)^{-1}\sum_{i=1}^a \sum_{j=1}^b c^2 \bar{x}_{ij.}' \bar{x}_{ij.}]}, \tag{2.8}$$

where $\hat{u}'\hat{u}$ denotes the residual sum of squares from the regression of Y on X, and $\bar{x}_{ij.}$ denotes the $(1 \times p)$ vector having the ℓth element $\bar{x}_{ij.\ell}$, $\ell = 1, 2, \ldots, p$. Then, the non-negative estimator of σ_τ^2 is

$$\hat{\sigma}_\tau^2 = \tilde{\sigma}_\tau^2 \qquad \text{if } \tilde{\sigma}_\tau^2 > 0,$$
$$= 0 \qquad \text{otherwise.}$$

The estimators (2.7) and (2.8) for σ_ϵ^2 and σ_τ^2 are unbiased and consistent. They can be substituted into (2.6) to obtain the transformation factor. The estimated generalized least-squares estimator (2.4), obtained by using the estimators $\hat{\sigma}_\epsilon^2$ and $\hat{\sigma}_\tau^2$, remains unbiased for θ in a wide range of situationsand and is consistent, asymptotically efficent (c.f. Fuller and Battese). A simple test of $\sigma_\tau^2 = 0$ is possible because $\hat{u}'\hat{u}$ and $\hat{\sigma}_\epsilon^2$ are independent. We have

$$\frac{\hat{u}'\hat{u}/\nu_1}{\hat{\sigma}_e^2} \sim F_{\nu_1,\nu_2}, \tag{2.9}$$

where $\nu_1 = N - p$ and $\nu_2 = N - ab - 1$, with F denoting Fisher's F distribution.

3. PREDICTIONS AND COMPARISONS OF TIME TO 10-PRECENT g_m DEGRADATION FOR DIFFERENT MATERIALS

One of the purposes of degradation studies is to predict the time (t_F) to certain percent degradation, say 10-percent of transconductance g_m degradation at normal use condition. If the device has degraded beyond this level, the usefulness of the product is questionable. Hence, one might say that the component has failed. For the ease of reference, we call t_F as "failure time". The mean time to failure at normal use condition is estimated by utilizing a calibration of regression model (2.1) (c.f. Martinelle, 1970)

$$\log \hat{t}_F = \hat{\gamma}^{-1}(\mu - \hat{\alpha} - \hat{\beta}\, V_0^{-1}) \text{ and } \hat{t}_F = \exp(\log \hat{t}_F),$$

where $\hat{\alpha}$, $\hat{\beta}$ and $\hat{\gamma}$ are the GLS estimators of the parameters in (2.1), $\mu = \mathrm{E}(\log Y)$ (e.g. 10% for g_m degradation) and V_0 is the normal use drain voltage. The notation $\log \hat{t}_F$ stands for the estimation of the nature logarithm of mean lifetime. An interval estimator ($\log \hat{t}_L$, $\log \hat{t}_U$), called "fiducial limits" (c.f. Williams, 1959), is obtained by reversing the $100(1-\alpha)\%$ confidence interval for the true mean value of Y. That is, we solve the followng two equations for the unknown limits $\log t_L$ and $\log t_U$

$$\mu = (\hat{\alpha} + \hat{\beta}\, V_0^{-1} + \hat{\gamma}^{-1}\log t_{UL}) \pm t_{N-p,\alpha/2}[(\hat{\sigma}_e^2 + \hat{\sigma}_\tau^2)\, x_0'(X'\, \hat{V}^{-1} X)^{-1} x_0]^{1/2},$$

where $t_{N-p,\alpha/2}$ is the $(1-\alpha/2)$th percentile of t distribution with $N-p$ degree of freedom, $x_0 = (1,\ 1/V_0,\ \log t_{\mathrm{UL}})'$ and $t_{\mathrm{UL}} = t_{\mathrm{U}}$ or t_{L} depending on the "+" or "−" sign in the equation. Note that $(X'\hat{V}^{-1}X)^{-1}$ can be easily obtained from the variance-covariance matrix of the estimation of θ in the transformed model (2.5). Next, we estimate $\mathrm{var}(\hat{t}_{\mathrm{F}})$ and $\mathrm{var}(\log\hat{t}_{\mathrm{F}})$ (if they exist) for latter use. An intuitive estimate of $\mathrm{var}(\log\hat{t}_{\mathrm{F}})$ is given by

$$\hat{\mathrm{var}}(\log\hat{t}_{\mathrm{F}}) = [(\log\hat{t}_{\mathrm{U}} - \log\hat{t}_{\mathrm{L}})/(2\,t_{N\text{-}p})]^2, \tag{3.1}$$

and $\hat{\mathrm{var}}(\hat{t}_{\mathrm{F}})$ will be constructed correspondingly with $\log\hat{t}$ replaced by $\hat{t}$. Alternatively, one might apply the Taylor expansion on the regression parameters,

$$\hat{t}_{\mathrm{F}} - t_{\mathrm{F}} \simeq -\,t_{\mathrm{F}}\gamma^{-1}\Big[(\hat{\alpha} - \alpha) + (\hat{\beta} - \beta)\ V_0^{-1} + \gamma^{-1}\eta(\hat{\gamma} - \gamma)\Big],$$

where $\eta = \mu - \alpha - \beta\,V_0^{-1}$. Then, another estimator of $\sigma_t^2 = \mathrm{var}(\hat{t}_{\mathrm{F}})$ can be obtained based on the following approximation.

$$\mathrm{var}(\hat{t}_{\mathrm{F}} - t_{\mathrm{F}}) \simeq \hat{t}_{\mathrm{F}}^2\,\gamma^{-2}\,[\mathrm{var}(\hat{\alpha}) + V_0^{-2}\,\mathrm{var}(\hat{\beta}) + \gamma^{-2}\eta^2\mathrm{var}(\hat{\gamma}) +$$

$$2\,V_0^{-1}\mathrm{cov}(\hat{\alpha},\,\hat{\beta}) + 2\gamma^{-1}\eta\,\mathrm{cov}(\hat{\alpha},\,\hat{\gamma}) + 2\,V_0^{-1}\gamma^{-1}\eta\,\mathrm{cov}(\hat{\beta},\,\hat{\gamma})]. \tag{3.2}$$

Similarly, an estimate of $\mathrm{var}(\log\hat{t}_{\mathrm{F}})$ can be obtained.

The large sample estimation of the pth-percentile and its $100(1-\alpha)\%$ confidence interval of the failure time are of the form $\hat{t}_p = \hat{t}_{\mathrm{F}} + z_p\,s_t$ and $\hat{t}_p \pm z_{\alpha/2}[p(1-p)/f^2(\hat{t}_p)]^{1/2}$ respectively, where s_t is the estimation of σ_t, the standard deviation of t_{F}, z_p and $z_{\alpha/2}$ are the pth and the $(1-\alpha/2)$th percentile of the standard normal distribution, respectively and f is the probability density function of

$N(0, \sigma_t^2)$.

