The following exercises were taken from Dr. D. Dickey WEB site for ST512

http://www.stat.ncsu.edu/people/dickey/courses/st512/

1) I ran a regression model with interaction obtaining

\[ Y = 100 + 2C - 3T + 0.2C*T \]  \( \rightarrow \) \( \beta = 2 \)

where C is the amount of some chemical, T is time in days, and Y is yield. My error mean square was 12 and my error sum of squares was 240.

\[ MSE = S S E / d f_e \]
\[ 12 = 2.40 / d f_e \]
\[ d f_e = 20 \]

a) How many data points did I have? \( n = d f_e + 1 \)
\[ n = 20 + 1 = 24 \]

b) For what amounts of chemical C, if any, will the predicted Y increase over time? \( \beta = 0.2C; C > 2/0.2 = 15 \)

C > 15

b) If I remove the interaction term, the error sum of squares will become 340. Find, if possible, the t test for the interaction coefficient.

\[ t = -2 \]

d) How many degrees of freedom does this interaction t test have?

2) I ran a regression of yield Y on PH (soil pH), M (soil moisture), and N (available soil nitrogen) getting these sequential (type I) and partial (type II) sums of squares from the commands PROC REG: MODEL Y = PH M N;

<table>
<thead>
<tr>
<th>SOURCE</th>
<th>TYPE I (sequential)</th>
<th>TYPE II (partial)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PH</td>
<td>500</td>
<td>140</td>
</tr>
<tr>
<td>M</td>
<td>80</td>
<td>60</td>
</tr>
<tr>
<td>N</td>
<td>20</td>
<td>20</td>
</tr>
</tbody>
</table>

Find the following if possible (if any is not possible, put "NP")

a) The (corrected) model sum of squares 600 for the regression described above.

b) The error sum of squares NP from the regression described above.

c) The (corrected) model sum of squares 580 in the regression of Y on PH and M

d) The (corrected) model sum of squares in the regression of Y on PH and N

3) A multiple regression of Y on X1 and X2 includes an intercept. The resulting equation from the regression is
The error mean square is 25 and the (corrected) regression sum of squares is 382.5. Answer the following if possible from the given information

a) (*) Find the missing entry (C). \( C = \frac{\text{12}}{\text{2}} \).

b) Give the number of observations on Y. \( n = 30 \).

c) Give the error degrees of freedom \( 27 \).

d) Give sum(Yi-Y) \( = 1057.5 \).

e) Give \( X'Y = \begin{bmatrix} 285 \\ 40 \\ -20 \end{bmatrix} \).

f) Estimate the mean Y for \( X_1 = 3 \) and \( X_2 = 2 \). \( \hat{Y} = \frac{7}{2} \).

g) We obtained \( b_2 = -6 \). Estimate the variance of \( b_2 \). \( 2.5 \).

h) We also obtained \( b_1 = 3 \) and so \( b_1 - b_2 = 9 \). Estimate the variance of \( b_1 - b_2 \). \( 110 \).

i) Give the rank of \( X \). \( 4 \).

i) Give \( Y = 9.5 \).

4) I fit a regression model to predict root weight \( R \) as a function of stem height \( S \) and leaf area \( L \). Here is my regression equation with standard errors in parentheses below the coefficients:
\[ R = 10 + 3S + 2L \]

(3) (1.2) (1.2)

I had 20 data points and my error mean square was 2. The corrected total sum of squares is 65. All questions refer to this regression.

a) Draw a picture (graph) to represent \( R = 10 + 3S + 2L \).

b) In your picture, what corresponds to the 10 in the equation above?

\[ y = \alpha + \beta_1 x_1 + \beta_2 x_2 \]

\[ \mu_{y|x_1, x_2} = \frac{\beta_0}{\sigma^2} \]

\[ \omega_{x_1, x_2} \]

\[ x_1 \geq 0, \ x_2 \leq 0 \]

c) Explain what the coefficient 3 in the above equation tells you about your graph.

d) Another worker is surprised by the coefficient 3 in this study. He tells you that the coefficient should be 4 according to theory. Give a \( t \) statistic to test his hypothesis.

\[ t = -0.8333 \]

e) How many degrees of freedom does the above \( t \) statistic have?

\[ df = 20 - 1 - 2 = 17 \]

f) In SAS we would get an overall \( F \) test for our model. Letting \( B_0 \), \( B_1 \) and \( B_2 \) be the theoretical coefficients being estimated in our model, write down the hypothesis being tested.

