Circular CAR Modeling of Vector Fields *

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Abstract

As hurricanes approach landfall, there are several hazards for which coastal populations must be prepared. Damaging winds, torrential rains, and tornadoes play havoc with both the coast and inland areas; but, the biggest seaside menace to life and property is the storm surge. Wind fields are used as the primary forcing for the numerical forecasts of the coastal ocean response to hurricane force winds such as the height of the storm surge and the degree of coastal flooding. Unfortunately, developments in deterministic modeling of these forcings have been hindered by extreme computational expenses.

In this paper, we present a multivariate spatial model for vector fields, that we apply to hurricane forcing winds. More specifically, a circular conditional autoregressive (CCAR) representation of the vector direction, and a spatial conditional model for the

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vector length given the direction is presented, while a Bayesian framework is being used for inference from this model. We apply our framework for vector fields model hurricane surface wind fields. A case study of Hurricane Floyd of 1999 compares our CCAR model to prior methods that decompose wind speed and direction into its N-S and W-E cardinal components.

1 Introduction

Across many areas of research, one may come into contact with data that has been collected in a vector format. One such type of data is wind fields. Studying wind fields is important in environmental research. For example, with the current insurgence of support for cleaner energy, many are setting their eyes on harnessing the power of the wind. Unfortunately, it is not practical to place wind turbines in random locations. Researchers are currently trying to pinpoint locations where it would be beneficial for these turbines to be placed. To do that, they need to be able to model the wind speed, direction, and duration at different sites. Another example is the wind fields generated by a hurricane. Residents living along the coastal area of the southeast United States and Gulf Coast are presented with many hazards during a landfalling hurricane. With populations in these areas increasing, it is imperative that as storms approach the coastline we have the means necessary to give these citizens the information needed to prepare for possible landfall conditions. Storm winds, torrential rain, and spawned tornadoes each can harm both life and property but the single largest threat to coastal areas is the storm surge. With homes and businesses being either right at or just a few feet above sea level, this inundation of water pushed by the landfalling storm can quickly take lives and destroy property.

With the development of accurate forecasts for these storm surges, we could improve upon the preparedness of these communities. Research has shown that the effectiveness of these forecasts depends upon accurate modeling of wind forcings. At present, there has not been a model adopted for the forecasting/mapping of these hurricane winds for the specific purpose of improving storm surge forecasts. Some researchers believe that this is due to the computational expense of such models. However, there have been some models and methods developed to assist with modeling these hurricane wind fields. Holland (1980), Depperman (1947), and DeMaria et al. (1992) each presented models that have been termed as axis-symmetric. These models are based upon a cyclostrophic wind balance and place the key dependence on the distance a location is from the storm circulation center. These models are simple to understand and apply; however, they do not describe the true asymmetrical structure of the winds within hurricanes. It is a known fact that the winds within the northeast quadrant of the storm are typically stronger than those in other locations within a storm, and, researchers have investigated why these deviations from symmetry appear. Causes of this asymmetry can be attributed to friction, environment, vertical shear, etc. These and other possible sources were discussed by Chen and Yau (2003), Ross and Kurihara (1992), Shapiro (1983), and Wang and Holland (1980).

Xie et al. (2010) looked into the effect asymmetry of a storm has on the storm surge. Using the Coastal Marine Environmental Prediction System (CMEPS), developed at North Carolina State University, they simulated storm surge under different conditions. They found that there was significant difference in water levels when the asymmetry of hurricane wind fields was changed while holding all other parameters such as maximum wind speed, radius of maximum winds, and minimum pressure constant. Xie et al. commented that there has been improvements made to improve the storm surge forecasts. However, they point out that all of these advancements were made under the assumption that the wind forcing fields were accurate.

With the knowledge that hurricanes do have asymmetrical tendencies, there have been models proposed that would attempt to incorporate asymmetric structures. Dating back to 1985, researchers like Georgiou (1985) have proposed these such models. Xie et al. (2006) looked at the wind model developed by Holland and attempted to model its error utilizing a Gaussian process. Reich and Fuentes (2007) took this approach one step further and removed the assumption of the Gaussian process. With the incorporation of a stick-breaking prior, they were able to develop a model approach that was more general that the previous.

Unfortunately, this model placed the assumption that the cross-dependence between the N-S and W-E wind components was constant across space. This may not be the fact.

The common statistical modeling approach for wind vectors decomposes the wind fields into the u and v components (cartesian representation), where u corresponds to the N-S and v to the W-E wind component. Figure 1 plots the data for Hurricane Floyd on September 14, 1999. Modeling u (Figure 2a) is challenging because it displays heavy-tails (non-normality), increased variability, and a shorter spatial range (non-stationarity) near the storm center. Joint modeling of u and v is also complicated because their correlation varies dramatically in different parts of the spatial domain. In contrast, the logarithm transformation of wind speed and wind direction, respectively, vary relatively smoothly in space and do not have a complicated joint relationship (Figure 3). Therefore, in this paper we propose to model hurricane wind fields using polar coordinates.

Modeling wind using polar coordinates presents challenges of its own. The literature on spatial modeling of angles is limited. Morphet, in his Utah State University thesis, (2009) presents some frequentist methods as well as enhanced the visualization of circular-spatial data through the development of an R package. Morphet developed a circular kriging solution that was based on fitting a new defined cosineogram. Morphet also presented a method of simulating from a circular random field that was a transformation of a Gaussian random field.