Another goal of the degradation study is to compare different types of devices. For example, Stinebaugh *et. al.* (1989) study five different types of plasma-deposited silicon nitride materials. It is of interest to test whether the failure time (t_F) for different materials at the normal use condition are all the same. The repeated-measurements model can be used to test the hypothesis H_0: $E(\log t_{F1}) = E(\log t_{F2}) = \cdots = E(\log t_{Fm})$, where m is number of types of devices. Let us define $\xi_i = E(\log t_{F1} - \log t_{Fi+1})$, $i = 1, \ldots, m-1$ and $Q = \hat{\underset{\sim}{\xi}}'[\hat{V}(\hat{\underset{\sim}{\xi}})]^{-1}\hat{\underset{\sim}{\xi}}$, where $\hat{\underset{\sim}{\xi}} = (\hat{\xi}_1, \ldots, \hat{\xi}_{m-1})'$ with $\hat{\xi}_i = \log \hat{t}_{F1} - \log \hat{t}_{Fi+1}$ and $\hat{V}(\hat{\underset{\sim}{\xi}})$ is an estimate of the variance-covariance matrix of $\hat{\underset{\sim}{\xi}}$. We rewrite the hypothesis as H_0: $\xi_1 = \xi_2 = \cdots = \xi_{m-1} = 0$. Then, the distribution of the statistic Q may be approximated by a χ^2 with $m-1$ degree of freedom. Because the parameters of the model (2.1) tend to be different for different materials (c.f. Figure 4 and 5 of Stinebaugh *et. al.*), we fit the model (2.1) to these distinct materials separately. We have

$$\hat{\xi}_i = \log \hat{t}_{F1} - \log \hat{t}_{Fi+1},\ \ \mathrm{var}(\hat{\xi}_i) = \mathrm{var}(\log \hat{t}_{F1}) + \mathrm{var}(\log \hat{t}_{Fi+1}),\ i = 1, \ldots, m-1, \quad (3.3)$$

and

$$\mathrm{cov}(\hat{\xi}_i, \hat{\xi}_j) = \mathrm{var}(\log \hat{t}_{F1}),\ \ i, j = 1, \ldots, m-1;\ i \neq j. \quad (3.4)$$

Using (3.1), (3.3) and (3.4), we can compute the sample statistic Q. Similarly, it is possible to test the hypothesis that the parameters α, β and γ are the same for different materials.

It is often of interest to see whether or not one or more predictions differ from the average of all of the predictions, and if they are different which type of material is the most reliable (*i.e.* longest lifetime). In this context, the analysis of means (ANOM) developed by Ott (1967), Schilling (1973) and Nelson (1988) might be useful. Let us define the deviations of the prediction from the grand mean as

$$\hat{\lambda}_i = \log \hat{t}_{Fi} - \log \hat{\bar{t}}_F,\ i = 1, \ldots, m, \quad \text{where} \quad \log \hat{\bar{t}}_F = \sum_{i=1}^{m} \log \hat{t}_{Fi}/m.$$

Simple algebraic manipulation of the variances of $\log \hat{t}_{Fi}$ and $\log \hat{\bar{t}}_F$ leads to

$$\mathrm{var}(\hat{\lambda}_i) = (1 - 2/m)\,\mathrm{var}(\log \hat{t}_{Fi}) + \sum_{j=1}^{m} \mathrm{var}(\log \hat{t}_{Fj})/m^2,$$

where an approximation for $\mathrm{var}(\log \hat{t}_{Fi})$ is given in (3.1). Under the hypothesis H_0: $\mathrm{E}(\log t_{F1}) = \mathrm{E}(\log t_{F2}) = \cdots = \mathrm{E}(\log t_{Fm})$, $\hat{\underset{\sim}{\lambda}}$ has an asymptotic m-dimensional normal distribution with mean zero and variance-covariance matrix $\mathrm{var}(\hat{\underset{\sim}{\lambda}}\hat{\underset{\sim}{\lambda}}')$, where $\hat{\underset{\sim}{\lambda}} = (\hat{\lambda}_1, \ldots, \hat{\lambda}_m)'$. In our case, the number of observations for the estimation of a regression model (2.1) is $N = abc$, which is typically large enough (about 120 in the experiment of Stinebaugh *et. al.*) for this asymptotic result. The difference of $\mathrm{var}(\log \hat{t}_{Fi})$, $i = 1, \ldots, m$ implies the unequal variances of $\hat{\lambda}_i$.

To develop the ANOM procedure, we use the following inequality

$$\Pr(|\hat{\lambda}_1| \le cs_1, \ldots, |\hat{\lambda}_m| \le cs_m) \ge 1 - \sum_{i=1}^{m} \Pr(|\hat{\lambda}_i| > cs_i), \tag{3.5}$$

where s_i is the estimated standard deviation of $\hat{\lambda}_i$. Hence, wishing to find a confidence rectangle for $\log t_{Fi} - \log \bar{t}_F$, $i = 1, \ldots, m$ with the confidence level $1 - \alpha$, we may determine a constant c such that the right-hand side of (3.5) equals $1 - \alpha$. The desired confidence rectangle, or the decision limits for ANOM procedure, is then

$$\log \hat{\hat{t}}_{\mathrm{F}} \pm z_{\alpha/(2m)}\, s_i, \quad \text{where } s_i = \left[\hat{\mathrm{var}}(\hat{\lambda}_i)\right]^{1/2},$$

and $z_{\alpha/(2m)}$ is the $[1 - \alpha/(2m)]$th percentile of the standard normal distribution.

4. VARIANCE FUNCTION ESTIMATION

In previous sections, we assumed that the variances of the random variables y_{ijk} in model (2.1) are the same for all stress levels and successive time points. In some accelerated life test experiments, the variation of the tested devices increases when the stress level decreases to the normal use condition. Glaser (1984) employs a linear model of the standard deviation σ in terms of temperature and tensile stresses to describe the ALT results of various composite materials. In this section, we first consider a simple linear model of σ_ϵ^2 in terms of drain voltage V_{D} to handle the possible pattern of increasing variances. That is, we assume that

$$\sigma_\epsilon^2 = \zeta_1 + \zeta_2(1/V_{\mathrm{D}}),$$

and the covariance between successive measurements σ_τ^2 is independent of the stress.

The parameters in model (2.1) are estimated according to the following procedure. Initially, we assume that the variances are all equal and compute the initial estimates of the regerssion parameters θ using the estimation method described in Section 2. We then estimate $\sigma_{\epsilon i}^2$ from the residuals at each stress level. The covariance σ_τ^2 is estimated from a modification of (2.8)

$$\tilde{\sigma}_T^2 = \frac{\hat{u}'\hat{u} - (bc-p)\sum_{i=1}^{a}\hat{\sigma}_{ei}^2}{N - \text{tr}\left[(X'X)^{-1}\sum_{i=1}^{a}\sum_{j=1}^{b} c^2\,\bar{x}'_{ij.}\,\bar{x}_{ij.}\right]}.$$

The transformation used in Section 2 is then modified for the unequal variances as

$$\hat{\delta}_i = 1 - \left[\hat{\sigma}_{ei}^2/(\hat{\sigma}_{ei}^2 + c\hat{\sigma}_T^2)\right]^{1/2},\;\; i = 1, \ldots, a. \tag{4.1}$$

We note that the squared residuals $\{r_{ijk}^2\}$, from the estimation of transformed model (2.5), have approximate expectation σ_{ei}^2 and variance proportional to σ_{ei}^4 (c.f. Amemiya, 1977; Jobson and Fuller, 1980). This suggests the weighted least squares estimator that minimizes in ζ_1 and ζ_2,

$$\sum_{i=1}^{a}\left[\sum_{j=1}^{b}\sum_{k=1}^{c}\left\{r_{ijk}^2 - (\zeta_1 + \zeta_2/V_{\mathrm{D}i})\right\}^2\right] / \;\tilde{\sigma}_{ei}^4,$$

where $\tilde{\sigma}_{ei}^4$ is a perliminary estimator for σ_{ei}^4, for example, squared sample variance at stress $V_{\mathrm{D}i}$. Other methods of variance function estimation can be found in Davidian and Carroll (1987). After obtaining the estimators of ζ_1 and ζ_2, we obtain the GLS estimator (2.4) using the estimated σ_{ei}^2 to update the estimation of θ. And, repeat this procedure iteratively until it converges. When it converges, the estimate of θ would be equivalent to the one obtained by the method of maximum likelihood.