\[ H_0 : \beta_1 = \beta_2 = 0 \]

and the calculated \( F \) statistic \[ \frac{MSSR}{MSE} = 7.55 \]

g) Suppose the \( F \) test from problem 6 has \( p \) value .035. I know from this that I should reject my null hypothesis. Explain what this \( p \) value represents (using a drawing of the \( F \) distribution if you like).
h) To see if I can omit S from the model, I can use either a t or F test.
   Find the calculated F test statistic.
   \[ F = \frac{t^2}{s^2} = \frac{5^2}{25} = 25 \]

i) Compute, if possible, the increase in error sum of squares that would
   result if S were omitted from the model.
   \[ \text{error sum of squares increases in } 50 = \text{MSE} \times F = 2 \times 25 \]

5) A regression of Y on a column of 1's, X1, and X2 is run to estimate the
   parameters of the model
   \[ Y = B_0 + B_1 X_1 + B_2 X_2 + e \]

   The estimated regression equation based on n=20 observations is
   \[ \hat{Y} = 30 + 12 X_1 - 5 X_2 \]

   that is, b0=30, b1=12, and b2=-5. Suppose we also know the \( \text{inv}(X'X) \) matrix
   is

   \[
   \begin{bmatrix}
   0.13333 & -0.13333 & 0.10000 \\
   -0.13333 & 0.23333 & -0.20000 \\
   0.10000 & -0.20000 & 0.20000
   \end{bmatrix}
   = \begin{bmatrix}
   4 & -4 & 3 \\
   -4 & 7 & -6 \\
   3 & -6 & 6
   \end{bmatrix}
   \]

   and the error sum of squares is 90.

   a) Estimate sigma squared, the variance of e:
   \[ \hat{\sigma}^2 = \text{MSE} = \frac{90}{17} \approx 5.29 \]

   b) Estimate the covariance between b1 and b2:
   \[ \text{COV}(b_1, b_2) = \text{MSE} \left( \begin{bmatrix}
   -6 \\
   30
   \end{bmatrix} \right) = \frac{-18}{17} \]

   c) I am thinking about what would happen if I ran this experiment over
      again. If my new sample has b1 > B1, should I expect b2 > B2 or
      b2 < B2? Explain your answer.

   d) I want to compute a 95% prediction interval for an individual Y at
      X1=3, X2=3. Fill in the two missing entries below with numbers.
6) I am interested in the relationship between \textit{YIELD} in potted plants as a function of soil Ph, temperature (\textit{TEMP}) and moisture (\textit{WATER}). I observe yield and soil conditions for several plants and run this regression:

\[
(a) \text{ PROC REG; MODEL YIELD = PH TEMP WATER / SS1;}
\]

The results are at the end of the quiz and all questions refer to this printout. As usual, I assume a model

\[
\text{Yield} = \beta_0 + \beta_1 \text{PH} + \beta_2 \text{TEMP} + \beta_3 \text{WATER} + \epsilon
\]

and I use \textbf{X} and \textbf{Y} to denote the usual regression matrices.

a) How many rows \(20\) and columns \(4\) does the \textbf{X} matrix have?

b) Give an estimate of \(\beta_1 = 25.87\) and calculate the \(t\) statistic for testing \(H_0: \beta_1 = 18\) (\(t = \frac{1.41}{1.56}\)). How many degrees of freedom does \(t\) have? \(20-1-3 = 16\)

\[
\frac{25.87 - 18}{5.51} = 1.41
\]

c) Calculate, if possible, an \(F\) for testing the null hypothesis that the coefficients of \textit{TEMP} and \textit{WATER} can both simultaneously be set to 0.

\[
F = \frac{(642.40 + 74.61)/2}{76.48} = \frac{717.01}{76.48} = 9.40
\]

d) The \(P\)-value 0.3380 for \textit{WATER} is somehow related to the statistic \(t = 0.988\). Explain this relationship, using a picture if you like.

\[
\frac{9.88}{-1} = 9.88
\]

e) Calculate (or just find) \(b'X'Y\) where \(b = (X'X)^{-1} (X'Y)\).

\[
b'X'Y = \frac{Model SS}{SS + \hat{\mu}^2}
\]