The Bayesian approach for hurricane modeling has several advantages, including a convenient framework for simultaneously modeling several data sources (e.g., satellite and buoy data) and natural measures of uncertainty for model parameters, which are crucial inputs to deterministic hurricane and storm surge models. Ravindran, in his North Carolina State University thesis, (2002) approaches circular data from a Bayesian perspective utilizing wrapped distributions. Ravindran states that likelihood-based inference for these wrapped distributions can be very complicated and not be computationally efficient. Suggesting a Markov Chain Monte Carlo (MCMC) method with a data augmentation step, he feels that the computational issues can be resolved. An extension of the method is given for time-correlated

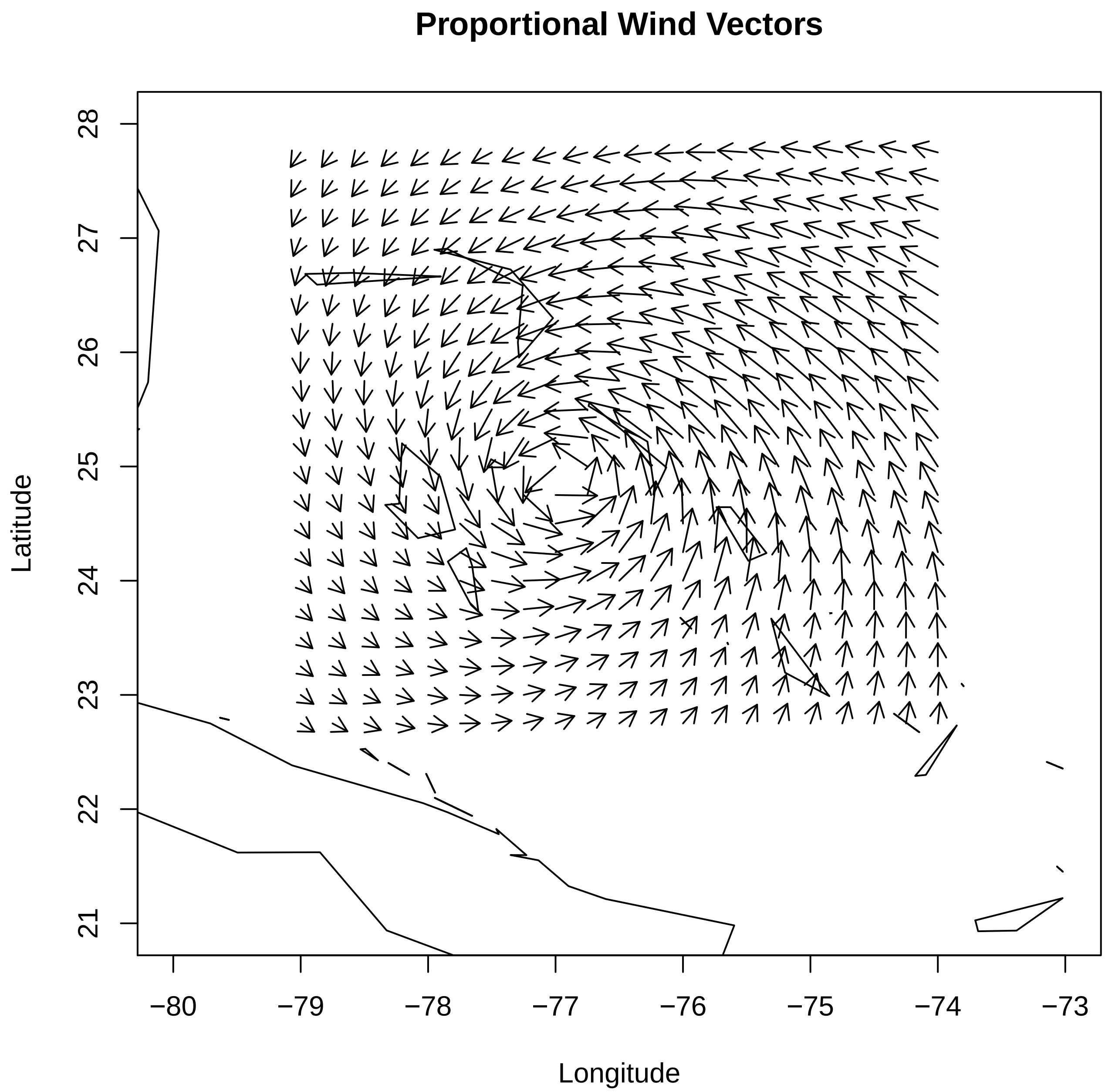

Figure 1: Plot of wind vectors of Hurricane Floyd (09/14/1999 at 12noon local time).

data. To our knowledge, we present the first hierarchical Bayesian model for spatial circular data.

