STATISTICAL EVALUATION OF VITAL RATES FOR SMALL GROUPS

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Introduction

In recent years much attention has been devoted to the use of statistical indices of health-related characteristics of populations. In particular, records of vital events such as births and deaths have been used to compute rates which serve to compare one population to another, or to monitor changes over time. These vital rates are frequently used as health status indicators for geographic, racial, ethnic, or other groups. Comparisons are made either between different groups at a fixed point in time, or between time points for a particular group. For example, infant, perinatal, or neonatal mortality rates are often used as indices of both health and adequacy of health care. Inferences are made concerning differences in observed rates among groups, or changes in observed rates over time.

In many of these studies, little recognition is given to the fact that the observed rate for a given group at a given time is a random variable, and may be regarded as differing by some chance amount from a hypothetical "true" or expected rate. For relatively large population groups---for example, in comparing vital rates of different nations, or in comparing rates of individual states to the U.S. rate---the variance is negligibly small, and observed differences may be taken as true differences. However, if comparisons of rates are to be made among relatively small groups, or among groups of greatly varying size, such as in examining counties within a state, the possibility of large variance components in the observed rates

requires the use of statistical methods in evaluating differences, and consideration of errors of estimation.

Because of this uncertainty or error inherent in the observed rates, caution should be used in collecting such data, and in basing actions on the results. In truth, though they may seem interesting to study, their information may be quite limited.

One example of a commonly used health status indicator based on vital records is the infant mortality rate. This paper examines several statistical techniques for evaluation of infant mortality rates, as applied to small-group data, focusing on the problem of drawing inferences when the denominator (number of live births) is only moderately large. The problems of estimating the true underlying rate, detecting a change in rate over time, and detecting a difference in rate between two groups are discussed. The techniques discussed are applied to data from counties in Michigan. Although the discussion focuses on infant mortality, the techniques may be applied to any rates.

Statistical Model

It is assumed that the number of infant deaths in a given population in a fixed time period is a binomial random variable X. A parameter n is the number of live births, and p is assumed to be the probability of an infant's death within one year. The probability of k infant deaths, given n live births, is

$$P[X=k|n] = \frac{n!}{k!\ (n-k)!}p^k\ (1-p)^{n-k}$$

The observed ratio X/n is the maximum likelihood estimate of p and is proportional to the usual infant mortality rate R (R=(X/n)x(1000)).

The binomial assumption implies that there is a constant probability p of infant death, and that successive deaths are statistically independent events. Although these assumptions are invalid in particular cases, the binomial model provides a reasonable and simple statistical basis for analysis.

In the following discussion rates are expressed sometimes as proportions and sometimes as rates per 1000. Thus, a rate usually expressed as 26.3 (per 1000 live births) may be given as a proportion, .0263. This proportion is an estimate of the probability p of an infant death, which is here interpreted as the true underlying rate of infant mortality.

Regarding X as a binomial random variable, the observed rate X/n has an estimated variance of $s^2/n = \hat{p}(1-\hat{p})/n$, where $\hat{p} = X/n$, and the standard error is $[\hat{p}(1-\hat{p})/n]^{1/2}$. The larger the number of births, the smaller this error will be. As an example of the relationship between the number of births and the error of estimation, we can determine the required size of n for any desired bound on the standard error. For example, suppose the rate is assumed to be approximately .025 (25 per 1000). If the standard error is to be less than .001 (1 per 1000) the number of live births must be almost 24,500. If p is approximately .030, this required minimum is almost 30,000. Thus a considerable population size is required for a fine degree of accuracy in estimating rates or in determining small differences. When a population is of such a size that the number of live births is 1000 or less caution is needed in drawing inferences about mortality from changes in the observed rate, or differences between two populations.