More general models, when σ_T^2 and σ_e^2 are both functions of V_{D} may be considered. For example, for the cable insulation data collected at voltage stress levels 52.5, 55.0 and 57.5 kilovolts in Stone (1978), the variances and covariance of the nature logarithm of the inception time and failure time are as follows: (.86326, .72433, .25602), (.79492, .71210, .20378), (.82803, .71541, .22759). One might use linear functions of voltage stress to model σ_e^2 and σ_T^2 of this bivariate degradation data. Estimation of the parameters of the models for σ_T^2 and σ_e^2 may be obtained using a generalized least squares procedure suggested in Chapter 4 of Fuller (1987). The least squares procedures consists of minimizing a weighted distance between the sample residual variance covariance matrix and its expected value.

It is possible that the variance of observations is a function of time. For example, the variance of the experimental results provided by Takeda increases as time increases (c.f. Figure 2). Lu and Meeker (1989) observe similar trend in their study on the data of fatigue crack length. A simple approach to handle the trend on variances is to utilize a variance function such as $\mathrm{Var}(y_k) = \sigma_k^2 = \zeta_1 + \zeta_2 \log_{10} t_k$, where $k = 1, \ldots, c$. To model the trend of covariance, one can plot the correlation coefficients versus the distances of time points (between the successive measurements of the same specimen) to see what kind of relationship might be suitable to describe the correlation over time-lags. For example, based on the data from Takeda (c.f. Figure 3), we consider the following function to model the correlations:

$$\mathrm{Corr}(y_i,\, y_j) = \rho_{ij} = \zeta^{t_i - t_j},\ \ i = 1, \ldots, c,\ \ j = 1, \ldots, c,\ \ i \neq j. \tag{4.2}$$

Then, the covariance between y_i and y_j is $\rho_{ij}\,\sigma_i\,\sigma_j$. With this variance-covariance function on model (2.1) the GLS estimators of regression coefficients can be obtained by using the iteration procedure similar to the one described above.

5. EXAMPLES

In this section we first give an example to demostrate the estimation of the regression coefficients and variance-covariance components in the repeated-measurements model (2.1) based on the data provided by Dr. Takeda. In the second example, we consider the experimental results of Dr. Stinebaugh to compare different types of materials by using the predicted time to 10% g_m degradation. A follow-up example illustrates the use of variance function to model the pattern of increasing variances.

Example 1. In a recent study of Dr. Takeda on the device degradation due to hot-carrier injection. The fabricated MOSFET's are test devices in n-channel MOS VLSI circuits and the results are plotted in Figure 1, where 35 successive measurements of the g_m degradation (in percent) are collected from 5 devices at drain voltage 3.8 volts and gate voltage 1.4 volts. Corresponding to model (2.1), we consider $\log_{10} y_{jk} = \alpha + \gamma \log_{10} t_{jk} + u_{jk}$, $j = 1, \ldots, 5$ for replications and $k = 1, \ldots, 35$ for successive measurements and the random error u_{jk} is as defined in (2.3). Using (2.7) and (2.8), we obtain estimates of the regression coefficients and variance components as follows: $\hat{\alpha} = -1.06390$ (with s.e. .059375), $\hat{\beta} = .464652$ (with s.e. .00039869), $\hat{\sigma}_e^2 = .0033336$ and $\hat{\sigma}_\tau^2 = .028468$ and estimated transformation factor $\hat{\delta} = .94226$. We then have $\hat{\text{var}}(\log Y_{jk}) = \hat{\sigma}_e^2 + \hat{\sigma}_\tau^2 = .031802$ and $\hat{\rho} = \hat{\sigma}_\tau^2 / (\hat{\sigma}_e^2 + \hat{\sigma}_\tau^2) = .89518$. We note that the estimate of the correlation coefficient ρ between the successive measurements is quite high. Since the observed value of the test statistic (2.9) is 7.911, we reject the hypothesis $\sigma_\tau^2 = 0$ at 1% level of significance.

Based on the plot (see Figure 2) of the variances of residuals at successive time points, we employ a linear function $\text{Var}(y_{jk}) = \sigma_k^2 = \zeta_1 + \zeta_2 \log_{10} t_{jk}$, where $j = 1, \ldots, 5$, $k = 1, \ldots, c$ to model the variances. Guilding from the plot (c.f. Figure 3) of correlation coefficients against time distances, we utilize the function (4.2) to obtain the correlations as well as covariances. The parameter $\zeta = \exp(\xi)$ in (4.2) is estimated as .88836. With these additional considerations in the variance and covariance of model (2.1), we obtain the GLS estimators of the model parameters by using the iteration procedure described in Section 4. Table 1 reports the final calculation results.

Example 2. We consider the experiment conducted by Stinebaugh *et. al.* (1989) in AT&T Laboratories. The drain voltages were set at 7.0, 7.25 and 7.5 volts. And, the normal use drain voltage is 5.5 volts. At each voltage, at least four devices in each category of transistors were tested. Several successive measurements were observed in order to monitor the degredation of the transistor caused by hot-carrier effects. The experiment is stopped when 11-percent decrease in g_m was measured. Other stresses, gate voltage and temperature, are set at 3.2 volts and 25°C, respectively, for the entire experiment. Because the experiments were done several years ago, only a small portion of the original data of replications is saved. However, the investigators encourage us to obtain the average of the replications from the data points plotted in the relevant figures of their paper and then simulate the replications from estimated variances and covariances.

Table 2 gives the average percentages of g_m degradation for five different materials at 7.5 volts drain voltage. Table 3 gives the failure time (time to 10 precent g_m degradation) for three different drain vlotages and all types of meterials. The numerical numbers are taken to match the plots in Figure 4 and 5 of Stinebaigh *et. al.*. Taking the number of replications $b = 4$ and assuming that the variances of all materials are the same, we use the average of degradation data to estimate the variance components σ_ϵ^2 and σ_τ^2 as $\tilde{\sigma}_{\epsilon 0}^2 = 0.0075840$ and $\tilde{\sigma}_{\tau 0}^2 = 0.00027587$ from (2.7) and (2.8), respectively. Because the slope γ of the model (2.1) does not change for different voltages, we compute the intercept $\log A_i = \alpha + \beta(1/V_{D_i})$ using the relationship $\log A_i = \log 10\% - \gamma \log t_{F_i}$, where t_{F_i} is the failure time reported in Table 3 and $\gamma = 0.483$, 0.509, 0.435, 0.719, 0.954 are slopes from the observations obtained at 7.5 volts drain voltage for materials 1 to 5 respectively. Finally, we simulate a data set from model (2.1) with the mean values given in Table 2 and variance and covariance $\tilde{\sigma}_{\epsilon 0}^2$ and $\tilde{\sigma}_{\tau 0}^2$. Figure 4 gives the plots of simulated data for materials a-SINF:H and a-SIN:H. The statistics of simulated degradation data are summarized in Table 4.