In this paper, we present a new statistical modelling framework for spatial vector fields,

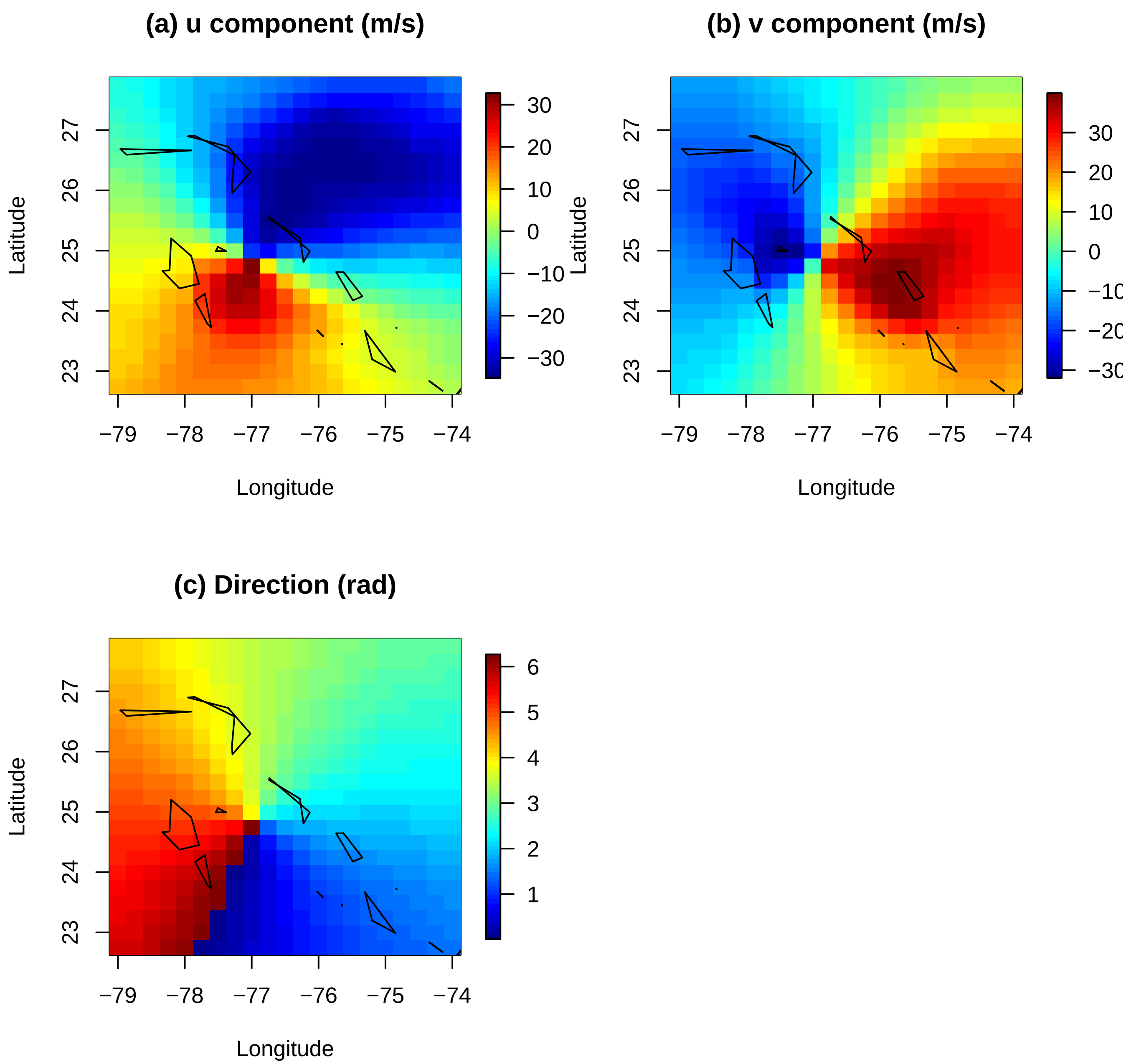

Figure 2: (a) u component. (b) v component. (d) the direction (θ).

that we implement to hurricane wind fields. The proposed approach does not require the decomposition of the wind vector into its cardinal components. With the assistance of wrapped distributions, we model the angle of the wind direction using a CCAR. The wind