Table 1 illustrates the possible errors of estimation, for various values of n, and for observed infant mortality rates of R=20,25, and 30. With R=(X/n)x(1000), the standard error of the estimate is s_R = (s)x(1000),

where s is the standard error of X/n. To allow for the variance of the observed rate, the interval $R \pm 2s_R$ may be used as an interval estimate of the true underlying rate. The table gives lower and upper limits for these intervals. For small numbers of births (n) the limits are very wide and little useful information is contained in the data (for $n \geq 500$ these limits correspond to approximate 95% confidence limits). Not until the number of births becomes fairly large, say $n \geq 20{,}000$ can the rates be estimated with any degree of precision. Conclusions regarding actions to be taken by health officials must be approached with extreme caution to avoid unnecessary alarm, wasted time, and expense.

In the following sections, several methods of statistical analysis are described and problems of inference are discussed. The examples used to illustrate these problems refer to infant mortality rates, but the conclusions apply equally well to perinatal or neonatal rates, and with some modification, to other vital rates as well. To compare counties within a state to one another, a method derived from techniques of quality control is introduced. Next, the problem of finding stable estimates or tests for small populations is considered. Sequential tests and estimates are presented, in which data of several years are accumulated to improve the precision of the estimate. Properties of these procedures are illustrated and discussed. Methods of Bayesian estimation are explored as a possible solution where sequential procedures are not sufficient.

Method of Quality Control

Suppose one wishes to determine whether county infant mortality rates differ from a regional rate or from the state rate by an amount sufficient to warrant corrective action. A technique which is widely used in industry has easy application in this context.

Table 2 gives the infant mortality rates for Michigan counties in 1966. The overall Michigan rate is 22.6 per 1000 (or .0226). Based on this state rate, the estimated variance of a county mortality rate for a county with n live births is .0221/n. The variance estimate will be used in determining whether an observed county rate is significantly larger or smaller than the state rate. The difference for a particular county is measured in units of the standard error for that county. The smaller the number of births, the larger this error will be.

In Table 2, the Z score associated with each county rate is derived by subtracting the state rate and dividing by the standard error for the county. (1) Thus, all values are expressed in the same units of measurement. The extent to which a county rate differs from the state rate depends on the extent to which Z differs from zero.

According to the Normal distribution, if measurements are expressed in standard units, and if the measurements arise from the same or similar populations, then 99% of the time, the measurements should be between -2.57 and +2.57; 95% of the time they should be between -1.96 and +1.96. In industrial work, when measurements fall outside of these limits, the process is considered to be "out of control", and corrective action is taken. Often, ± 1.96 are considered "warning limits" and ± 2.57 are considered "action limits". Many variations are possible; for example, action may be taken only if two consecutive values are "out of control".

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(1). In comparing county rates to the state rate, the state rate has been regarded as a fixed (non-random) value. In fact, the state rate itself is subject to random variation, which calls for some modification in the variance estimate used. However, if N is the total number of live births in the state, the correct variance estimate is less than the one used here by an amount of $p(1-p)/N$, which is usually small enough to ignore.

In studying infant mortality rates, one may only be interested in those rates that are significantly high. In this case the 99% upper limits are 2.33 and 1.65, respectively, assuming that the Normal distribution is approximately accurate for these variables. In Table 2, the Z scores can easily be examined to determine which ones are significantly high. The rankings of the Z scores differ considerably from the ranking of the original observed infant mortality rate. In Table 2, a county with 1268 live births has a rate of 30.8 and a Z score of 1.96, outside of the warning limits, while a county with 177 live births has a rate of 39.5 but a Z score of only 1.59.

This methodology, while useful in some contexts, points out some of the problems that arise in small-area estimation. First, as shown, some counties will not be outside of the control limits, while other counties, with smaller observed rates (but larger Z scores), will be outside the control limits. While this is perfectly consistent according to statistical theory, it may be difficult to explain to a non-statistical administrator. A second problem arises from the probability of error. As pointed out above, the rates are random variables, subject to chance variation. The probability of error in concluding that infant mortality is "out of control" is limited to .01 or .05 for each county by the structure of the test used, but with a large number of counties there is a high total probability that some of the Z scores will be above the control limits when in fact all of the counties are "in control". In industrial work, this problem is not serious, since it means only that an occasional batch of items will be rejected when the overall quality is actually acceptable. But in health planning, it might mean that a county or other agency embarks on an expensive program unnecessarily.