Model: \texttt{MODEL1}
Dependent Variable: YIELD

Analysis of Variance

<table>
<thead>
<tr>
<th>Source</th>
<th>DF</th>
<th>Sum of Squares</th>
<th>Mean Square</th>
<th>F Value</th>
<th>Prob&gt;F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model</td>
<td>3</td>
<td>2006.05659</td>
<td>668.68553</td>
<td>8.743</td>
<td>0.0012</td>
</tr>
<tr>
<td>Error</td>
<td>16</td>
<td>1223.74341</td>
<td>76.48396</td>
<td></td>
<td></td>
</tr>
<tr>
<td>C Total</td>
<td>19</td>
<td>3229.80000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Root MSE</td>
<td></td>
<td>8.74551</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>R-square</td>
<td></td>
<td>0.6211</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dep Mean</td>
<td></td>
<td>114.10000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Adj R-sq</td>
<td></td>
<td>0.5501</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C.V.</td>
<td></td>
<td>7.66478</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Parameter Estimates

| Variable | DF  | Parameter | Standard Error | T for H0: Parameter=0 | Prob > |T| |
|----------|-----|-----------|----------------|------------------------|--------|---|
| INTERCEP | 1   | -18.656243| 26.26313587    | -0.710                 | 0.4942 |
| PH       | 1   | 25.866095 | 5.56684983     | 4.666                 | 0.0003 |
| TEMP     | 1   | -0.549719 | 0.19337876     | -2.843                | 0.0118 |
| WATER    | 1   | 0.491309  | 0.49743915     | 0.988                 | 0.3380 |

Variable DF Type I SS

| INTERCEP | 1   | 260376    |
| PH       | 1   | 1289.049922|
| TEMP     | 1   | 642.396128 |
| WATER    | 1   | 74.610536  |

7) I regressed Y on X1 and X2 getting these Type I and Type II sums of squares from PROC REG. What would have happened if I had run the regression in the opposite order (PROC REG; MODEL Y = X2 X1; )

Variable DF Type I SS Type II SS

| INTERCEP | 1   | 260376 |
| PH       | 1   | 1289.049922 |
| TEMP     | 1   | 642.396128 |
| WATER    | 1   | 74.610536 |

ST512 SUMMER 2011
8) Here is the result of running PROC REG on some data using the /I SS1 SS2 options to deliver sums of squares and the inverse of X'X (I have deleted the extraneous data that usually borders this inverse). Note that the inverse is in two parts.

a) Fill in the missing numbers below:

Model: MODE1

X'X Inverse

<table>
<thead>
<tr>
<th>Variable</th>
<th>DF</th>
<th>Type I SS</th>
<th>Type II SS</th>
</tr>
</thead>
<tbody>
<tr>
<td>INTERCEPT</td>
<td>1</td>
<td>90000</td>
<td>18000</td>
</tr>
<tr>
<td>X2</td>
<td>1</td>
<td>(600)</td>
<td>(700)</td>
</tr>
<tr>
<td>X1</td>
<td>1</td>
<td>(400)</td>
<td>(900)</td>
</tr>
</tbody>
</table>

X'X Inverse

<table>
<thead>
<tr>
<th>INTERCEPT</th>
<th>X1</th>
<th>X2</th>
</tr>
</thead>
<tbody>
<tr>
<td>INTERCEPT</td>
<td>2.3701839387</td>
<td>-0.126883817</td>
</tr>
<tr>
<td>X1</td>
<td>-0.126883817</td>
<td>0.0120466582</td>
</tr>
<tr>
<td>X2</td>
<td>-0.023663287</td>
<td>0.000731109</td>
</tr>
<tr>
<td>X3</td>
<td>-0.083663841</td>
<td>0.0007429646</td>
</tr>
<tr>
<td>X4</td>
<td>-0.004940533</td>
<td>-0.000075675</td>
</tr>
</tbody>
</table>

Answer:

Model SS

1500

<table>
<thead>
<tr>
<th>Variable</th>
<th>DF</th>
<th>Type I SS</th>
<th>Type II SS</th>
</tr>
</thead>
<tbody>
<tr>
<td>INTERCEPT</td>
<td>1</td>
<td>90000</td>
<td>18000</td>
</tr>
<tr>
<td>X2</td>
<td>1</td>
<td>(600)</td>
<td>(700)</td>
</tr>
<tr>
<td>X1</td>
<td>1</td>
<td>(400)</td>
<td>(900)</td>
</tr>
</tbody>
</table>