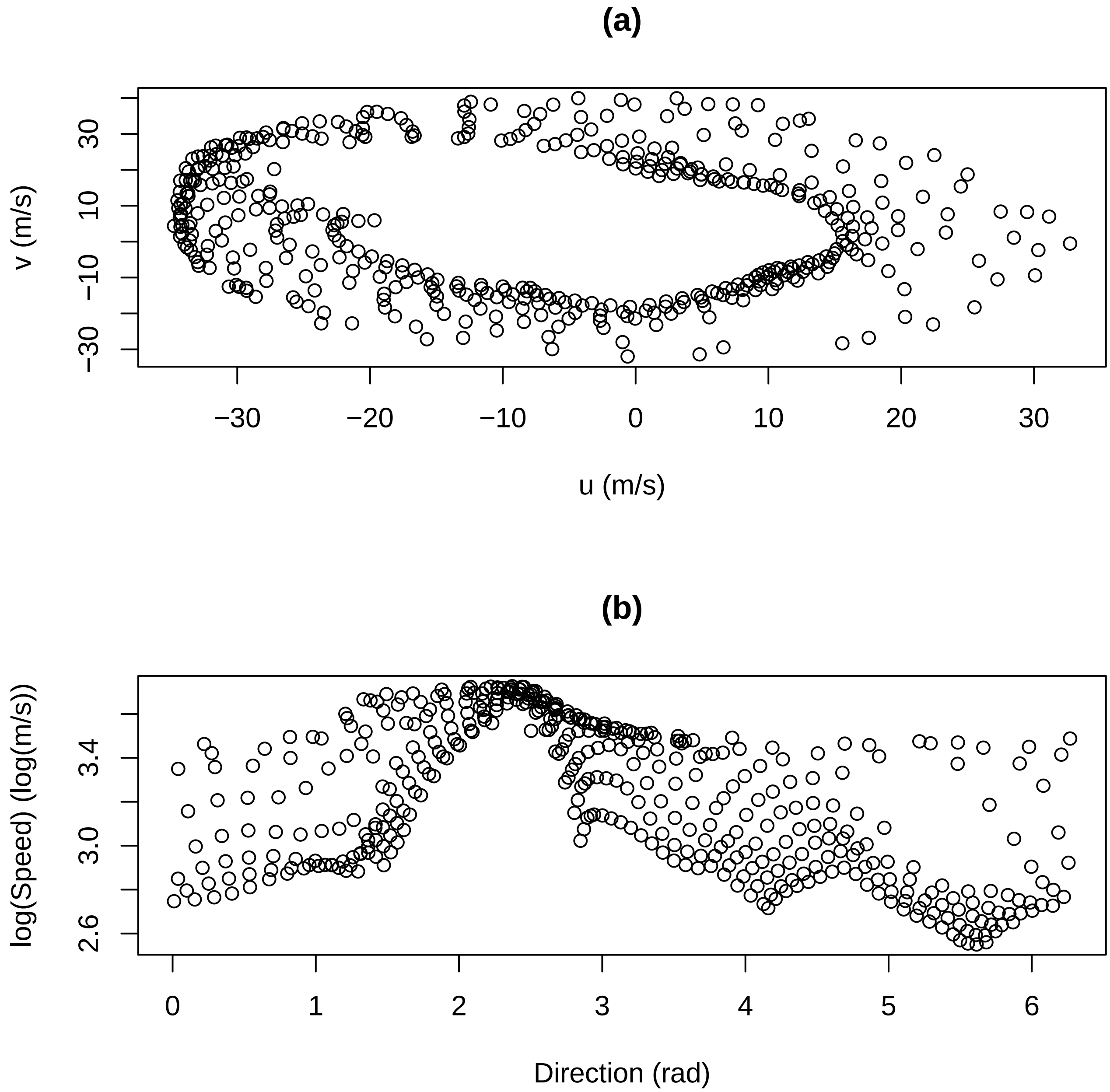

Figure 3: Scatterplots of Hurricane Floyd data. (a) u and v components. (b) speed (ω) and direction (θ).

speed and wind direction at a particular location within a storm tend to be less correlated than the u and v components. Then, it is easier to explain the spatial cross-dependence of wind vectors using polar coordinates. We use this polar coordinate mapping of the wind fields to assist in the quantification of the asymmetry of the hurricane winds. This paper is organized as follows. In Section 2 we review circular statistics. In Section 3 we describe the new CCAR methodology. In Section 4 we apply our methods to Hurricane Floyd. And, we conclude with results and some final remarks in Section 5 and 6 respectively.

2 Circular Statistics

In many different fields, one can find data that incorporates the use of angles. Since the 1970s, there have been advancements in the analysis of such data with, according to Fisher (1993), a "vigorous development" of methods in the 1980s. Angles are vastly different than their linear counterparts. Computation of summary statistics, performing analysis, and simply displaying the data all must take into account the periodic nature that 0 and 2π represent the same angle. Thus the standard approaches to model distributions and calculate moments have to be augmented when working with angles. This section describes the common approaches to obtain moments and distributions of angles.

2.1 Sample Moments

We begin with the calculation of the mean. With linear data, the sample mean is $\bar{x} = \sum_{i=1}^{n} \frac{x_i}{n}$. When $x_i = \theta_i$ is an angle, this is not appropriate because this ignores similarity of values near zero and near 2π. The change here would be to incorporate vector addition, as explained in Fisher (1993). We begin by calculating three values: $C = \sum_{i=1}^{n} \cos\theta_i$, $S = \sum_{i=1}^{n} \sin\theta_i$, and $R^2 = C^2 + S^2$. With these calculations, the value (direction) of $\bar{\theta}$ is $\bar{\theta} = \arctan\frac{S}{C}$.

In the previous equations, $R \in (0, n)$ is commonly referred to as the resultant length of the resultant vector. Thus, we can calculate the mean resultant length $\bar{R} = \frac{R}{n}$. Fisher (1993) states that $\bar{R} = 1$ represents all the points were overlapping; however, he is quick to add that $\bar{R} = 0$ does not imply uniform dispersion around the circle. The main usage of $\bar{R}$

is in the calculation of sample circular variance, $V = 1 - \bar{R}$. Similar to the interpretation of linear variance, a small circular variance does imply that the distribution of data was more concentrated. Differences between linear and circular variance fall in that $V \in [0, 1]$ and calculation of standard deviation is not just a square root. Sample circular standard deviation is defined as $\upsilon = \{-2\log(1-V)\}^{\frac{1}{2}}$. These calculations are needed for calculating posterior means and standard deviations from MCMC output.

2.2 Circular Distributions

Fisher (1993) describes seven distributions that can be placed on circular data. He begins with the simplest setup by assuming that all possible directions on the circle are equally likely. Under this uniform distribution, the probability density function is $f(\theta) = \frac{1}{2\pi}, \quad 0 \leq \theta < 2\pi$. This is no different that our typical understanding of the uniform distribution. The mean of this distribution is undefined and $\bar{R} = 0$, where in this case the circular dispersion is infinite.