When a county has a very large number of live births, the standard error of estimation of the rate is very small, and differences in infant mortality rates are easily detected. Thus, in Table 2, a county with more than 50,000 live births has a Z score of 3.98, by far the largest in the series, although the observed rate of 25.2 is well below those of other counties. In Table 1, it can be seen that with R=25 and n=50,000, the interval estimate is only 2.8 units long. In such a case, if the observed rate differs from the state rate by only a small amount, the Z score will be beyond the control limits. However, since the standard error is so small, judgements about whether corrective action is needed can be based on differences in the observed rates themselves.

An alternative approach to the one suggested here is to compare county rates to a selected "desirable" rate, rather than to a state or regional rate. One could get considerably different verdicts about which counties were in or out of control by varying the "desirable" value, but the rankings of the Z scores would remain the same.

Finally, a major problem in this procedure is the power of the tests used. The power associated with a particular difference is the probability of detecting that difference, and is a measure of how well the test detects the differences it is supposed to detect. It is a function of (a) the level of the action and warning limits, (b) the size of the difference and (c) the number of live births. Table 3 gives approximate values of the power of the suggested test for an action limit of 2.33 and for various sizes of n and various size differences, denoted as Δ p. The power is based on the Normal approximation, with variance estimated as .02/n (for higher variance, the power is reduced). One can see that these procedures are not very powerful. With a change of 5 deaths per 1000 live births (Δ p=.005),

10,000 live births are needed to detect the change with probability .889. This means that with n only moderately large, one may conclude that action is not necessary when in fact it really is needed.

The method of quality control can be useful in assessing observed rates for small population groups, if the problems discussed above are taken into consideration. It is more useful to the state or regional administrator who is evaluating the overall status of a large collection of small population groups than it is to the administrator associated with only one of the groups. If his group is a modest size, he knows that there is considerable chance of error in drawing inferences based on observed statistics.

Sequential Procedures

According to Table 2, 37 counties in Michigan had observed rates in 1966 which were greater than the overall rate for Michigan of 22.6. However, only 7 of these 37 counties have Z scores exceeding the warning limit of 1.645, and only 4 exceed the action limit of 2.33. As Table 4 shows, the power of the test being used to detect differences is quite low when the number of live births is moderately large, hence we may fail to take corrective action when it is needed. In this case, a sequential procedure may be appropriate to monitor the proportion of infant deaths in suceeding years and to determine when and if the accumulated evidence indicates that the rate is significantly high.

Frequently health officers wish to compare the infant mortality rate within a county, beginning in some particular year, to the county's own rate as averaged over several preceding years. In this case, they may wish to determine whether the accumulated rate is either higher or lower than the historical rate, and a two-sided sequential test is appropriate.

If a new health program has been initiated, a one-sided sequential test may be used to detect whether there is a decrease.

In the sequential test, the base rate being used for comparison is expressed as an interval [p',p"]. The test determines at each stage whether the cumulative observed rate is significantly greater than p" or significantly less than p' with the interval from p' to p" regarded as a zone of indifference. A decision can be reached in the minimum amount of time possible, given the observations generated by the group being studied. The sequential test also has the advantage of allowing arbitrary selection of the two error probabilities, probability of incorrect assumption of increase, and probability of incorrect assumption of decrease. This overcomes the problem of lack of power due to a small number of observations, at the cost of a longer waiting period before a decision is reached. The error probabilities to be tolerated can be chosen on the basis of practical considerations, according to the seriousness of the action that would result from either decision.