Fitting model (2.1) to the degradation data for five different materials separately, we estimate the parameters and summarize the results in Table 5. The prediction (and its variance) of time to 10% g_m degradation ("failure time") at normal use drain voltage is presented in Table 6. We notice that the material 1 and material 5 have the largest and smallest failure time predictions. Large sample test indicates that the expected failure time for material one is greater than that of meterial 5 (and material 3) at significance level 0.01.

To test the difference between these materials, we apply the ANOVA procedure to the hypothesis H_0: $\mathrm{E}(\log t_{F1}) = \mathrm{E}(\log t_{F2}) = \cdots = \mathrm{E}(\log t_{F5})$. The test statstic $Q = \hat{\underset{\sim}{\xi}}'[\hat{\mathrm{V}}(\hat{\underset{\sim}{\xi}})]^{-1}\hat{\underset{\sim}{\xi}}$ proposed in Section 3 is computed as 1685.749 based on the results given in Table 6. Comparing with the table value of χ^2_4, we conclude that there is a difference between the failure time of these five materials at the significance level 0.01. Next, we apply the ANOM procedure to test if one or more log failure time predictions differ from the average of all predictions. The result is shown in Figure 5. We conclude that all predictions of these materials are different from their average at significance levels 0.01.

Example 3. In this example we modify the Example 2 to the case of constant covariance and $\sigma^2_{ei} = \eta_1 + \eta_2/V_{Di}$ for model (2.1). We simulate a data set with seven drain voltage stress levels: 8.0, 7.75, 7.5, 7.25, 7.0, 6.75, 6.5 volts. The covariance is set as $\sigma^2_\tau = .00027585$, and the parameters of variance function are $\eta_1 = -.394$ and $\eta_2 = .315$. The sample values (the initial estimates) of the variance σ^2_{ei} are calculated from (2.7) as 0.000093, 0.001504, 0.003312, 0.004340, 0.005534, 0.007107 and 0.009801 for seven design levels 8.0 to 6.5 volts. Table 7 gives the final results of the parameter estimation based on the procedure developed in Section 4 for the material 1. In Figure 6 we plot the sample values of σ^2_{ei} along with the estimated variance function and its predicted value .01821 at 5.5 volts drain voltage. In Figure 7, we plot the prediction of mean failure time with its fiducial limits for materials 1, 3 and 5. The values of $\hat{\sigma}^2_\tau$ are .000263 and .000213 for materials 3 and 5. Because the small values of variances σ^2_{ei} and σ^2_τ chosen in data generations, we note that the standard deviations of these predictions are quite small.

If we ignore the trend of the increasing variances and fit the data based on equal variance assumption, the value of $\hat{\sigma}_\epsilon^2$ is 0.004506 which is quite different from 0.01821 estimated based on the model with the linear variance function. Although the estimation of the mean failure time, 21.4286 (compared to 21.4245) and the regression parameters, 14.4402, −123.45 and .481 (compared to 14.4414, −123.47, .481) are not too far from the ones obtained with constant variance function, their interval estimations as well as the predictions of precentile failure time are quite different. For example, the 95% fiducial limits of mean time to failure is (21.3411, 21.5166) compared to (21.2671, 21.6949), and the 10th percentile of failure time at normal use condition is 21.3244 compared to 21.2259 based on variance function.

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Table 1. Parameter Estimation for the Repeated-Measurements Model with Variance-Covariance Function

Iteration	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\eta}_1$	$\hat{\eta}_2$	$\hat{\xi}$
0	−1.06390	.46465			
1	−1.09556	.48973	−.015946	.013658	−.11838
2	−1.10358	.48985	−.007868	.011290	−.11838
3	−1.10383	.48987	−.007625	.011217	−.11838
4	−1.10383	.48987	−.007625	.011217	−.11838
S.E.	(.0039019)	(.00060468)	(.0060221)	(.0018904)	(.0051247)

Table 2. Average Percentages of g_m Degradation[a] Measured at Successive Time Points

Material[b]											
1	x[c]	14	44	120	220	580	950	1400	1775	2650	3300
	y	0.45	0.85	1.45	1.9	2.4	3.1	3.1	4.2	6.1	7.1
2	x	14	65	280	410	610	780	1000	1333	1600	1930
	y	.9	1.7	3	3.85	5.1	6	7.2	8.3	9.5	11
3	x	50	200	400	500	780	1250	1500	1800	2225	
	y	2	3	4	5	6	7	8	9	10	
4	x	14	40	60	100	200	300	400	500		
	y	.9	2	3	4	6	8	10	13.6		
5	x	14	30	40	50	60	70	80	90	200	
	y	.9	1.8	2.4	3.5	4	4.75	5	6.5	10	

[a] The drain voltage is set at 7.5 volts.

[b] Materials 1 to 5 are denoted as a–SINF:H, AMI Process D, a–SION:H, AMI PROCESS B and a–SIN:H in Stinebaugh *et. al.* (1989).

[c] y is the percent of g_m degradation and x is the successive time point (in minutes).

Table 3. Time (in minutes) to 10 Percent g_m Degradation

	Drain Voltages (volts)		
Material[a]	*7.5*	*7.25*	*7.0*
1	7864.8	26000	89000
2	2054.7	5250	8200
3	2700	6000	17860
4	360	900	1870
5	187	350	700

[a] See Footnote b of Table 1.

Table 4. Summary Statistics of Simulated Degradation Data

Material	$\log y$[a]	$\log y/V_D$	$\log y \cdot \log t$	$\log t$	$\log t/V_D$	$\log t \cdot \log t$
1	107.16	14.791	956.60	870.63	120.64	6805.13
2	179.96	24.837	1367.58	819.78	113.43	5907.89
3	184.47	25.462	1420.92	793.20	109.83	6039.71
4	142.42	19.661	895.159	538.79	74.62	3197.08
5	136.72	18.871	700.331	512.55	71.00	2524.79

[a] All these quantities are summations over subscripts i, j and k,
e.g. $\log y = \sum_{i=1}^{a} \sum_{j=1}^{b} \sum_{k=1}^{c} \log y_{ijk}$.