Fisher continues by stating, that if we begin by assuming that X is a random variable from the real line, we can construct a random variable on the circle and determine its density. Let's assume that X has some probability density function $g(x)$ and cumulative density function $G(x)$, we will define $\theta = X[mod 2\pi]$. The probability density function $f(\theta)$ is found by us wrapping $g(x)$ around a unit circle. Thus $f(\theta) = \sum_{k=-\infty}^{\infty} g(\theta + 2k\pi)$ with a corresponding cumulative density function of $F(\theta) = \sum_{k=-\infty}^{\infty}[G(\theta + 2k\pi) - G(2k\pi)]$. The Wrapped Normal distribution is of particular interest in modeling wind fields.

3 Hierarchical Bayesian spatial model for a vector field

We assume that the response in grid cell $i = 1, ..., n$ is a vector defined by its speed and direction (ω_i, θ_i). Empirical analysis seems to indicate that a log transform of the vector speed allowed conditional normality to be a reasonable assumption, $y_i = \log(\omega_i) \in \mathcal{R}$. Our statistical framework is as follows

$$y_i|\theta_i \sim N(\mathbf{X}_i^T \boldsymbol{\beta}_1 + \mu_{1i} + g(\theta_i), \sigma_1^2) \tag{1}$$

$$\theta_i \sim WN(\mathbf{X}_i^T\boldsymbol{\beta}_2 + \mu_{2i}, \sigma_2^2)$$

where $\mathbf{X}_i^T\boldsymbol{\beta}_1$ and $\mathbf{X}_i^T\boldsymbol{\beta}_2$ represent the contribution to each mean by covariates, $g(\theta_i)$ captures the relationship between vector direction and log vector length, and μ_{i1} and μ_{2i} are spatial effects. There are several possibilities for the functional relation between the θ_i and y_i. One could assume a linear mean model $g(\theta_i) = b\theta_i$ but this is not appropriate, since conceptually we should have $g(0) = g(2\pi)$. Another is the standard approach for circular/linear association is $g(\theta_i) = b\cos(\theta_i)$ (Fisher, 1993). To specify more complicated circular/linear relationship, rather than including higher-order polynomials, one could include higher frequencies $g(\theta_i) = \sum_{k=1}^{M} a_k \sin(k\theta_i) + \sum_{k=1}^{M} b_k \cos(k\theta_i)$. A more detail explanation about the Wrapped Normal distribution follows.

Modeling θ_i is challenging due to the restrictions that $\theta_i \in [0, 2\pi)$ and that its density at 0 and 2π should be equal since these are the same angle. We model θ_i by extending the wrapped normal (WN) distribution to the spatial setting. The WN distribution is commonly used in circular statistics (Fisher, 1993). It is the symmetric and unimodal distribution obtained by wrapping a normal distribution on the real line around a circle. The density is

$$f(\theta_i) = \sum_{j=-\infty}^{\infty} \phi(\theta_i | 2\pi j + \mu_i, \sigma^2), \tag{2}$$

where $\phi(\cdot|m, s^2)$ is the $\mathrm{N}(m, s^2)$ density function. In the wrapped normal model, the mean direction is $E(\tilde{\theta}_i) = \mu_i$ and $\sigma^2 > 0$ controls the variability. We denote this model as $\theta_i \sim \mathrm{WN}(\mu_i, \sigma^2)$.

The WN distribution alleviates several difficulties in modeling spatially-referenced angles. Unfortunately, the WN density (2) cannot be evaluated directly because it includes an infinite sum with no closed form. However, we are able to analyze this model using MCMC methods after introducing auxiliary variables for the wrap number, $K_i \in \{..., -2, -1, 0, 1, 2, ...\}$. The auxiliary model is

$$\begin{aligned} \theta_i | K_i &\sim \mathrm{TN}_{[0,2\pi)}\left(2\pi K_i + \mu_i, \sigma^2\right) \qquad (3) \\ P(K_i = j) &= \Phi(2\pi | 2\pi j + \mu_i, \sigma^2) - \Phi(0 | 2\pi j + \mu_i, \sigma^2), \end{aligned}$$

where $\text{TN}_A(m, s^2)$ denotes the truncated normal distribution with domain A, location m, and scale s, and $\Phi(\cdot|m, s^2)$ is the distribution function of a normal with mean m and standard deviation s. The truncated normal density can be written

$$p(\theta_i|K_i = j) = \frac{\phi(\theta_i|2\pi j + \mu_i, \sigma^2)}{\Phi(2\pi|2\pi j + \mu_i, \sigma^2) - \Phi(0|2\pi j + \mu_i, \sigma^2)} = \frac{\phi(\theta_i|2\pi j + \mu_i, \sigma^2)}{P(K_i = j)}. \tag{4}$$

Therefore, marginally over K_i,

$$p(\theta_i) = \sum_{j=-\infty}^{\infty} \frac{\phi(\theta|2\pi j + \mu_i, \sigma^2)}{P(K_i = j)} \text{P}(K_i = j) = \sum_{j=-\infty}^{\infty} \phi(\theta_i|2\pi j + \mu_i, \sigma^2), \tag{5}$$

as desired.