Model SS

1500
\[
\begin{align*}
\text{X3} & \quad \text{X4} \\
\text{INTERCEP} & \quad -0.083663841 \quad -0.004940533 \\
\text{X1} & \quad 0.0007429646 \quad -0.000075675 \\
\text{X2} & \quad 0.0005471731 \quad -0.000942185 \\
\text{X3} & \quad 0.0071878077 \quad 0.0001485814 \\
\text{X4} & \quad 0.0001485814 \quad 0.0006775134 \\
\end{align*}
\]

Dependent Variable: Y

Analysis of Variance

\[
\begin{align*}
\text{Source} & \quad \text{DF} & \quad \text{Sum of Squares} & \quad \text{Mean Square} & \quad F \text{ Value} & \quad \text{Prob} > F \\
\text{Model} & \quad 4 & \quad 34603.17258 & \quad 8650.79315 & \quad 10.47076 & \quad (826.25) & \quad 0.0001 \\
\text{Error} & \quad (23) & \quad \frac{240.93}{(24)} & \quad 10.47076 & \quad 0.0001 \\
\text{C Total} & \quad (27) & \quad 34844.00000 & \quad \quad & \quad \quad & \quad \quad \\
\end{align*}
\]

Root MSE \quad 3.23585 \\
Dep Mean \quad 186.00000

R-square \quad 0.9931

Parameter Estimates

\[
\begin{align*}
\text{Variable} & \quad \text{DF} & \quad \text{Parameter Estimate} & \quad \text{Standard Error} & \quad T \text{ for H}_0: \text{Parameter}=0 & \quad \text{Prob} > |T| \\
\text{INTERCEP} & \quad 1 & \quad 109.396250 & \quad 4.98172872 & \quad 21.959 & \quad 0.0001 \\
\text{X1} & \quad 1 & \quad 6.434904 & \quad \frac{0.3551}{(18.121)} & \quad 0.0001 \\
\text{X2} & \quad 1 & \quad 2.684596 & \quad 0.17803728 & \quad 15.079 & \quad 0.0001 \\
\text{X3} & \quad 1 & \quad -5.410582 & \quad 0.27433882 & \quad -19.722 & \quad 0.0001 \\
\text{X4} & \quad 1 & \quad 1.937675 & \quad 0.08422635 & \quad 23.006 & \quad 0.0001 \\
\end{align*}
\]

(continued on next page)

\[
\text{Se} (\hat{\beta})=\frac{\text{MSE} \cdot (X'X)^{-1}}{2n} \quad \text{SUMMER 2011}
\]
<table>
<thead>
<tr>
<th>Variable</th>
<th>DF</th>
<th>Type I SS</th>
<th>Type II SS</th>
</tr>
</thead>
<tbody>
<tr>
<td>INTERCEPT</td>
<td>1</td>
<td>958688</td>
<td>5049.202823</td>
</tr>
<tr>
<td>X1</td>
<td>1</td>
<td>2362.216057</td>
<td>3439.005860</td>
</tr>
<tr>
<td>X2</td>
<td>1</td>
<td>21940</td>
<td>2380.802</td>
</tr>
<tr>
<td>X3</td>
<td>1</td>
<td>4784.29</td>
<td>4072.785802</td>
</tr>
<tr>
<td>X4</td>
<td>1</td>
<td>5541.71</td>
<td>5541.709506</td>
</tr>
</tbody>
</table>

\[ 34603.17258 \approx (2362.216 + 21940 + 5541.71) \]

\[ \text{MSE} = \frac{23.006^2 \times 10.47076}{4} \]

\[ \text{SS for } X_4 = 5541.922 \]

(a) Compute the rank of \( X'X \). Rank = \( \frac{4}{4} \)

(Note: It was an increase.)
4(a)

\[ y = 10 \text{ when } x_1 = 0, x_2 = D \]
\[ Y = \beta_0 + \beta_1 C + \beta_2 T + \beta_3 C \times T \]

\[ Y = 100 + 2C - 3T + 0.2C \times T \]

1. \( p = 3 \)

\[ \text{MSE} = 12 \quad \text{MSE} = \frac{\text{SSE}}{dfe} \]

\[ \text{SSE} = 240 \]

\[ 12 = 240/dfe \quad \Rightarrow \quad dfe = 240/12 = 20 \]

\[ dfe = n - 1 - p = 20 \quad \Rightarrow \quad n = 20 + 1 + 3 = 24 \]

\[ n = 24 \]

b) \[ y = 100 + 2C + (-3 + 0.2C)T \]