Let n_1, n_2, n_3,... denote the numbers of live births in successive years for the group being studied, and let X_1, X_2, X_3,... be the numbers of infant deaths. The estimated rate up to and including the m^{th} year is the accumulated ratio $(X_1+X_2+...X_m)/(n_1+n_2+...+n_m) = \hat{p}_m$. A sequence of intervals $[L_1,U_1]$, $[L_2,U_2]$, $[L_3,U_3]$,.. is established. As long as the cumulative estimated rate in a given year is within the appropriate interval for that year, judgement is reserved until more evidence can be accumulated. However, if in the m^{th} year, $\hat{p}_m$ is less than L_m, the rate is judged to be significantly less than p', or if $\hat{p}_m$ is greater than U_m, the rate is judged to be significantly greater than p". The intervals become shorter as the data accumulate, and a difference becomes easier to

detect.

The threshold values L_m and U_m have the form: $L_m = (a_1/N_m)+b$ and $U_m=(a_2/N_m)+b$, where N_m is the total number of births up to and including the m^{th} year. The values a_1, a_2, and be depend on the values of p' and p" chosen for the test, and on the two error probabilities: α, the probability of incorrectly deciding the rate is greater than p", and β, the probability of incorrectly deciding the rate is less than p' [2]. It is simple to construct graphs to study this procedure over time. The boundaries of the decision regions are hyperbolas and appear as in Figure 1. (For very large values of N_m, the boundaries converge to the value b, and the interpretation of the test changes). In this example, p'=.0200 and p"=.0250; α =.05 and β = .10. The boundaries of the decision regions are $U_m=(12.661/N_m)$ +.02247 and $L_m = (-9.861/N_m)$ + .02247. Data for two counties in Michigan have been used to illustrate the procedure, using births and infant mortality for the years 1960-69. Data for County 1 are plotted with x in the graph.

Data for County 1
(Plotted with x in Figure 1)

		1960	61	62	63	64	65	66	67	68	69
	Births	444	381	423	383	396	328	380	345	347	358
Infant	Deaths	17	14	13	11	11	14	10	11	10	7
	N_m	444	825	1248	1631	2027	2355	2735	3080	3427	3785
	$\hat{p}_m$	.0383	.0376	.0353	.0337	.0326	.0340	.0329	.0328	.0324	.0309

(2)

$$a_1 = \frac{\log \frac{1-\beta}{\alpha}}{\log \frac{p_1}{p_o} - \log \frac{1-p_1}{1-p_o}}; \quad a_2 = \frac{\log \frac{\beta}{1-\alpha}}{\log \frac{p_1}{p_o} - \log \frac{1-p_1}{1-p_o}} \quad b = \frac{\log \frac{1-p_o}{1-p_1}}{\log \frac{p_1}{p_o} - \log \frac{1-p_1}{1-p_o}}$$

It is clear from the figure that $\hat{p}_1$ is well within the acceptable range (indicating that the test should be continued), $\hat{p}_2$ is barely within, and the rest of the estimates are all outside of the "continue sampling" region. In practice, after the third year (1962) some action would have been taken.

Data for the second county are given below.

Data for County 2
(Plotted with o in Figure 1)

	1960	61	62	63	64	65	66	67	68	69
Births	176	202	165	153	150	127	113	144	138	142
Infant Deaths	5	11	7	2	4	3	3	2	4	5
N_m	176	378	543	696	846	973	1086	1230	1368	1510
P_m	.0284	.0423	.0424	.0359	.0343	.0329	.0322	.0301	.0300	.0305

For this county, while the cumulative observed infant mortality rate remains relatively high, the test does not indicate that action should be taken. The number of births is so small that even a cumulative ten year rate has a rather large standard error.

The results in this section show how cumulative data may be used to reach a decision when the number of births in a single year is not large enough to yield a statistically significant result. However, in some cases, even the cumulative number of births may be too small for valid inference.

Moving Average

Even when no test of increase or decrease is involved, cumulative data methods can be used to improve the accuracy (that is, reduce the standard error) of an estimate. This is frequently done by using a "moving average" of a few years data; for example, data from the present and the previous two years are used to obtain an estimate for the present year. If the true underlying rate of infant mortality is not changing, this will yield a better

estimate of the underlying rate.