Clearly $\sum_{j=-\infty}^{\infty} P(K_i = j) = 1$. However, implementing this prior in standard software such as `WinBUGS` (`http://www.mrc-bsu.cam.ac.uk/bugs/`) is challenging since K_i has an infinite domain and non-standard prior. An equivalent representation of (3) is

$$\begin{aligned} \theta_i|z_i &\sim \text{TN}_{[0,2\pi)}\left(2\pi\left(\lfloor -z_i/(2\pi)\rfloor + 1\right) + \mu_i, \sigma^2\right) \\ z_i &\sim N(\mu_i, \sigma^2), \end{aligned} \tag{6}$$

where $\lfloor -z_i/(2\pi)\rfloor$ is defined as the largest integer less than $-z_i/(2\pi)$. Here we replace K_i's prior in (3) with the two-stage model $K_i = \lfloor -z_i/(2\pi)\rfloor + 1$ and $z_i \sim N(\mu_i, \sigma^2)$, which gives the same prior probabilities for K_i since

$$\begin{aligned} P(K_i = j) &= P(\lfloor -z_i/(2\pi)\rfloor + 1 = j) \\ &= P(j - 1 < -z_i/(2\pi) < j) \\ &= P(-2\pi j < z_i < -2\pi(j-1)) \\ &= \Phi(-2\pi(j-1)|\mu_i, \sigma^2) - \Phi(-2\pi j|\mu_i, \sigma^2) \\ &= \Phi(2\pi|2\pi j + \mu_i, \sigma^2) - \Phi(0|2\pi j + \mu_i, \sigma^2). \end{aligned} \tag{7}$$

As shown in the Appendix, this representation is conducive to standard software packages because it only requires standard parametric distributions.

The spatial random effects μ_{1i} and μ_{2i} have a proper conditionally autoregressive prior ("CAR"; Banerjee et al., 2004). The CAR covariance is specified through spatial adjacencies.

Let $i \sim j$ indicate that cells i and j are spatial neighbors and m_i be the number of spatial neighbors of cell i. The CAR model for the log vector lengths μ_{1i} is defined through the full conditional distribution of μ_{1i} given μ_{1j} at all other cells with $j \neq i$. The full conditional distribution is Gaussian with

$$\begin{aligned} E\left[\mu_{1i}|\mu_{1j}, j \neq i\right] &= \rho_1 \sum_{j \sim i} \mu_{1j}/m_i \\ V\left[\mu_{1i}|\mu_{1j}, j \neq i\right] &= \tau_1^2/m_i. \end{aligned} \tag{8}$$

The full conditional mean is proportional to the average of the spatial neighbors, $\rho_1 \in [0,1]$ controls the degree of spatial association, and the variance is controlled by $\tau_1^2 > 0$. We denote the model $\boldsymbol{\mu}_1 = (\mu_{11}, ..., \mu_{1n})^T \sim \text{CAR}(\rho_1, \tau_1^2)$. Similarly, the spatial angle effects are $\boldsymbol{\mu}_2 = (\mu_{21}, ..., \mu_{2n})^T \sim \text{CAR}(\rho_2, \tau_2^2)$.

To complete the Bayesian model, specify uninformative priors for the hyperparameters. We use independent $\text{N}(0, 100)$ priors for the elements of $\boldsymbol{\beta}_1$ and $\boldsymbol{\beta}_2$, independent InvGamma(0.5,0.0005) prior of the variances σ_1^2, σ_2^2, τ_1^2, and τ_2^2, and independent Unif(0,1) priors of the CAR association parameters ρ_1 and ρ_2. MCMC sampling is performed using `WinBUGS`. We run two independent chains of length 20,000, and discard the first 5,000 samples from each chain as burn-in. Convergence is monitored using trace plots and autocorrelations for the deviance and several representative parameters. For the data in Section 4, sampling takes around twenty-five minutes on an ordinary PC.

4 Analysis of Hurricane Floyd

We use satellite data obtained from NOAA to characterize wind fields. The data are publicly available at www.ncdc.noaa.gov/oa/rsad/seawinds.html. This is the best source of hurricane wind satellite data that we currently have. It is obtained by combining different satellites, and it is stored across the globe on a grid of 0.25 degree squares. We will focus on the September 14th noon observance of Hurricane Floyd, a category three storm, from the 1999 hurricane season. Our area of interest is a 41x41 grid centered approximately on the storm's center of circulation. Our data is given in the cartesian decomposition format therefore we transform to the polar scale, $\omega_i = \sqrt{U_i^2 + V_i^2} > 0$ and $\theta_i = \arctan \frac{u_i}{v_i} \in [0, 2\pi)$. For our CCAR model, recall that $y_i = \log \omega_i$.

We compare the following two models:

1. **Circular model**: $\theta_i \sim \text{WN}(\mathbf{X}_i^T \boldsymbol{\beta}_2 + \mu_{2i}, \sigma_2^2)$ and $y_i|\theta_i \sim \text{N}(\mathbf{X}_i^T \boldsymbol{\beta}_1 + \mu_{1i} + g(\theta_i), \sigma_1^2)$

2. **U/V model**: $U_i \sim \text{N}(\mu_{ui}, \sigma_u^2)$ and $V_i \sim \text{N}(a\mu_{ui} + \mu_{vi}, \sigma_v^2)$

where $\mathbf{X}_i$ includes covariates such as radial distance from center of storm r_i, latitude of location i, longitude of location i, and the sine and the cosine of the inflow angle at cell i across circular isobars towards the storm center $\phi_i \in [0, 2\pi)$. The spatial terms μ_{ui}, μ_{vi}, μ_{1i}, and μ_{2i} have independent CAR priors. For hyperpriors we use independent $\text{N}(0, 100)$ priors for the mean parameters $\boldsymbol{\beta}_1$ and $\boldsymbol{\beta}_2$, InvGamma(0.05,0.0005) priors for the variances σ_1^2, σ_2^2, σ_u^2, and σ_v^2, and Uniform(0,1) prior for CAR association parameters. We will also assume that $g(\theta_i) = 0$.