Table 5. Parameter Estimation for the Repeated-Measurements Model

Material	*Parameter*		$(X' V^{-1} X)^{-1}$			
1	α	14.568	17.455	-130.33	.07547	$\hat{\sigma}_e^2$ = .006074
	β	-124.52		980.32	−.68848	$\hat{\sigma}_T^2$ = .000277
	γ	.48433	symmetric		.00270	δ = .17145
2	α	8.628	57.196	-416.43	.04877	$\hat{\sigma}_e^2$ = .005250
	β	-77.234		3043.35	−.53793	$\hat{\sigma}_T^2$ = .002312
	γ	.51721	symmetric		.00373	δ = .56981
3	α	11.292	19.651	-149.83	.14222	σ_e^2 = .005598
	β	-93.615		1159.57	-1.39443	σ_T^2 = .000269
	γ	.45468	symmetric		.00684	δ = .16453
4	α	14.147	26.746	-199.04	.13346	σ_e^2 = .006362
	β	-121.20		1493.71	-1.27485	σ_T^2 = .000682
	γ	.72471	symmetric		.00758	δ = .26629
5	α	16.742	21.873	-168.54	.29607	σ_e^2 = .006209
	β	-144.61		1316.24	-2.7728	σ_T^2 = .000316
	γ	.94520	symmetric		.01827	δ = .17183

Table 6. Predictions of Time to 10% g_m Degradation

	Material				
	1	2	3	4	5
$\log \hat{t}_F$	21.421	14.921	17.665	14.064	12.500
$\log \hat{t}_{FL}$	21.026	14.263	17.245	13.725	12.324
$\log \hat{t}_{FU}$	21.818	15.579	18.088	14.404	12.757
$\log \hat{\mathrm{var}}\,(t_F)$[a]	39.699	27.802	32.316	24.663	20.690
$\log \hat{\mathrm{var}}\,(t_F)$[b]	39.645	27.659	32.255	24.623	20.673
$\hat{\mathrm{var}}\,(\log \hat{t}_F)$[a]	.04082	.11270	.04625	.03000	.01220

[a] computed from Eq. (3.1). [b] computed from Eq. (3.2).

Table 7. Parameter Estimation for the Repeated-Measurements Model with Variance Function

Iteration	$\log \hat{t}_F$ [a]	$\hat{\alpha}$ [b]	$\hat{\beta}$	$\hat{\gamma}$	$\hat{\sigma}^2_T$	$\hat{\eta}_1$ [c]	$\hat{\eta}_2$ [d]
1.	21.4273	14.448	-123.53	.481	.000109	–.04004	.3211
2.	21.4147	14.402	-123.16	.481	.000140	–.03978	.3191
3.	21.4303	14.460	-123.62	.482	.000158	–.03981	.3193
4.	21.4295	14.456	-123.59	.481	.000170	–.03976	.3188
5.	21.4193	14.420	-123.31	.481	.000176	–.03965	.3180
6.	21.4204	14.428	-123.37	.481	.000180	–.03965	.3180
⋮	⋮		⋮		⋮	⋮	⋮
11.	21.4182	14.419	-123.30	.481	.000184	–.03961	.3177
12.	21.4220	14.432	-123.40	.481	.000184	–.03963	.3179
13.	21.4336	14.473	-123.72	.482	.000185	–.03973	.3185
14.	21.4303	14.461	-123.63	.482	.000183	–.03971	.3184
15.	21.4242	14.439	-123.46	.481	.000183	–.03966	.3180
16.	21.4245	14.441	-123.48	.481	.000184	–.03966	.3180

[a] $\log \hat{t}_F$ is the estimate of the log time to 10% g_m degradation.

[b]

For iteration 23, $$(X' \hat{V}^{-1} X)^{-1} = \begin{bmatrix} 2.6723 & -20.6661 & .032718 \\ & 162.7805 & -.297876 \\ \text{symmetric} & & .001155 \end{bmatrix}.$$

[c] η_1 and η_2 are the parameters of the variance function $\sigma^2_{ei} = \eta_1 + \eta_2 / V_{D_i}$.

[d] The transformation factors δ_i, $i = 1, \ldots, 7$ are (.7762, .3452, .2259, .1659, .1289, .1043, 0864) which are computed from Eq. (4.1).

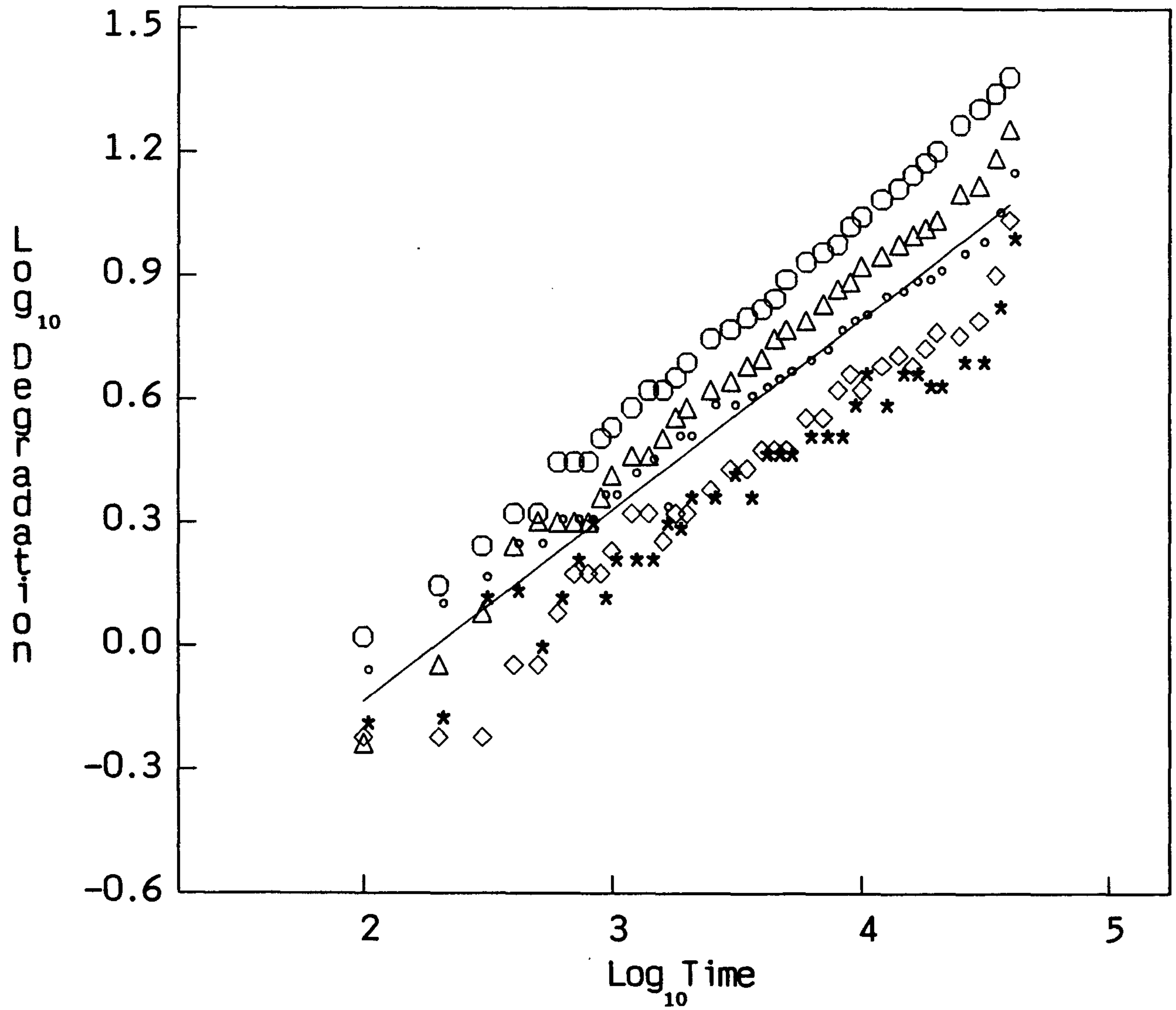

Figure 1. Plot of Percent $\Delta g_m/g_{mo}$ versus Time in Seconds (Data Set from Takeda).