A moving average is usually obtained by simply averaging the observed annual rates for the three year period. Thus if $\hat{p}_1, \hat{p}_2$, and $\hat{p}_3$ are three consecutive annual estimates, based on n_1, n_2, and n_3 births, the average is $\hat{p}_A = (\hat{p}_1+\hat{p}_2+\hat{p}_3)/3$, which has variance.

$$s_A^2 = \frac{1}{9}\,[p(1-p)]\left[\frac{1}{n_1} + \frac{1}{n_2} + \frac{1}{n_3}\right].$$

If the number of births is small, this estimate can be improved by a very simple device which reduces the variance of the estimate, thus centering it more closely on the true value. The improved estimate is obtained by summing the number of deaths for the three years and dividing by the total number of births: $\hat{p}_B = (X_1 + X_2 + X_3)/(n_1 + n_2 + n_3)$. The improved estimate has variance

$$s_B^2 = p(1-p)/(n_1+n_2+n_3).$$

which can be shown to be smaller than s_A^2. (The estimate $\hat{p}_B$ is the same as the one used in the sequential test).

For example, using the data from County No. 2 in the previous section, the variance of the three year estimate using data from 1960-61-62 is .0018547 p(1-p) using $\hat{p}_A$, and .0018416 p(1-p) using $\hat{P}_B$. The improvement will be greatest for counties with very small numbers of births, say less than 100. The accuracy of the estimate for a given cumulative number of births can be judged by using Table 1, with n equal to the total number of births.

Bayesian Estimation

For areas of very small population, even the sequential or moving

average methods suggested in the previous sections may not yield estimates which are statistically accurate enough for valid inference. An alternative method of estimation, not commonly used in health statistics, is to assume some a-priori knowledge about the true underlying rate of infant mortality, and to make use of this knowledge in a systematic manner. The a-priori knowledge may be based on some independent information such as a state or regional rate, or an historical observed rate, and is combined with the data for the area being studied in order to obtain a more accurate estimate. This, in effect, combines two independent sources of information, and the procedure may be varied by the amount of weight placed on the a-priori information. The amount of weight given to the observed data for the area being studied is determined by the variance for these data alone, which is again a function of the number of live births.

The prior information is expressed in the form of a probability distribution for the true underlying rate $\hat{p}$. The most convenient distribution, for mathematical reasons, is the Beta. Its density function may be expressed in the form

$$f(p;\, m,z) = \frac{(m+1)!}{z!\,(m-z)!}\qquad p^{z}\,(1-p)^{m-z}\ ,\ 0<p<1$$

$$= 0 \text{ otherwise,}$$

where m and z are integers, $0<z<m$. This distribution has mean $(z+1)/(m+2)$ and variance $[(z+1)(m-z+1)]/[(m+2)^2(m+3)]$. In this form, the probability density function may be interpreted as the distribution of p based on a-priori information that m births results in z infant deaths.[3] Suppose now that

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(3) Novick, Melvin R., and Grizzle, James E., "A Bayesian Approach to the Analysis of Data from Clinical Trials", JASA, Vol. 60, No. 309, March 1965, pp. 81-96.

observations for the current year give X deaths in n births. The Bayes estimate p which minimizes the expected squared deviation from the true underlying value of p is

$$\hat{p} = \frac{X+z+1}{n+m+2}$$

This is the mean of a revised or a-posteriori Beta distribution with parameters z'=z+X and m'=m+n. The revised distribution is a probability distribution for p based on a combination of a-priori and newly observed information.