5 Results

We compare the performance of our CCAR model to the standard U/V model through 5-fold cross validation. With the category of the storm dependent on the magnitude of the fastest wind vector, our focus is in the calculation of the wind speed and direction. For each of these models the posterior mean of $\hat{\omega}$ is calculated and then summarized using the mean square error (MSE). The posterior mean of direction, $\hat{\theta}$, is calculated using the methods described in Section 2, $\hat{\theta} = \arctan \frac{S}{C}$ where S and C are the sums of the $\sin \theta_i$ and $\sin \theta_i$ across all iterations respectively. $\hat{\theta}$ is compared using two different metrics. First using the mean absolute cosine error (MACE), where closer to 1 indicates a better model. The other metric we use is the mean cosine difference error (MCDE), in this case closer to 0 is better.

$$\text{MACE} = \frac{1}{n} \sum_{i=1}^{n} |\cos(\theta_i) - \cos(\hat{\theta}_i)| \tag{9}$$

Model	MSE(ω)	MACE	MCDE
U/V	29.89 (3.20)	0.078 (0.008)	0.983 (0.003)
CCAR	0.96 (0.11)	0.092 (0.008)	0.930 (0.009)

Table 1: Comparison of the U/V model and CCAR model for wind speed and direction.

$$\text{MCDE} = \frac{1}{n}\sum_{i=1}^{n}\cos(\theta_i - \hat{\theta}_i) \tag{10}$$

Table 1 gives the calculated MSE, MACE, and MCDE values for the U/V model and the CCAR model along with their standard errors. We see that there is pronounced significant improvement, of almost 30 times, in the modeling of the wind speed. When we compare the direction, we see that the two models are comparable.

We also plotted the posterior means of the u_i and v_i for each model. Figure 4a when compared to Figure 5a show that the U/V model may be over smoothing. Looking at the third quadrant of each figure and comparing it to the response in Figure 2a we do not see the spike in intensity close to the center in the U/V image that does appear in the CCAR image.

6 Discussion and Remarks

In this paper, we present an innovative multivariate Bayesian spatial model for vector fields, that we apply to hurricane forcing winds. We introduce for the first time in the literature a spatial version of circular distributions, a circular conditional autoregressive (CCAR) model for the vector direction. We implemented our framework for vector fields to better characterize hurricane surface wind fields. A case study of Hurricane Floyd of 1999 compared our CCAR model to prior methods that decompose wind speed and direction into its N-S and W-E cardinal components.

We analyze, in our case study, only responses from a single source, blended satellite data. A second source of data, buoy measurements, can also be included to be combined with the satellite data. In our CAR model, we utilized the standard proximity neighborhood

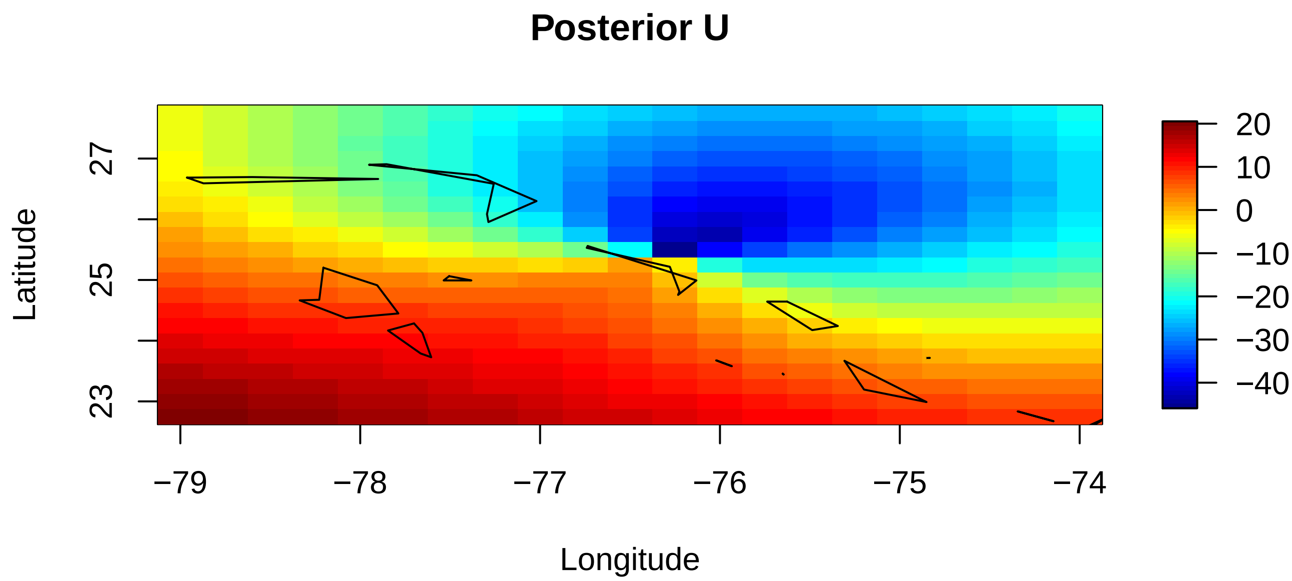

Figure 4: Posterior means of U and V for the U/V model.