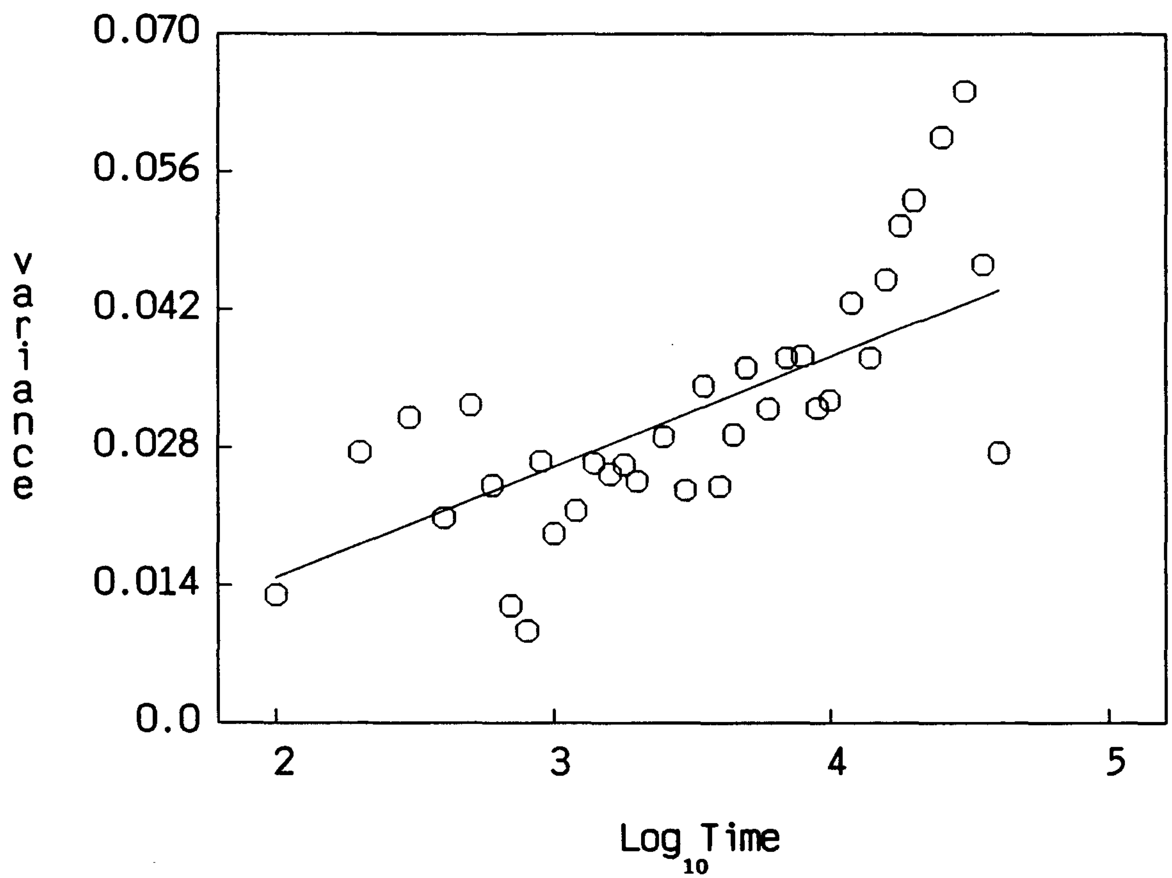

Figure 2. Variance Function Estimation for the Observations from Takeda.

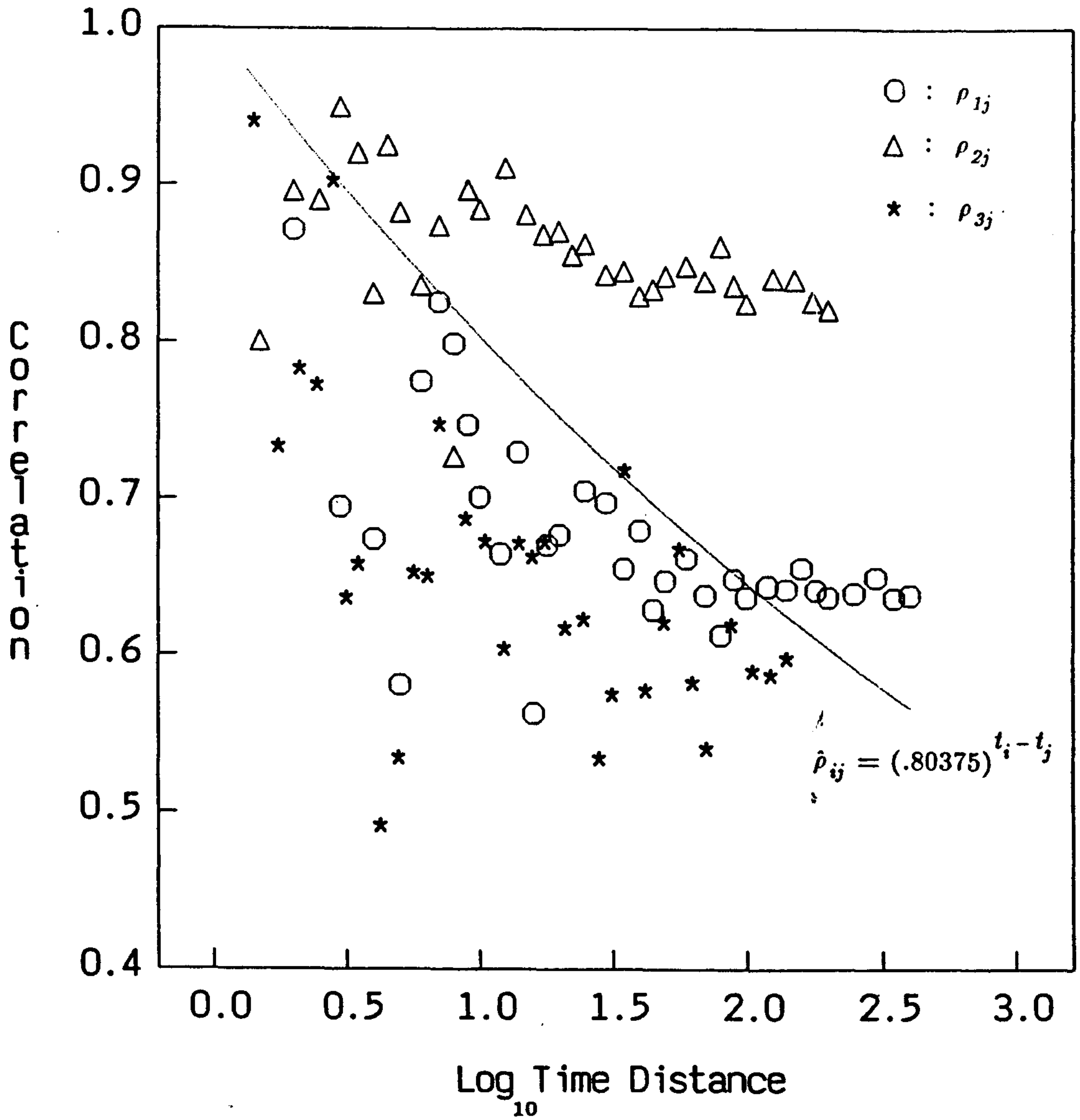

Figure 3. Correlation Estimation for the Observations from Takeda.