Want slope for \( T \) positive

\[ -3 + 0.2C > 0 \]

\[ \frac{C}{0.2} > 3 \]

\[ C > 15 \]

c) Model without \( C \times T \):

\[ Y = \beta_0 + \beta_1 C + \beta_2 T \]

Error
\[ \text{SS} = \text{SSE} = 348 \]

\[ \text{SS}[E] = 240 \quad \text{SS}_E = 348 \]

\[ R_{[\beta_3 / \beta_2, \beta_1, \beta_0]} = 348 - 240 = 108 \]

\[ H_0: \beta_3 = 0 \quad R(H_0) = 108 \quad \quad df = 1 \]

\[ H_1: \beta_3 \neq 0 \]

\[ F = \frac{348 - 240}{12} \]

\[ F = \frac{108}{12} \]

\[ \text{F has numerator df} = 1 \]

\[ \text{denominator df} = 20 \]

\[ F = \frac{108}{12} = 9 \]

\[ t = \sqrt{F} = \sqrt{9} = 3 \]

\[ t = 3 \]

\[ df = 20 - dfe \]
2) Model \[ Y = p + M + N \]

Source (Sec) (Partial)
\[ \begin{array}{ll}
\text{pH} & 500 \\
M & 80 \\
N & 20 \\
\end{array} \]

\[ \begin{array}{ll}
\text{SS} & 140 \\
\text{MS} & 60 \\
\text{SS} & 20 \\
\end{array} \]

a) Model SS = SS[R] = 500 + 80 + 20
SS[R] = 600

b) Error SS = SS[E] NOT Possible

c) Model SS = SS[R] = 500 + 80 = 580

d) Model SS = SS[R] = 500 + 80 = 580

3) Model \[ Y = x_1, x_2 \]

\[ \begin{align*}
y &= \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \epsilon \\
y &= 10 + 3x_1 - 6x_2
\end{align*} \]

\[ \text{MSE} = 2.5 \quad \text{SS}[R] = 38.25 \]

a) \[ \begin{align*}
(5)(-1/15) + (10/3)(C) + (10/3)(-1/10) = 1
\end{align*} \]
Sowe for C
\[ C = \left[ \frac{1 - (5)(-1/15) + (10/3)(-1/10)}{(10/3)} \right] = 0.5 \]

b) \[ (5)(-1/15) + (10/3)(1/2) + (10/3)(-1/10) = 1 \]

b) \[ dfe = n - 1 - 2 = 30 - 1 - 2 = 27 \]
Q3  d) \[ \sum (y_i - \bar{y})^2 = SS[Total] = SS[E] + SS[R] \]

\[ SSE = MSE \times df_e = 25 \times 27 = 675 \]

\[ SS[R] = 382.5 \]

\[ SS[Total] = 675 + 382.5 = 1057.5 \]

e) \[ X'Y = \begin{bmatrix} \sum y_i \\ \sum y_i x_i \\ \sum y_i x_i \end{bmatrix} \]

\[ \hat{\beta} = (X'X)^{-1} (X'Y) = \begin{bmatrix} 10 \\ 3 \\ -6 \end{bmatrix} \]

\[ (X'X) \hat{\beta} = \frac{(X'X)(X'X)^{-1}(X'Y)}{I} = X'Y \]

\[ (X'X) \beta = \begin{bmatrix} 30 & 5 & 5 \\ 5 & 10/3 & 10/3 \\ 5 & 10/3 & 40/3 \end{bmatrix} \begin{bmatrix} 10 \\ 3 \\ -6 \end{bmatrix} = \begin{bmatrix} 285 \\ 40 \\ -20 \end{bmatrix} \]
9) \[ \hat{\mu}_{\text{Y|x_1}} = 3, x_2 = 7 = 10 + 3(3) - 6(2) = 7 \]