structure. As future work we are planning to introduce a neighborhood structure that can be more representative of the true neighbors within hurricane wind fields. Our case study analyzed only one time point of Hurricane Floyd's track towards the US East Coast. Our model could be altered to account for time series data while still accounting for the spatial

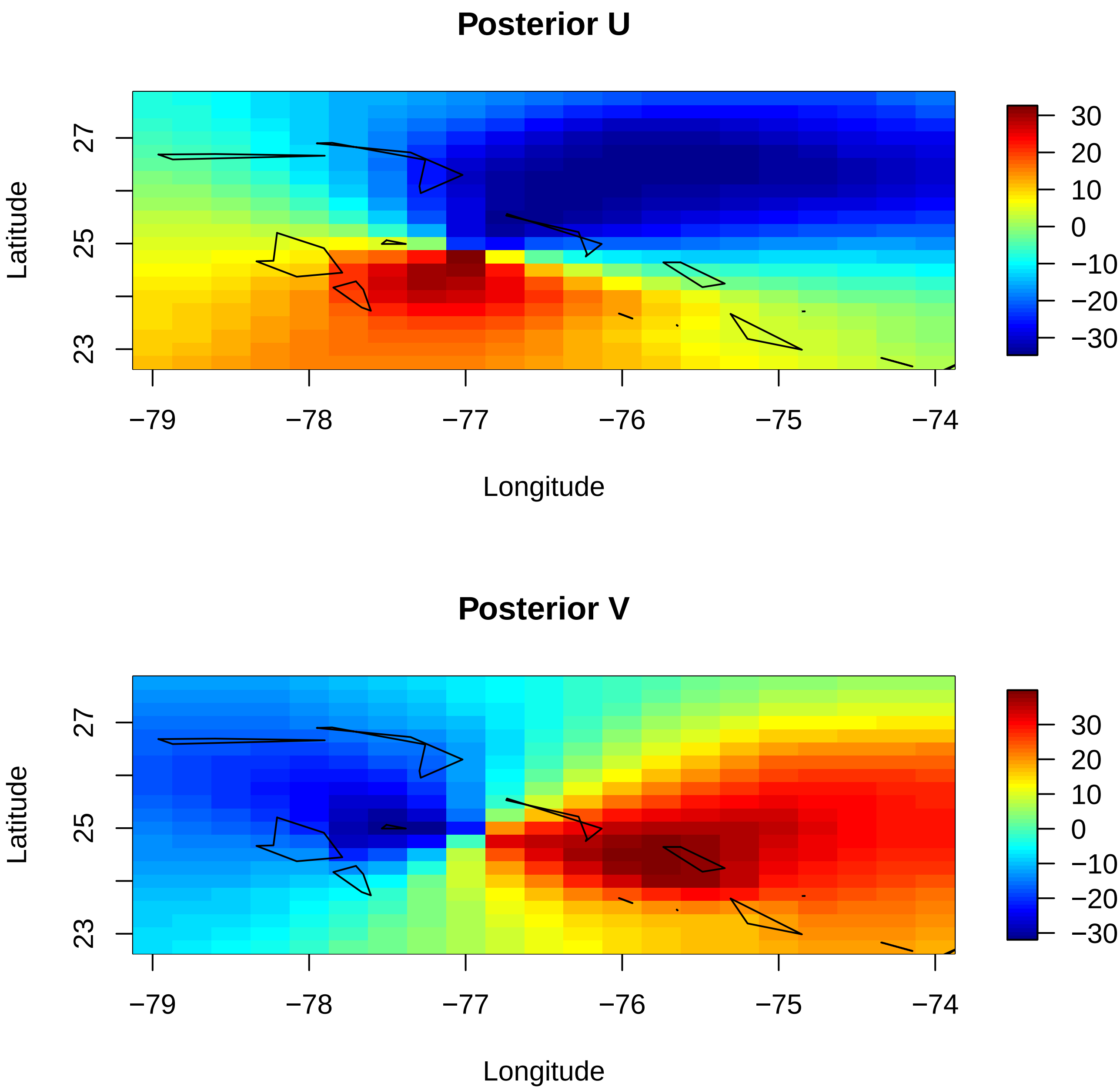

Figure 5: Posterior means of U and V for the CCAR model

structure across our region of the Atlantic Basin.

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Appendix - The WN density in WinBUGS

In this section we give the `WinBUGS` code used to specify the WN density for the vector angles. The remaining code for the vector length model, and all hyperpriors are omitted since they are straight-forward to code in `WinBUGS`. The WN likelihood is given by

```
for(i in 1:n){
  y[i]~dnorm(meany[i],tau2)
  one[i]<-1
  one[i]~dbern(denom[i])
  meany[i]<-mu2[i] + 2*pi*K[i]
  K[i]<-trunc(-z[i]/(2*pi))+1
  z[i]~dnorm(mu2[i],tau2)
  U[i]<-phi(sqrt(tau)*(2*pi-mu2[i]))
  L[i]<-phi(sqrt(tau)*(0-mu2[i]))
```

```
  denom[i]<-c/(U[i]-L[i])
}
```

where $c = exp(-200)$ is a small constant used in the "ones trick" to ensure that `denom[i]`$\in (0, 1)$. The product of the normal density for `y[i]` and the Bernoulli density for `one[i]` gives the truncated normal density in (4). The combination of models for `K[i]` and `z[i]` executes the auxiliary model in (6).