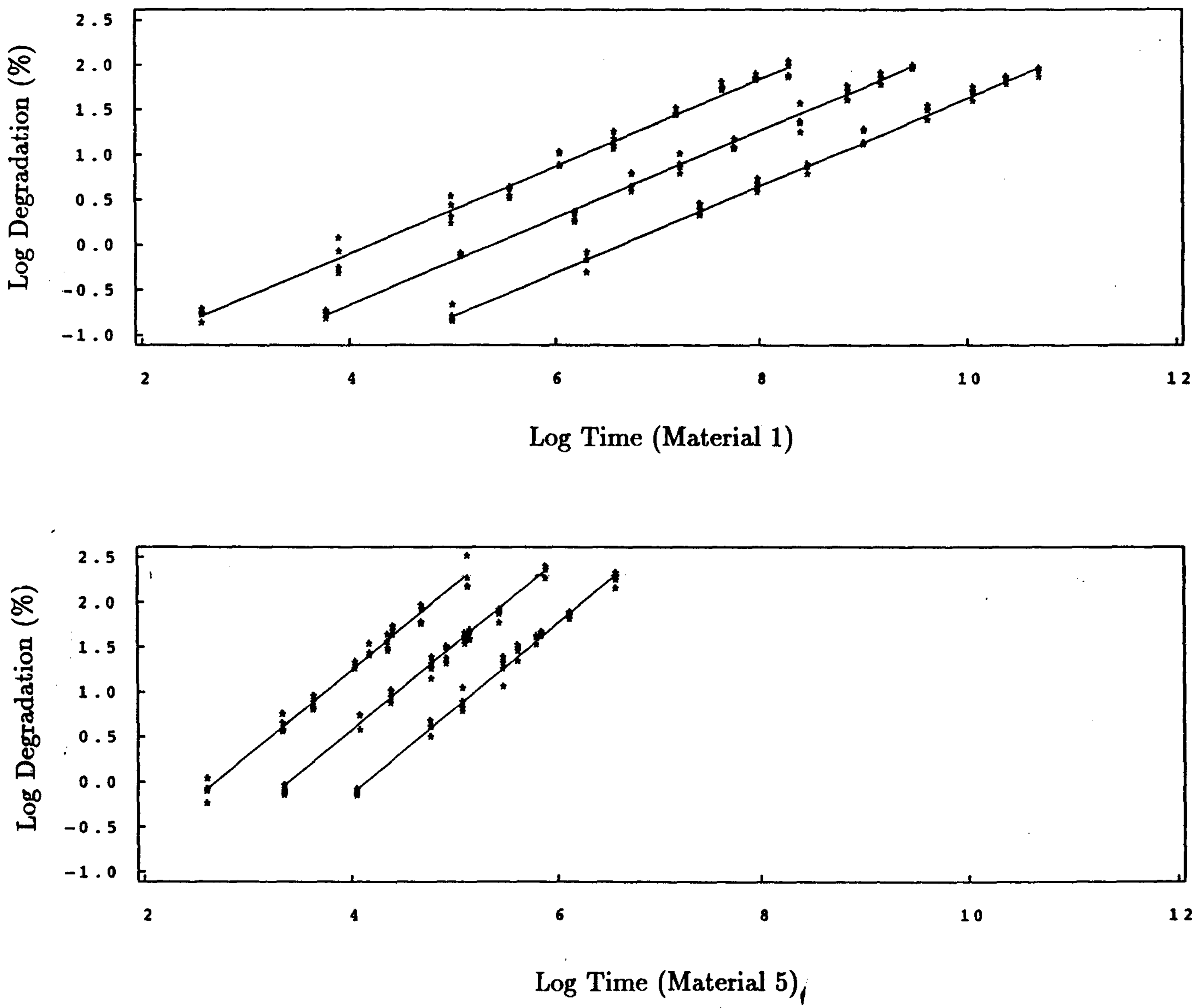

Figure 4. Plot of Percent $\Delta g_m/g_{mo}$ versus Time in Minutes.

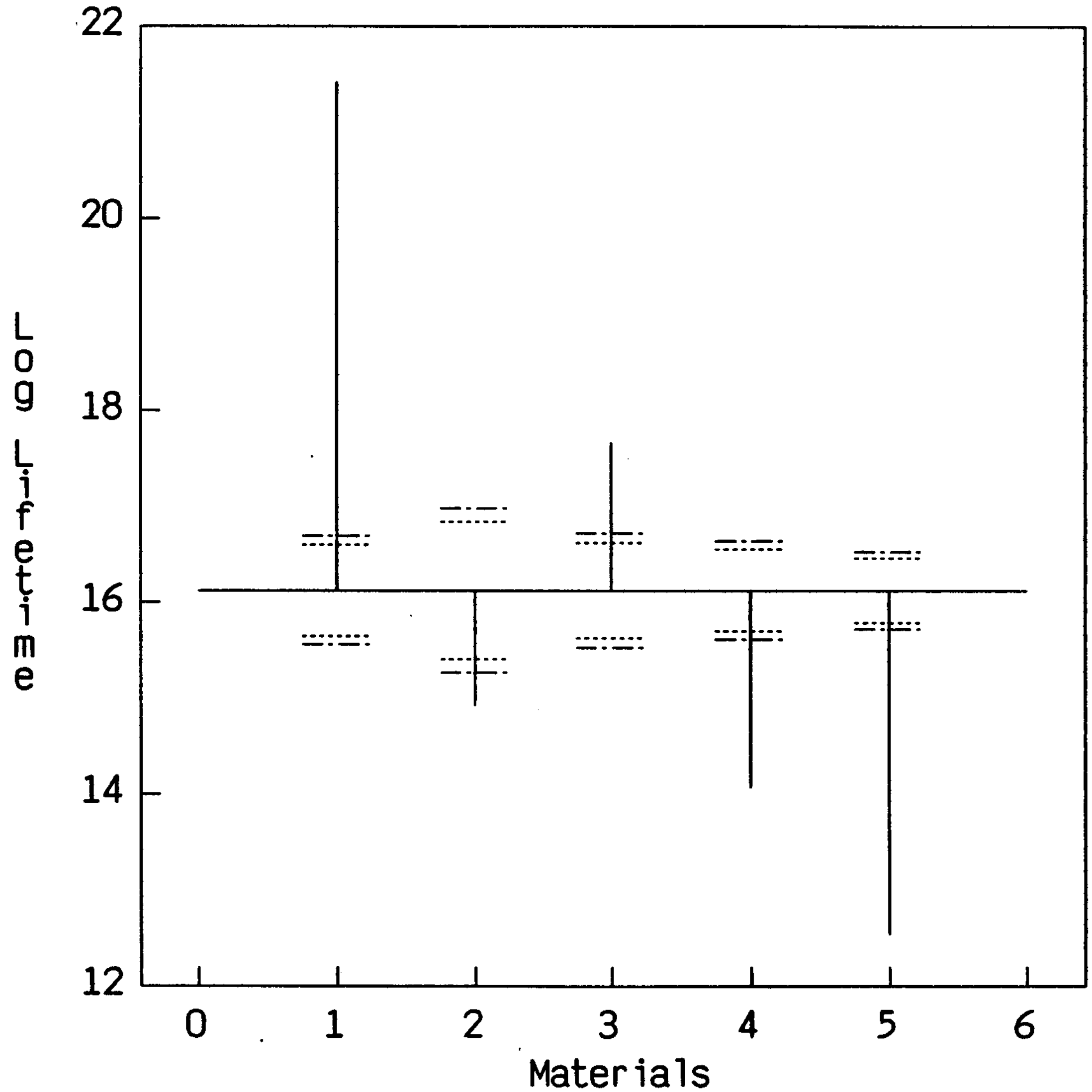

Figure 5. Analysis of Means for the Degradation Data.