9) \[ b_2 = -6 \quad \text{var}(b_2) = \text{MSE} \left( x'x \right)^{-1} \]
\[ = 25 \times (0.1) \quad = 2.5 \]

h) \[ b_1 = 3 \quad b_1 - b_2 = 9 \]
\[ \text{var}(b_1 - b_2) = \]
\[ \begin{bmatrix} 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} -1/10 & -1/15 & 0 \\ -1/15 & 1/2 & -1/10 \\ 0 & 1/10 & 1/10 \end{bmatrix} \begin{bmatrix} 0 \\ -1 \\ -1 \end{bmatrix} = \text{Var}(\hat{\beta}) \quad \text{MSE}(x'x)^{-1} \]
\[ \begin{bmatrix} -1/15 & 4/10 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} = 4/10 \]

i) \[ \text{rank}(x) = \text{rank}(x'x) = 4 \]

j) \[ \bar{y} = \frac{285}{30} = 9.5 \]
(4) Model: \( R = S \ L \)
\[ \hat{R} = 10 + 3.5 + 2 \cdot L \]
\[ (3) \quad (1.2) \quad (1.2) \]

\( n = 20 \quad MSE = 2 \quad SS_{\text{Total}} = 65 \)

a)

b) \[ \hat{y} = \hat{\beta}_1 x_1 + 0, x_2 = 0 = 10 \]

c) For a given value of \( L \), slope of line along \( S \) axis is 3.
Keeping \( L \) constant, \( R \) increases in 3 units for one-unit increase in \( S \).

d) \[ H_0: \beta_1 = 4 \]
\[ H_1: \beta_1 \neq 4 \]
\[ t = \frac{\hat{\beta}_1 - 4}{SE(\hat{\beta}_1)} = \frac{3 - 4}{1.2} = -0.8333 \]
\( df = \text{error df} = 20 - 1 - 2 = 17 \)
\[ t(17, 0.05/2) = 2.11 \]
\[ | -0.8333 | < 2.11 \quad \text{do not Reject } H_0. \]
\[ SS[\text{Total}] = 65 \]
\[ MSR = \frac{SS[E]}{dfe} = \frac{SS[E]}{17} \]
\[ SS[E] = 2 \times 17 = 34 \]
\[ SS[\text{Total}] = SS[R] + SS[E] \]
\[ 65 = SS[R] + 34 \]
\[ SS[R] = 65 - 34 = 31 \]
\[ MSR[R] = \frac{31}{2} = 15.1 \]
\[ F = \frac{15.1}{2} = 7.55 \]
\[ F\text{-test p-value} = 0.035 \]

h) \[ H_0 : \beta_2 = 0 \]
\[ H_1 : \beta_2 \neq 0 \]
\[ t = \frac{-6}{1.2} = 5 \]
\[ t\text{-test} 17\text{df} \]
\[ F = t^2 = 5^2 = 25 \]
\[ F\text{-test with}(1, 17)\text{df} \]
(4) i) Model \( R = L S S \)

\[ SS[E] = 34. \]

\[ H_0: \beta_2 = 0 \quad F = 25 = \frac{\text{Type III } SS}{\text{for } S / L} \]

\[ \frac{R[\beta_2 | \beta_1]}{MSE} = 25 \times 2 = 50 \]

(5) \( Y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + e \)

\[ \hat{Y} = 30 + 12 x_1 - 5 x_2 \]

\[ \hat{\beta}_0 = b_0 = 30 \]

\[ \hat{\beta}_1 = b_1 = 12 \]

\[ \hat{\beta}_2 = b_2 = -5 \]

\[ n = 20 \]

\[ SS[E] = 90 \]

\[ df_e = 20 - 1 - 2 = 17 \]

\[ MSE = \frac{90}{17} = 5.29 \]

\[ \text{cov} (\hat{b}_1, \hat{b}_2) = MSE \left( \frac{6}{30} \right) = \frac{90}{17} \left( \frac{-6}{30} \right) = -\frac{18}{17} \]
d) 95% prediction interval for individual \( y \)

\[
\hat{y} = 30 + 12(3) - 5(3) = 30 + 36 - 15 = 51
\]

\[
\text{var}(\hat{y}) = \text{MSE} + \begin{bmatrix}
1 & 3 & -5 \\
4 & -4 & 3 \\
-4 & 7 & -6
\end{bmatrix} \begin{bmatrix} 1 \\ \frac{1}{30} \\ 3 \end{bmatrix} = 11.43
\]

\[
\begin{bmatrix} \frac{1}{30} & 23 & 47 & -45 \\
3 & -5 & (\frac{1}{30}) & 343 \end{bmatrix}
\]

6) b) \( \beta_1 = 18 \)

\[
t = \frac{\hat{\beta}_1 - \beta_0}{\text{se}(\hat{\beta}_1)}
\]

\[
t = \frac{25.87 - 18}{5.57} = 1.41
\]