Suppose the estimates in Michigan for 1966 are to be weighted by the information that the overall state rate is 22.6. This may be expressed, for example, by using a Beta distribution for p with parameters z=2 and m=131 (this distribution has mean .02256 and standard deviation .0128). The Bayes estimate for each county would be $\hat{p} = (X+3)/(n+133)$. These estimates (x1000) appear in Table 2 and can be compared to the unweighted estimates (X/n)x(1000). The Baye's estimates have the following properties: (1) the larger the parameter m is chosen in the Beta distribution, for a given mean value, the greater the weight placed on the prior information in the estimate, and (2) for a given Beta distribution, the larger the number of births in the county, the less the estimate is affected by the prior information. Thus it can be seen that for counties with large numbers of births, the two estimates are almost the same, whereas if the number of births is small, the Bayes estimate is considerably modified in the direction of the overall state rate.

The Bayes method of estimation may also be used iteratively, with each year's observed births and infant deaths being used to revise the previous estimate. After k years of observations, the estimate will be

$$\frac{X_1 + X_2 + \ldots\ldots + X_k + z + 1}{n_1 + n_2 + \ldots\ldots + n_k + m + 2}$$

As the number of observed births increases, more and more weight is given to the observed data as compared to the a-priori information. If the true underlying value of p is not changing, the estimate will converge to this value. The procedure cannot be used to detect changes in the pattern of infant mortality, since new observations are weighted by previous observations and the a-priori information in making the estimates.

Summary

When statistical indices such as infant mortality rates are computed for relatively small population groups, the variance of the estimated rate must be taken into consideration in making inferences about differences or changes. Statistical techniques such as quality control methods, sequential procedures, or Bayes estimates may sometimes be helpful in studying these rates. However, for small areas, or small population groups, such as counties within a state, the variance inherent in these rates may rule out their use as valid health status indicators.

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Table 1

Interval Estimates for Rate

$R = (X/n)x(1000)$

n	R=20	S_R	R=25	S_R	R=30	S_R
100	[-8.0 to 48.0]	14.0	[-6.2 to 56.2]	15.6	[-4.1 to 64.1]	17.1
500	[7.5 to 32.5]	6.3	[11.0 to 39.0]	7.0	[14.7 to 45.3]	7.5
1000	[11.1 to 28.9]	4.5	[15.1 to 34.9]	5.0	[19.2 to 40.8]	5.2
5000	[16.0 to 24.0]	2.0	[20.6 to 29.4]	2.2	[25.2 to 34.8]	2.4
10000	[17.2 to 22.8]	1.4	[21.9 to 28.1]	1.6	[26.6 to 33.4]	1.7
20000	[18.0 to 22.0]	1.0	[22.8 to 27.2]	1.1	[27.6 to 32.4]	1.2
50000	[18.7 to 21.3]	.65	[23.6 to 26.4]	.70	[28.5 to 31.5]	.75
100000	[19.1 to 20.9]	.45	[24.0 to 26.0]	.50	[28.9 to 31.1]	.55
150000	[19.3 to 20.7]	.35	[24.2 to 25.8]	.40	[29.1 to 30.9]	.45
200000	[19.4 to 20.6]	.30	[24.3 to 25.7]	.35	[29.2 to 30.8]	.40