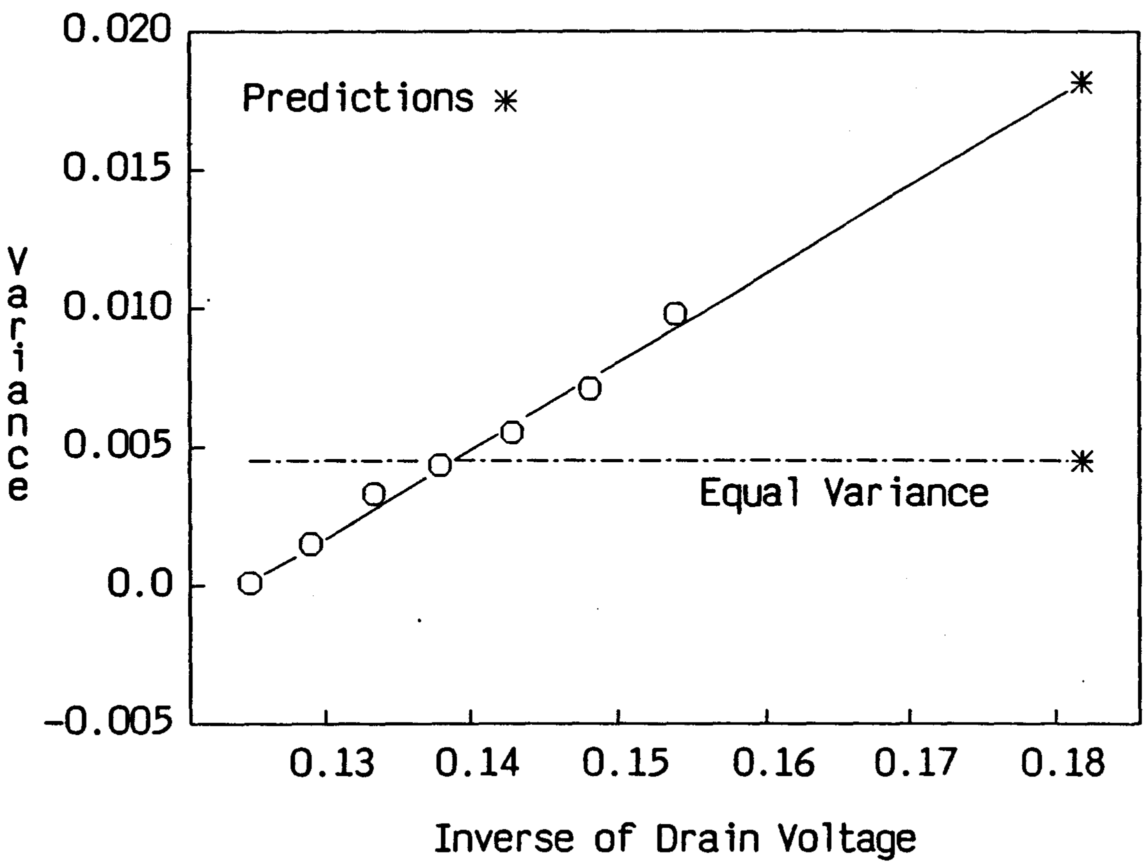

Figure 6. Variance Function Estimation.

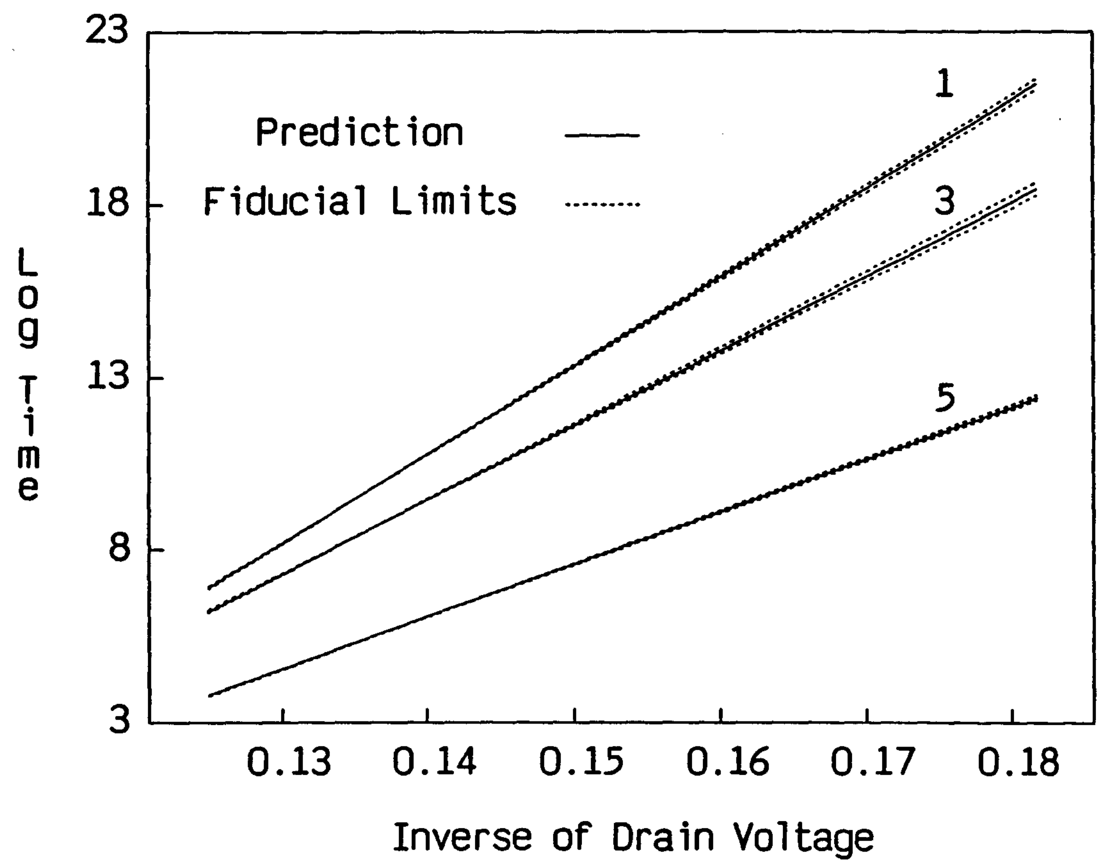

Figure 7. Plot of Time to 10-percent g_m Degradation in Minutes versus Inverse of Drain Voltage (for Three Materials).