Exam date and time: **THURS., 12/14, 8:15-11:00, HA 100**

Material covered on exam: Chapters 3 and 4 from text

- ✓ Chapter 3 §3.1 - §3.4
- ✓ Chapter 4: §4.1- 4.5
- ✓Factorials, permutations, combinations
- ✓Webassign homework 9 - 12; lecture worksheets 17-24

Needed for exam:

- ✓$8\frac{1}{2}" \times 11"$ sheet; calculator (I will supply tables)

Sample Questions

Questions 1, 2, and 3 refer to the following:

A marketing research firm wishes to determine if the adult men in Laramie, Wyoming would be interested in a new upscale men's clothing store. From a list of all residential addresses in Laramie, the firm selects a simple random sample of 100 and mails a brief questionnaire to each.

1. In the above survey, what is the sample?
   a. all adult men in Laramie, Wyoming
   b. all residential addresses in Laramie, Wyoming
   c. the members of the marketing firm that actually conducted the survey
   d. the adult men at the 100 addresses the survey was mailed to

2. In the above survey, what is the population of interest?
   a. all adult men in Laramie, Wyoming
   b. all residential addresses in Laramie, Wyoming
   c. the members of the marketing firm that actually conducted the survey
   d. the 100 addresses the survey was mailed to

3. In the above survey, what is the chance that all 100 homes in a particular neighborhood in Laramie end up being the sample of residential addresses selected?
   a. the same as for any other set of 100 residential addresses
   b. exactly 0 since random samples will spread out the addresses selected
   c. reasonably large due to the "cluster" effect
   d. 100 divided by the size of the population of Laramie

4. Which of the following is <u>not</u> a potential source of bias in a survey?
   a. undercoverage
   b. nonresponse
   c. the use of convenience sampling
   d. the use of simple random sampling

5. What is the difference between an <u>observational study</u> and an <u>experiment</u>?
   a. In an observational study one observes individuals and measures variables of interest but does not attempt to influence the responses. In an experiment, one deliberately imposes some treatment on individuals inorder to observe their responses.
   b. In an observational study, great emphasis is placed on how observations are to be taken. In an experiment, great emphasis is on what is to be observed.

c. In an observational study, all data are obtained by careful observation. In an experiment, all data are obtained by careful control.
d. There is no difference. These are two names for the same thing.

6. Control groups are used in an experiment in order to what?
   a. Control the subjects of a study so as to insure that all participate equally
   b. Control the effects of other variables, such as the placebo effect, that are not being measured or observed directly but that still affect the response
   c. Guarantee that someone other than the investigators, who have a vested interest in the outcome, control how the experiment is conducted.
   d. Achieve a proper and uniform level of randomization

7. What is the probability of getting a license plate that has a repeated letter or digit if you live in a state where the license plate scheme is four letters followed by two numerals?

8. The length of human prenancies from conception to birth varies according to a normal distribution that is approximately normal with mean 266 days and standard deviation 16 days.
   a. What is the probability that a pregnancy lasts less than 240 days?
   b. What is the probability that a pregnancy lasts between 240 and 270 days?
   c. How long do the longest 20% of pregnancies last?

9. The quartiles of any probability density curve are the points with area 0.25 and 0.75 to their left under the curve.
   a. What are the quartiles of the standard normal distribution?
   b. How many standard deviations away from the mean do the quartiles lie in any normal distribution?
   c. What are the quartiles for the lengths of human pregnancies?

10. The deciles of any distribution are the points that mark off the lowest 10% and the highest 10%. The deciles for any probability density curve are therefore the points with area 0.1 and 0.9 to their left under the curve.
    a. What are the deciles of the standard normal distribution?
    b. The heights of young women are approximately normal with mean 64.5 inches and standard deviation 2.5 inches. What are the deciles of this distribution?

11. Four radar systems are arranged so that they work independently of each other. Each system has a 0.9 chance of detecting an approaching airborne object. Find the probability that at least one radar system will fail to detect an approaching object.

12. A manufacturer of hand soap has introduced a new product. An extensive survey indicates that 40% of the people have seen advertising for the new product. It also showed that 20% of the people in the survey had tried the new product. In addition, 15% of those in the survey had seen it advertised and had tried the product. What is the probability that a randomly chosen person would have seen the advertising for the new product or have tried the product or both?

13. How many four-digit serial numbers can be formed if no digit is to be repeated within any number? (The first digit may be a zero).

3. 14. Let X denote a normal random variable with a mean of 100 and a standard deviation of 40. What is the probability that X falls between 144 and 170?