Table 2

Michigan Infant Mortality-1966

	Live Births	Observed Rate	Z Score	Z Rank	Bayes Estimate
1.	15	66.7	1.15	10	27.0
2.	86	46.5	1.49	9	31.9
3.	177	39.5	1.59	8	32.2
4.	610	39.3	2.78	2	36.3
5.	79	38.0	0.88	14	28.3
6.	58	34.5	0.61	20	26.1
7.	1520	32.2	2.58	3	31.4
8.	1268	30.8	1.96	6	30.0
9.	2659	29.7	2.47	4	29.4
10.	270	29.6	0.77	17	27.3
11.	345	29.0	0.80	16	27.2
12.	278	28.8	0.69	18	26.7
13.	208	28.8	0.60	21	26.4
14.	731	28.7	1.11	11	27.8
15.	595	28.6	0.98	12	27.5
16.	251	27.9	0.57	22	26.0
17.	2990	27.4	1.76	7	27.2
18.	4694	27.3	2.16	5	27.1
19.	113	26.5	0.28	30	24.4
20.	380	26.3	0.49	24	25.3
21.	892	25.8	0.64	19	25.4
22.	233	25.8	0.33	27	24.6
23.	788	25.4	0.53	23	25.0
24.	593	25.3	0.44	25	24.8
25.	356	25.3	0.34	26	24.5
26.	51753	25.2	3.98	1	25.2
27.	358	25.1	0.32	28	24.4
28.	3448	24.9	0.91	15	24.0
29.	9740	24.0	0.93	13	23.7
30.	1085	24.0	0.31	29	23.8
31.	167	24.0	0.11	33	23.3
32.	467	23.6	0.14	32	23.3
33.	297	23.6	0.11	34	23.2
34.	2565	23.4	0.27	31	22.2
35.	429	23.3	0.10	35	23.1
36.	915	23.0	0.08	36	22.9
37.	568	22.9	0.05	37	22.8
38.	712	22.5	-0.02	38	22.5
39.	833	21.6	-0.19	43	21.7
40.	7880	21.6	-0.59	57	21.6
41.	650	21.5	-0.19	42	21.7
42.	2386	21.4	-0.39	48	21.4
43.	94	21.3	-0.09	39	22.0
44.	141	21.3	-0.11	41	21.9
45.	565	21.2	-0.22	44	21.5
46.	95	21.1	-0.10	40	21.9
47.	15468	20.5	-1.75	76	20.5
48.	996	20.1	-0.54	55	20.4
49.	500	20.0	-0.39	49	20.5

Table 2 continued.....

	Live Births	Observed Rate	Z Score	Z Rank	Bayes Estimate
50.	252	19.8	-0.30	46	20.8
51.	509	19.6	-0.46	52	20.2
52.	617	19.4	-0.54	56	20.0
53.	2194	19.1	-1.10	65	19.3
54.	5237	18.7	-1.90	79	18.2
55.	162	18.5	-0.35	47	20.3
56.	2242	18.3	-1.37	70	18.5
57.	4039	18.3	-1.84	77	18.4
58.	55	18.2	-0.22	45	21.3
59.	948	17.9	-0.97	62	18.5
60.	169	17.8	-0.42	50	19.9
61.	12339	17.7	-3.67	83	17.8
62.	972	17.5	-1.07	64	18.1
63.	2217	17.1	-1.74	75	17.4
64.	3330	17.1	-2.13	82	16.5
65.	118	16.9	-0.42	51	19.9
66.	1258	16.7	-1.40	71	17.1
67.	800	16.3	-1.20	69	17.1
68.	126	15.9	-0.51	53	19.3
69.	533	15.0	-1.18	68	16.5
70.	1136	15.0	-1.73	74	15.8
71.	1192	14.3	-1.93	80	15.1
72.	784	14.0	-1.62	73	15.3
73.	73	13.7	-0.51	54	19.4
74.	365	13.7	-1.14	66	16.1
75.	158	12.7	-0.84	59	17.2
76.	82	12.2	-0.63	58	18.6
77.	165	12.1	-0.91	61	16.8
78.	182	11.0	-1.05	63	15.9
79.	550	10.9	-1.84	78	13.2
80.	101	9.9	-0.86	60	17.1
81.	131	7.6	-1.15	67	15.2
82.	181	5.5	-1.55	72	12.7
83.	276	3.6	-2.13	81	9.8

Table 3

Power of Control Procedure*

(2.33 Action Limits) $s^2 = .02/n$

n	Δp .0005	.001	.0015	.002	.0025	.003	.0035	.004	.0045	.005
100	.001	.012	.013	.014	.016	.017	.019	.020	.022	.024
900	.013	.017	.022	.028	.036	.046	.056	.069	.084	.102
2500	.016	.024	.036	.053	.074	.102	.138	.181	.230	.288
10000	.024	.053	.102	.181	.288	.421	.564	.695	.808	.889

*The entries are the probabilities of exceeding the action limits when the true value of the probability exceeds the overall state rate by an amount Δp.

Figure 1

Regions for Sequential Test