15. Suppose the amount of tar in cigarettes is normally distributed with a mean of 3.5 mg and a standard deviation of 0.5 mg.
    a. What proportion of cigarettes have a tar content that exceeds 4.25 mg?

b. In order to advertise as a low tar brand, a manufacturer must prove that their tar content is below the 25th percentile of the tar content distribution. Find the 25th percentile of the distribution of tar amounts.

16. Determine which of the following functions is in fact a probability distribution function.

a. $p(x) = \frac{1}{4},\ x = 3, 4, 5, 6.$

b. $p(x) = \frac{x^2}{25},\ x = 0, 1, 2, 3, 4.$

c. $p(x) = \frac{5-x^2}{6},\ x = 0, 1, 2, 3.$

17. In a population of students the number of calculators owned is a random variable x with $p(0) = .2$, $p(1) = .6$, $p(2) = .2$. Find the expected value of this probability distribution.

18. An oil firm is to drill three wells, with each well having probability 0.2 of successfully producing oil. It costs the firm \$20,000 to drill each well. A successful well will bring in oil worth \$750,000. Let the random variable X_i be the oil firm's gain from well i, $i = 1, 2, 3$. The wells are in different geographic areas and so the drilling outcomes are independent.
    a. Find the firm's expected gain G from the three wells.
    b. Find the standard deviation of the firm's gain.

19. Let the random variable X denote the displacement in cubic inches (in^3) of the engine in a particular model of automobile. The size of the engine (that is, the cubic inch displacement) varies depending on the options chosen by the buyer of the automobile. It is known that $E(X) = 177$ in^3 and $\sigma_X = 22$ in^3. If X^*denotes the engine diplacement in cubic centimeters (cm^3), determine $E(X^*)$ and σ_{X^*}. Note that 1 in^3 $=$ 16.4 cm^3.

20. Let X be the number of accidents per week at a hazardous intersection; X varies with mean 2.2 and standard deviation 1.4. Let X_1, X_2, and X_3 be the number of accidents in each of 3 different weeks at this intersection. The $X_i's$ are independent and identically distributed, each with the same distribution as X. What is $\sigma_{(X_1+X_2+X_3)}$, the standard deviation of the sum $X_1 + X_2 + X_3$?

SOLUTIONS

1. d **2.** a **3.** a **4.** d **5.** a **6.** b **7.** .29335 **8a.** .052 **8b.** .547 **8c** 279.47 **9a** $-.675, .675$ **9b** .675 **9c** $266 - .675(16), 266 + .675(16)$ **10a** $-1.28, 1.28$ **10b** $64.5 - 1.28(2.5) = 61.3$, $64.5 + 1.28(2.5) = 67.7$.

11. The probability that no radar system fails to detect an airborne object (i. e. all four radar systems work) is $(.9)^4$, therefore the probability that at least one fails is $1 - (.9)^4$.

12. P(seen advertising$\bigcup$tried the product) $= .4 + .2 - .15 = .45$.

13. ${}_{10}P_4 = \frac{10!}{(10-4)!} = 10(9)(8)(7) = 5040$. **14.** .0956 **15.** a. .0668 b. z = -0.675 ; x = 3.16

16. a. is the only one since in b. $\sum_{\text{all x}} p(x) > 1$; in c. $p(3) < 0$.

17. $\mu = 0(.2) + 1(.6) + 2(.2) = 1$

18. a. $E(X_i) = .2{*}730,000 + .8{*}(-20,000) = 130,000$; $G = X_1 + X_2 + X_3$; $E(G) = E(X_1 + X_2 + X_3) = E(X_1) + E(X_2) + E(X_3) = 390,000$.
b. $Var(X_i) = [(730{,}000 - 130{,}000)^2{*}.2 + (-20{,}000 - 130{,}000)^2{*}.8] = 9 \times 10^{10}$;
$Var(G) = Var(X_1 + X_2 + X_3) = Var(X_1) + Var(X_2) + Var(X_3) = 27 \times 10^{10}$;

$SD(G) = \sqrt{Var(G)} = \sqrt{27 \times 10^{10}} = 519,615.24.$

19. $E(X^*)$ = 177 in^3* 16.4 cm^3/in^3 = 2902.8 cm^3;
σ_{X^*}= 22 in^3*16.4 cm^3/in^3 = 360.8 cm^3.

20. Since the $X_i's$ are independent, $Var(X_1 + X_2 + X_3) = Var(X_1) + Var(X_2) + Var(X_3) = 5.88$; so $\sigma_{(X_1+X_2+X_3)} = \sqrt{5.88} = 2.425$.